Sumi, T.
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Z/kZ-FINITENESS FOR CERTAIN S^1-SPACES

Toshio SUMI

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Introduction

Let G-$\mathcal{FDCW}$ denote the category of G-spaces having the G-homotopy type of a finitely dominated G-CW complex for a compact Lie group G. Lück [8] has introduced a functor Wa^G from G-$\mathcal{FDCW}$ into the category of abelian groups and has realized the equivariant finiteness obstriction as the element $w^G(X)$ in $Wa^G(X)$. That is, a finitely dominated G-CW complex X is G-homotopy equivalent to a finite G-CW complex if and only if $w^G(X)=0$. When G is the trivial group, there is an isomorphism from $Wa^G(X)$ to the reduced projective group $\tilde{K}_0(\boldsymbol{Z}[\pi_1(X)])$ which sends the element $w^G(X)$ to the Wall's finiteness obstruction ([14]).

Anderson [1] and Ehrlich [4] have studied a sufficient condition for $w^{\{1\}}(E)=0$ for some fibration $E\rightarrow B$ with fiber S^1. Munkholm, Pedresen [11], Lück [6, 7, 9] and others have studied the transfer map $\tilde{K}_0(\boldsymbol{Z}[\pi_1(B)])\rightarrow\tilde{K}_0(\boldsymbol{Z}[\pi_1(E)])$. The purpose of this paper is to get a sufficient condition for $w^L(X)=0$ for a S^1-space X and a finite cyclic group L.

We call G-maps f_0: $Y_0\rightarrow X$ and f_0: $Y_4\rightarrow X$ equivalent if there exists a commutative diagram

$$\begin{tikzcd}
Y_0 \arrow[r, hook] \arrow[drr, "f_0"'] & Y_1 \arrow[r, "\simeq_G"] \arrow[dr, "f_1"] & Y_2 \arrow[r, "\simeq_G"] \arrow[d, "f_2"] & Y_3 \arrow[dl, "f_3"'] & Y_4 \arrow[l] \arrow[dll, "f_4"] \\
 & & X & &
\end{tikzcd}$$

such that (Y_1, Y_0) and (Y_3, Y_4) are relatively finite G-CW complexes, and $Y_1\rightarrow Y_2$ and $Y_3\rightarrow Y_2$ are G-homotopy equivalences. The group $Wa^G(X)$ consists of equivalence classes $[f\colon Y\rightarrow X]$ of the set of G-maps $f\colon Y\rightarrow X$ with Y finitely dominated and $w^G(X)$ is the equivalence class containing the identity 1_X of X. The additive structure on $Wa^G(X)$ is given by a disjoint sum:

$$[f\colon Y\rightarrow X]+[g\colon Z\rightarrow X]=[f\amalg g\colon Y\amalg Z\rightarrow X]$$

Let K be a closed subgroup of G. For a K-space X, we define $\operatorname{ind}_K^G X$ as the orbit space $G\times_K X$ of the product space $G\times X$ with respect to the K-action $k\cdot(g, x)=(gk^{-1}, kx)$. For a K-map $f: X\to Y$, we have an induced map $1\times_K f$: $G\times_K X\to G\times_K Y$, denoted by $\operatorname{ind}_K^G f$. The induction functor ind_K^G induces a transformation Ind_K^G: $(Wa^K, w^K)\to(Wa^G, w^G)$. To consider G-maps as K-maps implies a transformation Res_K^G: $(Wa^G, w^G)\to(Wa^K, w^K)$.

Throughout this paper we denote by ν the restriction of the G-action of X to $G\times\{x_0\}$. If $G=S^1$ and X is connected, the order of the image of $H_1(\nu; \boldsymbol{Z})$ is independent of taking a point x_0 of X.

Our main results are as follows:

Theorem A. *Let X be a connected S^1-CW complex which has finitely many orbit types. Suppose that the S^1-map ν: $S^1\to X$ defined as above induces a monomorphism $H_1(\nu; \boldsymbol{Z})$ between 1-dimensional homology groups. Then there exist a proper subgroup K of S^1 and a K-CW complex Y such that $S^1\times_K Y$ is S^1-homotopy equivalent to X.*

Theorem B. *Let $G=S^1$ and let X be a connected G-space. If the above defined G-map ν: $G\to X$ induces an injective homomorphism $H_1(\nu; \boldsymbol{Z})$, then the restriction homomorphism* $\operatorname{Res}_H^G(X)$: $Wa^G(X)\to Wa^H(X)$ *is trivial for any proper subgroup H of G.*

This paper is organized as follows. Let $F\to E\xrightarrow{p} B$ be a G-fibration with fibre F ([15]). In [13], we have constructed a transfer $p^!$: $Wa^G(B)\to Wa^G(E)$. But this homomorphism does not always send $w^G(B)$ to $w^G(E)$. It is originated from that (Wa^G, w^G) is not a functiorial additive invariant for G-$\mathcal{FDCW}$. In section 1 we study a K-CW structure on $p^{-1}(1K)$ for a G-CW complex E which has a G-map $E\to G/K$. In section 2 we show that if $G\times_K X$ has the G-homotopy type of a finite G-CW complex then X has the K-homotopy type of a finite K-CW complex. In section 3, we prove Theorem A in the case where X is free. We use the fact that $\pi_1(X/K)$ has the subgroup $\pi_1(S^1/K)$ as a direct summand for some closed subgroup K of G. The last section consists of the proof of the main theorems. The proof of Theorem A is obtained from applying the free case.

1. K-CW structure on X of a G-CW complex $G\times_K X$

Let G be a compact Lie group. We study a space $p^{-1}(\{pt\})$ for a G-map p from a G-space onto an orbit space G/K of G. We note that it has a canonical K-action.

Proposition 1.1. (cf.[12]) *A G-map p: $E\to B$ is a G-fibration if and only if p^K: $E^K\to B^K$ is a fibration for any closed subgroup K of G.*

Proof. This follows essentially from Theorem 4.1 in [2].

Since $Y \to G\times_K Y \to G/K$ is a G-fibration ([15, 13]), we have that $Y^L \to (G\times_K Y)^L \to (G/K)^L$ is a fibration for any closed subgroup L of G.

We symbolize 1 as the identity element of G. For any G-map $p: X \to G/K$ it is a G-fibration with fibre $p^{-1}(1K)$. The following lemma is a key to show the main theorems. It implies that a G-map $p: X \to G/K$ with X a G-CW complex is G-homotopy equivalent to a G-fibration whose fibre is a K-CW complex.

Lemma 1.2. *Let X be a G-CW complex which has a G-map $p: X \to G/K$. A K-CW complex can be constructed from the G-CW structure of X such that the K-space $V=p^{-1}(1K)$ is homotopy equivalent to it. In particular it is a finite K-CW complex if X is a finite G-CW complex.*

Proof. Clearly we have $G\times_K V$ and X are G-homeomorphic. Then we construct a K-CW complex W and a K-homotopy equivalence $W \to V$ by induction on the dimension of cells of X. By the existence of the G-map p, we obtain that L is subconjugate to K for any isotropy subgroup L of G in X. We can regard a 0-cell $G/L\times e^0$ of X as $G\times_K K/aLa^{-1}\times e^0$ for $a\in G$ with $aLa^{-1}\leq K$. Suppose that $X=G\times_K Y\cup_\phi G/L\times e^n$ for some K-CW complex Y. Let C be a connected component of $(G/K)^L$ which contains $p^L\circ\phi(1L\times \dot{e}^n)$. Take $aK\in C$ and let $\psi: G/aLa^{-1} \to G/L$ be the canonical G-map. Then the pushout of

$$\begin{tikzcd} G/aLa^{-1}\times \dot{e}^n \arrow[r, hook] \arrow[d, "\phi\circ(\psi\times 1)"] & G/aLa^{-1}\times e^n \\ G\times_K Y & \end{tikzcd}$$

is G-homotopy equivalent to X. Then we can assume that $L\leq K$ and $p^L\circ\phi(1L\times \dot{e}^n)$ is contained in the connected component of $1K$.

$$\begin{tikzcd} & 1L\times e^n & 1L\times \dot{e}^n \arrow[l, hook'] \arrow[d, "\phi|_{1L\times \dot{e}^n}"] & \\ & Y^L \arrow[r] & (G\times_K Y)^L \arrow[r, "p^L"] & (G/K)^L \end{tikzcd}$$

Since the map $p^L\circ\phi|_{1L\times \dot{e}^n}$ is homotopic to a constant map, there is a map $\sigma: 1L\times e^n \to (G/K)^L$ such that σ coincides with $p^L\circ\phi$ over $1L\times \dot{e}^n$ and $\sigma(0)=1K$. We define a map $\tau: 1L\times e^n\times I \to (G/K)^L$ as $\tau(s,t)=\sigma((1-t)s)$. Since $(G\times_K Y)^L \to (G/K)^L$ is a fibration with fibre Y^L, there exists a homotopy $F: 1L\times \dot{e}^n\times I \to (G\times_K Y)^L$ such that $F_0=\phi|_{1L\times \dot{e}^n}$ and $F_1(1L\times \dot{e}^n)\subseteq Y^L$. This map can be canonically extended to a G-map Φ from $G/L\times \dot{e}^n\times I$ to $G\times_K Y$. Let W be a K-CW complex obtained from the following pushout.

$$\begin{array}{ccc} K/L\times \mathring{e}^n & \hookrightarrow & K/L\times e^n \\ \downarrow{\scriptstyle \Phi_1} & & \\ Y & & \end{array}$$

By the property of pushout, we get a K-map $k\colon W\to V$.

$$\begin{array}{ccccc} W & \hookrightarrow & G\times_K W & \longrightarrow & G/K \\ \downarrow{\scriptstyle k} & & \downarrow{\scriptstyle \mathrm{ind}_K^G k} & & \| \\ V & \hookrightarrow & X=G\times_K V & \xrightarrow{p} & G/K \end{array}$$

Since the G-map $\mathrm{ind}_K^G k$ is a G-homotopy equivalence, we have the K-map k is a K-homotopy equivalence.

Theorem 1.3. *Let $f\colon Y\to X$ be a K-map between K-CW complexes and $(V, G\times_K Y)$ be a G-CW pair. If there is a G-map $g\colon V\to G\times_K X$ which is an extension of the G-map $1\times_K f$, then there exists a K-map $k\colon W\to X$ unique up to K-homotopy equivalence which fullfills the following conditions.*

(1) *(W, Y) is a K-CW pair.*

(2) *There is a G-homotopy equivalence $h\colon (G\times_K W, G\times_K Y)\to (V, G\times_K Y)$ such that $\mathrm{ind}_K^G k$ and $g\circ h$ are G-homotopic.*

(3) *The number of the relative cells of (W, Y) equals that of $(V, G\times_K Y)$.*

2. Induction homomorphism

Let $D^G(X)$ be the set of equivalence classes of the set of G-maps $f\colon Y\to X$ where Y has the G-homotopy type of a G-CW complex. Here the equivalence relation is defined as in introduction. For a G-map $f\colon Y\to X$, we denote by $[f\colon Y\to X]$ its represented element of $D^G(X)$. The additive structure on $D^G(X)$ is given as the one of $Wa^G(X)$. A G-map from a finite G-CW complex to X represents the zero element of $D^G(X)$. Then $D^G(X)$ is a semigroup and we obtain a map $Wa^G(X)\to D^G(X)$ which preserves the abelian structures.

Lemma 2.1. *The element of $D^G(X)$ represented by the identity map of X is invertible if and only if X is finitely dominated.*

Proof. The "if" part is trivial and then we show the "only if" part. There is the commutative diagram

$$\begin{tikzcd} X\amalg Y_1 \arrow[r, "i"] \arrow[dr] & Y_2 \arrow[d] & Y_3 \arrow[l, "h"'] \arrow[dl, "r"] \\ & X & \end{tikzcd}$$

such that $(Y_2, X \amalg Y_1)$ is a relatively finite G-CW complex, Y_3 is a finite G-CW complex, and h is a G-homotopy equivalence. Let h^{-1}: $Y_2 \to Y_3$ be the G-homotopy inverse of h. Then r is a domination with section $h^{-1} \circ i$.

Proposition 2.2. *Let K be a closed subgroup of G. A K-space X is a finitely dominated K-space if and only if $G \times_K X$ is a finitely dominated G-space.*

Proof. Suppose $G \times_K X$ is dominated by a finite G-CW complex Y_3. There is a commutative diagram such that f_3 is a domination with section $h^{-1}ij$.

$$\begin{tikzcd}
G \times_K X \arrow[r, hook, "j"] & (G \times_K X) \amalg Y_1 \arrow[r, hook, "i"] \arrow[dr, "id \amalg f_1"'] & Y_2 \arrow[d, "f_2"] & Y_3 \arrow[l, "h"'] \arrow[dl, "f_3"] \\
& & G \times_K X &
\end{tikzcd}$$

We let $Z_1 = (pf_1)^{-1}(1K)$, $Z_2 = (pf_2)^{-1}(1K)$, and $Z_3 = (pf_3)^{-1}(1K)$ for short, where $p: G \times_K X \to X$ is the canonical projection. We have G-homeomorphisms h_l $(l = 1, \cdots, 3)$ such that the following diagram is commutative.

$$\begin{tikzcd}
X \amalg Z_1 \arrow[r, hook, "\bar{i}"] \arrow[d] & Z_2 \arrow[d] & Z_3 \arrow[l, "\bar{h}"'] \arrow[d] \\
(G \times_K X) \amalg (G \times_K Z_1) \arrow[r, hook] \arrow[d, equal, "h_1"] & G \times_K Z_2 \arrow[d, equal, "h_2"] & G \times_K Z_3 \arrow[l] \arrow[d, equal, "h_3"] \\
(G \times_K X) \amalg Y_1 \arrow[r, hook, "i"] & Y_2 & Y_3 \arrow[l, "h"']
\end{tikzcd}$$

By taking the Z_l's, the G-maps f_l induce K-maps $\bar{f}_l$: $Z_l \to X$ $(l = 1, \cdots, 3)$. Since $\bar{h}$ is a K-homotopy equivalence and the diagram

$$\begin{tikzcd}
X \amalg Z_1 \arrow[r, hook, "\bar{i}"] \arrow[dr, "id \amalg \bar{f}_1"'] & Z_2 \arrow[d, "\bar{f}_2"] & Z_3 \arrow[l, "\bar{h}"'] \arrow[dl, "\bar{f}_3"] \\
& X &
\end{tikzcd}$$

commutes, Z_3 dominates X. By Lemma 1.2, Z_3 has the K-homotopy type of a finite K-CW complex. This completes the proof.

Let Φ: $D^G(G \times_K X) \to D^K(X)$ be a homomorphism induced by a mapping assigning k: $W \to X$, described as in Theorem 1.3, to any G-map g: $V \to G \times_K X$. It is an inverse isomorphism of a homomorphism $D^K(X) \to D^G(G \times_K X)$ induced by ind_K^G. Since $G \times_K W \underset{G}{\simeq} V$, it follows from Proposition 2.2 that $\Phi(Wa^G(G \times_K X)) \subset Wa^K(X)$. Then we have:

Theorem 2.3. *Let K be any closed subgroup of G and let X be a K-space. The induction homomorphism* $\mathrm{Ind}_K^G(X)$: $Wa^K(X)\to Wa^G(G\times_K X)$ *is an isomorphism. In particular $G\times_K X$ has the G-homotopy type of a finite G-CW complex if and only if X has the K-homotopy type of a finite K-CW complex.*

3. Free S^1-spaces

In this section we study Theorem A for free S^1-spaces. We denote a group S^1 by G and let X be a connected free G-CW complex such that $H_1(\nu;\boldsymbol{Z})$ is injective. If the projection $X\to X/G$ is a principal G-bundle and the fundamental group of X is abelian, Anderson [1] has shown that the universal cover of X is $\pi_1(X/G)$-homeomorphic to the product of the universal cover of X/G and the real space $\boldsymbol{R}$ with some $\pi_1(X/G)$-action on $\boldsymbol{R}$. We show that for some $K\leq G$ and some CW complex V, the G-space X/K is G-homotopy equivalent to $G/K\times_{\{1_K\}}V=G/K\times V$.

Lemma 3.1. *Let X be a connected free G-space such that the G-map ν: $G\to X$ induces a monomorphism $H_1(\nu;\boldsymbol{Z})$. There is a finite subgroup K of G such that $\pi_1(X/K)$ isomorphic to $\pi_1(G/K)\oplus\pi_1(X/G)$.*

Proof. For any $K\leq G$, we have a short exact sequence:

$$1\to\pi_1(G/K)\to\pi_1(X/K)\to\pi_1(X/G)\to 1$$

We construct a splitting $\pi_1(X/K)\to\pi_1(G/K)$ for some $K<G$. By the assumption, there is an epimorphism μ: $\pi_1(X)\to\boldsymbol{Z}$ such that the following diagram commutes:

$$\begin{tikzcd}
\pi_1(G) \arrow[r, tail, "\nu"] \arrow[dr, "n"'] & \pi_1(X) \arrow[d, "\mu"] \\
 & \boldsymbol{Z}
\end{tikzcd}$$

Here n is multiplication by $n\geq 0$. Let K be a subgroup of G with order n.

$$\begin{tikzcd}
\pi_1(G) \arrow[r, "\nu"] \arrow[d, "n"] & \pi_1(X) \arrow[r, "\mu"] \arrow[d, tail, "p"] & \boldsymbol{Z} \\
\pi_1(G/K) \arrow[r, "\bar{\nu}"] \arrow[d, two heads, "\partial"] & \pi_1(X/K) \arrow[ur, dashed, "\bar{\mu}"'] \arrow[d, two heads, "\partial"] & \\
K \arrow[r, equal] & K &
\end{tikzcd}$$

By a chasing method, the equation $\bar{\nu}(m)p(y)=\bar{\nu}(m')p(y')$ implies that there is

$z\in\pi_1(G)$ satisfying $m=m'+nz$ and $y=\nu(z^{-1})y'$. Then we have

$$m+\mu(y)=m'+nz+\mu(\nu(z^{-1}))+\mu(y')=m'+\mu(y')\,.$$

For any $x=\bar{\nu}(m)p(y)\in\pi_1(X/K)$ we define as $\bar{\mu}(x)=m+\mu(y)$. Then the map $\bar{\mu}\colon\pi_1(X/K)\to\boldsymbol{Z}$ is a homomorphism with $\bar{\mu}\circ p=\mu$, since the image of $\bar{\nu}$ is a subgroup of the center of $\pi_1(X/K)$. Since both $\mu\circ\nu$ and n are multiplication by n, we have $\bar{\mu}\circ\bar{\nu}=1$ and $\bar{\mu}$ is the required splitting.

Propoistion 3.2. *Let X be as in Theorem A. If X is free, then there are a proper subgroup K of G and a CW complex V such that $G/K\times V$ and X/K are G-homotopy equivalent.*

Proof. Let K be a subgroup of G such that $\pi_1(X/K)\cong\pi_1(G/K)\oplus\pi_1(X/G)$. We denote by $p\colon V\to X/K$ the covering space corresponding to $\pi_1(X/G)\leq\pi_1(X/K)$. The G-map $G/K\times V\to X/K$, sending (gK, v) to $g\cdot p(v)$, induces an isomorphism of homotopy groups. By a Whitehead theorem of the equivariant version [10], it is a G-homotopy equivalence.

By Lemma 1.2 there is a K-CW complex Y such that $G\times_K Y$ and X is G-homotopy equivalent.

REMARK. Let Y be a K-space obtained from the G-homotopy pullback of the G-map p through the covering map $V\to X/K$. Then the G-map $G\times_K Y\to X$ induced by the given K-map $Y\to X$ is a G-homotopy equivalence.

4. Proof of Theorems A and B

In this section, we also denote S^1 by G.

Proposition 4.1. *Let X be as in Thoerem A and let H be a finite subgroup of G. Then $H_1(p\circ\nu;\boldsymbol{Z})$ is monic for the projection $p\colon X\to X/H$.*

Proof. As the H-action on X comes from a G-action by restriction, H acts trivially on $H_1(X;\boldsymbol{Z})$. Applying Theorem 2.4 [3, p. 120], we obtain that the projection induces an isomorphicm $H_1(p;\boldsymbol{Q})\colon H_1(X;\boldsymbol{Q})\to H_1(X/H;\boldsymbol{Q})$. Then $H_1(p;\boldsymbol{Z})$ is injective on any free abelian subgroup of rank one in $H_1(X;\boldsymbol{Z})$.

We note that $\pi_1(\nu)$ is monic does not imply that $\pi_1(p\circ\nu)$ is injective.

Proof of Theorem A. Let H be a cyclic subgroup of which order is a common multiple of order of all isotropy subgroups in X. Clearly X/H is a connected free G/H-CW complex. By the argument in the previous section, there are a free K/H-CW complex V and a G/H-homotopy equivalence $h\colon G/H\times_{K/H}V\to X/H$. We see canonically V as a K-CW complex. Then h induces a G-homotopy equivalence $h'\colon G\times_K V\to X/H$. The G-space W obtained from a

G-homotopy pullback of h' through the projection $X \to X/H$ is G-homotopy equivalent to X.

$$\begin{array}{ccc} W & \longrightarrow & X \\ \downarrow & & \downarrow \\ G \times_K V & \xrightarrow[G]{\simeq} & X/H \end{array}$$

On the other hand, by Lemma 1.2, there is a K-CW complex Y such that $G \times_K Y$ and W are G-homotopy equivalent. This completes the proof.

To prove Theorem B we may show the following:

Proposition 4.3. *Let X be a connected finitely dominated G-space which fulfills that $H_1(\nu; \boldsymbol{Z})$ is injective. Then the K-space X has the K-homotopy type of a finite K-CW complex for any finite subgroup K of G.*

Proof. X has the G-homotopy type of a G-CW complex with finitely many orbit types [5, Theorem 1.4]. By Theorem A and Proposition 2.2, there is a finitely dominated L-space Y such that $X \underset{G}{\simeq} G \times_L Y$. Since $G \times_L Y \to G/L$ is a K-fibration, we have the result. (See Theorem 3.6 [13].)

Theorem 4.4. *Let G be any compact Lie group and let K be a subgroup of G. Let X be a finitely dominated G-space with X/G connected. If the rank of the image of $H_1(W_G K \to X/K; \boldsymbol{Z})$ is not zero, then* $\mathrm{Res}_K^G(X)$: $Wa^G(X) \to Wa^K(X)$ *is a zero map.*

Proof. Let T be a maximal torus of $W_G K$. Since $H_1(T \to W_G K; \boldsymbol{Z})$ is epic, there is a proper subgroup $\bar{C}$ of $W_G K$ such that $\bar{C}$ is isomorphic to S^1 and $H_1(\bar{C} \to X/K; \boldsymbol{Z})$ is injective. Then there is a $\bar{C}$-map $f: X/K \to \bar{C}/\bar{L}$ for some finite subgroup $\bar{L}$ of $\bar{C}$. Let C (resp. L) be the preimage of $\bar{C}$ (resp. $\bar{L}$) under the projection $N_G K \to W_G K$. Clearly $h: \bar{C}/\bar{L} \cong C/L$. Since K is a normal subgroup of C, the projection $p: X \to X/K$ is a C-map. Then $h \circ f \circ p: X \to C/L$ is an equivariant C-fibration. The K-space C/L has a trivial K-action and its Euler characteristic is zero. If we apply Theorem 2.6 [13] to the equivariant K-fibration $h \circ f \circ p$, we conclude the proof.

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Department of Mathematics
Kyushu University 33
Fukuoka 812, Japan
E-mail: d75190a@kyu-cc.cc.
kyushu-u.ac.jp,
sumi@math sci. kyushu-u.
ac.jp