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CORRECTION TO "MAXIMAL SOLUTIONS OF THE SCALAR CURVATURE EQUATION ON OPEN RIEMANNIAN MANIFOLDS"

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In the proofs of Lemmas 2.1 and 2.2, the author used the "fact" that any compact Riemannian manifold with boundary can be deformed conformally to zero scalar curvature. However, this is not true in general, and the lemmas are incorrect.

The assertions of Lemmas 2.1 and 2.2 hold if we assume that the given manifold (M,g) satisfies the following condition:

(P₀) For any relatively compact domain $\hat{\Omega}$ of M,

there exists a positive smooth function ψ_0 defined on $\hat{\Omega}$ satisfying $L_g \psi_0 \ge 0$. Indeed, if we define $\hat{g} := \psi_0^{q-1} g|_{\hat{\Omega}}$ with ψ_0 given by the condition (P₀), then we have $S_{\hat{g}} \ge 0$. Now, to prove Lemmas 2.1 and 2.2 with (P₀), we have only to replace the displayed inequalities in the proofs by the following ones:

(Lemma 2.1.)

$$L_{q}\eta = L_{q}(\psi_{0}\xi) = \psi_{0}{}^{q}L_{\hat{q}}\xi = \psi_{0}{}^{q}(a_{n}\lambda_{1}(-\Delta_{\hat{q}}) + S_{\hat{q}})\xi > 0 \quad \text{in} \quad \hat{\Omega},$$

(Lemma 2.2.)

$$\begin{cases} a_n \Delta_{\hat{g}} - S_{\hat{g}} + f_0 \psi_0^{1-q} \frac{u^q - u_0^q}{u - u_0} \end{cases} (\psi_0^{-1} u - \psi_0^{-1} u_0) \le 0 \quad \text{in} \quad \Omega, \\ -S_{\hat{g}} + f_0 \psi_0^{-1-q} \frac{u^q - u_0^q}{u - u_0} \le 0 \quad \text{in} \quad \Omega. \end{cases}$$

Therefore, Lemmas 2.1 and 2.2, Theorem I, Proposition 2.3, and the first assertions of Theorems II and III are valid under the assumption (P_0), and Theorem IV is valid under the assumption $\lambda_1(L_{\bar{q}}) \ge 0$.

On the other hand, Lemma 3.1, the second assertions of Theorems II and III, Theorem V and Proposition 4.2 are correct without any additional assumptions, because the stronger condition (P) (as in Lemma 3.1) is satisfied in these cases.

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