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ALMOST IDENTICAL IMITATIONS OF (3, 1)-DIMENSIONAL MANIFOLD PAIRS AND THE BRANCHED COVERINGS

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0. Introduction

By a *good* (3, 1)-*manifold pair* (M, L) (or a *good* 1-*manifold* L *in a* 3-*manifold* M), we mean that M is a compact connected oriented 3-manifold and L is a compact proper smooth 1-submanifold of M such that any 2-sphere component of the boundary ∂M meets L with at least three points. For a compact connected oriented 3-manifold E, let $\partial_0 E$ be the union of all tori in ∂E and $\partial_1 E = \partial E - \partial_0 E$. Let $\mathrm{int}\, E = E - \partial E$ and $\mathrm{int}_0\, E = E - \partial_0 E$. A compact connected oriented 3-manifold E is said to be *hyperbolic* if $\mathrm{int}\, E$ (when $\partial_1 E = \emptyset$) or the double $\mathrm{D}\,(\mathrm{int}_0\, E)$ pasting along $\partial_1 E$ (when $\partial_1 E \neq \emptyset$) has a complete Riemannian structure of constant curvature -1. Then we define the *volume* $\mathrm{Vol}\, E$ of E to be the hyperbolic volume $\mathrm{Vol}\,(\mathrm{int}\, E)$ (when $\partial_1 E = \emptyset$) or the half hyperbolic volume $\mathrm{Vol}\,(\mathrm{D}(\mathrm{int}_0\, E))/2$ (when $\partial_1 E \neq \emptyset$), and the *isometry group* $\mathrm{Isom}\, E$ of E to be the hyperbolic isometry group $\mathrm{Isom}\,(\mathrm{int}\, E)$ (when $\partial_1 E = \emptyset$) or the quotient by τ of the following subgroup $\{f \in \mathrm{Isom}\,(\mathrm{D}(\mathrm{int}_0\, E)) \mid f\tau = \tau f\}$ (when $\partial_1 E \neq \emptyset$), where τ denotes the unique isometry of $\mathrm{D}(\mathrm{int}_0\, E)$ induced from the involution of $\mathrm{D}(\mathrm{int}_0\, E)$ interchanging the two copies of $\mathrm{int}_0\, E$ (cf. [22]). By Mostow rigidity theorem (cf. [23], [24]), $\mathrm{Vol}\, E$ is a topological invariant of E and $\mathrm{Isom}\, E$ is a unique (up to conjugations) finite subgroup of the diffeomorphism group $\mathrm{Diff}\, E$. Furthermore, there is a natural isomorphism $\mathrm{Isom}\, E \cong \mathrm{Out}\, \pi_1(E) = \mathrm{Aut}\, \pi_1(E)/\mathrm{Inn}\, \pi_1(E)$ and for any finite subgroup G of $\mathrm{Diff}\, E$ there is a natural monomorphism $G \to \mathrm{Out}\, \pi_1(E)$, so that G is isomorphic to a subgrpup of $\mathrm{Isom}\, E$. In a previous paper [8], for each good (3,1)-manifold pair (M, L), we have constructed an infinite family of almost identical imitations (M, L^*) of (M, L) such that the exterior $E(L^*, M)$ of L^* in M is hyperbolic. In this paper, we shall strengthen this result from the viewpoint of regular branched coverings.*)

DEFINITION: A good (3,1)-manifold pair (M, L) has the *hyperbolic covering property* if for any component unions L_0, L_1 (possibly, $\emptyset$) of L with $L_1 = L - L_0$,

*) By coverings, we will mean connected coverings.

any finite regular covering space $\widetilde{E}(L_0, M)$ of the exterior $E(L_0, M)$ of L_0 in M branched along L_1 is hyperbolic after *spherical completion*, that is, after adding a cone over each 2-sphere in $\partial\widetilde{E}(L_0, M)$, where we understand that $E(L_0, M)=M$ when $L_0=\emptyset$.

The spherical completion of $\widetilde{E}(L_0, M)$ is denoted by $\widetilde{E}(L_0, M)_\wedge$. The covering transformation group of $\widetilde{E}(L_0, M)$ acts on $\widetilde{E}(L_0, M)_\wedge$ by a natural extension. Let $q: (M^*, L^*)\rightarrow(M, L)$ be a normal imitation. For component unions $L_i, i=0, 1$, (possibly, $\emptyset$) of L with $L_1=L-L_0$, let $L_i^*=q^{-1}(L_i), i=0, 1$. Then the imitation map q induces a normal imitation map q_E: $E(L_0^*, M^*)\rightarrow E(L_0, M)$ by the definition of normal imitation. For any regular covering p: $\widetilde{E}(L_0, M)\rightarrow E(L_0, M)$ branched along L_1 with covering transformation group denoted by G, we see from [7, Property IV] that $\tilde{q}_E$ is a normal imitation map and p^* is a regular covering map branched along L_1^* with covering transformation group G in the following commutative diagram pulling back the covering map p and the imitation map q_E:

$$\begin{array}{ccc}\widetilde{E}(L_0^*, M^*) & \xrightarrow{\tilde{q}_E} & \widetilde{E}(L_0, M)\\ p^*\downarrow & & \downarrow p\\ E(L_0^*, M^*) & \xrightarrow{q_E} & E(L_0, M)\,.\end{array}$$

Since $\tilde{q}_E$ is ∂-diffeomorphic G-map, we can extend $\tilde{q}_E$ uniquely to a G-map

$$(\tilde{q}_E)_\wedge\colon \widetilde{E}(L_0^*, M^*)_\wedge \rightarrow \widetilde{E}(L_0, M)_\wedge$$

over the spherical completion, which is still a normal imitation map.

Definition: The covering map p^* is the *lift* of the covering map p (by the imitation map q_E). The imitation maps $\tilde{q}_E$ and $(\tilde{q}_E)_\wedge$ are the *lift* and *spherical completion lift* of the imitation map q_E (by the covering map p), respectively.

The main result of this paper can be stated as follows:

Main Theorem. *For any good* (3.1)*-mainfold pair* (M, L)*, there exists an infinite family* $\mathfrak{J}$ *of almost identical imitations* (M, L^*) *of* (M, L) *with hyperbolic covering property. Further, if we denote the imitation map* $(M, L^*)\rightarrow(M, L)$ *by* q*, then for any positive number* C *and any positive integer* N*, this family can have the following properties*:

(1) *There is a number* $C^+>C$ *such that* $\operatorname{Vol} E(L^*, M)<C^+$ *and* $\sup_{(M,L^*)\in\mathfrak{J}} \operatorname{Vol} E(L^*, M)=C^+$,

(2) *Let* L_0, L_1 *be any component unions* (*possibly* $\emptyset$) *of* L *with* $L_1=L-L_0$. *For the spherical completion lift* $(\tilde{q}_E)_\wedge$: $\widetilde{E}(L_0^*, M)_\wedge\rightarrow\widetilde{E}(L_0, M)_\wedge$ *of the imitation map* q_E: $E(L_0^*, M)\rightarrow E(L_0, M)$ (*induced from* q) *by any regular*

covering p: $\tilde{E}(L_0, M) \to E(L_0, M)$ *branched along* L_1 *with covering transformation group,* G, *of order* $< N$, *the group* G, *which is regarded as a subgroup of* Diff $\tilde{E}(L_0^*, M)_\wedge$, *is isomorphic to* Isom $\tilde{E}(L_0^*, M)_\wedge$. *In particular,* Isom $E(L^*, M) = \{1\}$. *Further, when* $L_1 = \emptyset$ (*i.e.,* $L_0 = L$), *we can take* $N = +\infty$.

When $L_1 = \emptyset$, (2) implies that G is conjugate to Isom $\tilde{E}(L_0^*, M)_\wedge$ in Diff $\tilde{E}$ $(L_0^*, M)_\wedge$, since $E(L^*, M)$ is hyperbolic. If we use Thurston's announcement result in [23, p. 379], [25] for the case $L_1 \neq \emptyset$, we see that the isomorphism $G \cong$ Isom $\tilde{E}(L_0^*, M)_\wedge$ in (2) can be always replaced by the following (2′):

(2)′ *G is conjugate to* Isom $\tilde{E}(L_0^*, M)_\wedge$ *in* Diff $\tilde{E}(L_0^*, M)_\wedge$.

To state a property occurring from our construction, we need the following definition:

DEFINITION: For a good (3, 1)-manifold pair (M, L), a tangle (i.e., a proper 1-manifold without loop component) t in a 3-ball $B \subset \text{int}\, M$ is a *basic tangle* for (M, L) if $t = B \cap \text{int}\, L$ and each component of L contains a component of t and t has at least 3 components. The good (3, 1)-manifold pair $(M', L') = (M - \text{int}\, B, L - \text{int}\, t)$ is the *complement* of (B, t).

The imitation map q: $(M, L^*) \to (M, L)$ in Main Theorem has the following property:

(3) *There is a 2-sphere* $S \subset \text{int}\, M$ *which splits the imitation map* q: $(M, L^*) \to (M, L)$ *into two almost identical imitation maps* q_B: $(B, t^*) \to (B, t)$ *and* q': $(M', L'^*) \to (M', L')$ *such that* (B, t) *is a basic tangle for* (M, L) *and* (M', L') *is the complement, and* (B, t^*) *and* (M', L'^*) *have the hyperbolic covering property. Further, we can previously take any basic tangle for* (M, L) *as* (B, t).

Before concluding this introduction, we remark that we shall alter the definition of almost identical imitation in [6], [8] into a slightly more improved definition. In §1 we discuss when branched covering spaces of a 3-manifold are simple and semi-simple. In §2 the improved definition of almost identical imitation is stated. In §3 we construct an almost identical imitation with hyperbolic covering property of a tangle in a 3-ball, which is generalized, in §4, to a good (3,1)-manifold pair. In §5 we prove Main Theorem. In §6 some applications are given. This manuscript has been prepared since 1987 and the present version has been written up during the author's visit to University of Melbourne in March-April 1991 under an exchanging program. The author would like to thank this exchanging program, particularly Professor Junzo Tao, for making his visit possible and Department of Mathematics, University of Melbourne, particularly Professor J. Hyam Rubinstein, for various hospitalities.

1. Basic lemmas for branched coverings. A graph Γ in a 3-manifold M is said to be *good* if the pair (M, Γ) is obtained from a good (3, 1)-manifold pair (M_0, L) by spherical completion associated with some 2-spheres in ∂M_0 (cf. [8]). For an integer $n \geq 3$, we denote by $v_n(\Gamma)$ the set of vertices of Γ with degree n. Let $v(\Gamma) = \cup_{n \geq 3} v_n(\Gamma)$.

DEFINITION: A smooth 2-sphere S in int M or in ∂M is an *n-pointed sphere* in (M, Γ) if S meeets Γ-$v(\Gamma)$ transversely with just n points and $S \cap v(\Gamma) = \emptyset$. Further, it is *essential* if $S_E = S \cap E(\Gamma, M)$ is incompressible and non-∂-parallel in the exterior $E(\Gamma, M)$ of Γ in M.

DEFINITION: Let D be a proper disk in M or a disk in ∂M. D is an *n-pointed disk* in (M, Γ) if int D meets Γ-$v(\Gamma)$ transversely with just n points and $D \cap v(\Gamma) = \partial D \cap \Gamma = \emptyset$. Further, it is *essential* if $D_E = D \cap E(\Gamma, M)$ is incompressible and non-∂-parallel in $E(\Gamma, M)$.

A good graph Γ in M is *trivial* if it is on a smooth proper disk or 2-sphere. A good graph Y in a 3-ball B is called a *trivial* Y-*graph* if $|Y \cap \partial B| = 3$ and there is a diffeomorphism of B sending Y to a cone over the set $Y \cap \partial B$. A good graph H in a 3-ball B is called a *trivial* H-*graph* if the pair (B, H) is diffeomorphic to a pair obtained from two copies of the pair (B, Y) of a trivial Y-graph Y in B by identifying the two copies of a 1-pointed disk D in (B, Y) with $D \subset \partial B$.

Lemma 1.1. *Let Γ be a good graph in a 3-manifold M. If a finite regular covering space $\tilde{M}$ of M branched along Γ is a 3-manifold, then $v_n(\Gamma) = \emptyset$ for all $n \geq 4$. Further, if M is a 3-ball and Γ is a trivial good tree graph, then $\tilde{M}$ is a handlebody.*

Proof. Let $(V, \Gamma \cap V)$ be a cone pair over an n-pointed sphere with $n \geq 3$ in (M, Γ). Since $\tilde{M}$ is a 3-manifold, the lift of V to $\tilde{M}$ consists of disjoint 3-balls. By the Riemann/Hurwitz formula (on a regular covering of S^2) (cf. Scott [21]), we have $n=3$. To see the latter half, we consider a handle decomposition of M consisting of 0-handles h_i^0 and 1-handles h_j^1 such that $h_i^0 \cap \Gamma$ is a trivial arc or trivial Y-graph in h_i^0 or $\emptyset$ for each i, and $h_j^1 \cap \Gamma$ is a core of the 1-handle h_j^1 or $\emptyset$ for each j. Then $\tilde{M}$ has a handle decomposition consisting of 0-handles being the lifting components of the h_i^0's and 1-handles being the lifting components of the h_j^1's. Since $\tilde{M}$ is connected, it is a handlebody. This completes the proof.

Let a_i, $i=1, 2, \cdots, r$, be disjoint arcs in S^1. Let D_0 be a disk in the interior of a disk D. For two points p_1, p_2 in int D_0, we consider a link L in the solid torus $S^1 \times D^2$ obtained from the link $S^1 \times \{p_1, p_2\}$ by replacing, in $a_i \times D_0$, the standard trivial 2-string tangle $a_i \times \{p_1, p_2\}$ with a trivial (i.e., rational) 2-string tangle for each i.

DEFINITION: This link L in $S^1\times D$ is called a *Montesinos link* in $S^1\times D$. When we identify $S^1\times D$ with a soild tours V in a lens space M such that $V'=\mathrm{cl}(M-V)$ is a solid torus, we call this link L in M a *Montesinos link* in M.

In case $M=S^3$, the Montesinos link L is a link considered by Montesinos [17].

Lemma 1.2. *Let $\tilde{M}$ be a regular covering space of a closed 3-manifold branched along a good graph Γ. If $\tilde{M}$ is an irreducible Seifert manifold and the exterior $E(\Gamma, M)$ is hyperbolic, then we have one of the following*:

(1) *$\tilde{M}$ has a spherical or Euclidean geometry (i.e., has S^3 or $S^1\times S^1\times S^1$ as a finite unbranched regular covering space).*

(2) *M is a lens space except $S^1\times S^2$ and there is a Montesinos link L in M such that $L\subset\Gamma$ and $L^c=\mathrm{cl}(\Gamma-L)$ is a 1-manifold with at most one loop component or $\emptyset$, and the covering $\tilde{M}\to M$ is the composite of a double covering $M_2\to M$ branched along L and a regular covering $\tilde{M}\to M_2$ branched along the lift L_2^c of L^c to M_2 where M_2 is a Seifert manifold over S^2 with each component of L_2^c a fiber.*

Proof. Assume that the Seifert manifold $\tilde{M}$ has no spherical or Euclidean geometry. Then we show that (2) is satisfied. By a result of Meeks/Scott [15], the covering transformation group G of $\tilde{M}$ preserves the fibers of the Seifert fibration. Hence G acts on the base space $\tilde{F}$ of the Seifert manifold $\tilde{M}$. If the orbit space $F=\tilde{F}/G$ is closed, then we see that $M=\tilde{M}/G$ is a Seifert manifold over F with Γ a set of fibers, so that the exterior $E(\Gamma, M)$ is a Seifert manifold, contradicting that it is hyperbolic. Hence F has a boundary. We take a collar N of any boundary component C in F so that $N-C$ is disjoint from the image of Γ under the natural projection $M\to F$ and the images of the points in $\tilde{F}$ represented by the exceptional fibers of $\tilde{M}$ under the projection $\tilde{F}\to F$. Let $\tilde{N}$ be a connected component of the lift of N in $\tilde{F}$, which is an orientable surface. Let $G_N=\{g\in G\,|\,g\tilde{N}=\tilde{N}\}$. Then there is an index 2 subgroup G'_N of G_N acting on $\tilde{N}$ orientation-preservingly, so that $\tilde{N}/G'_N$ is an annulus and the group G_N/G'_N acts on the annulus $\tilde{N}/G'_N$ as a reflection in a center circle. Let $\tilde{M}_N$ be the Seifert submanifold of $\tilde{M}$ with base space $\tilde{N}$. Note that the orbit space M'_N of $\tilde{M}_N$ by G'_N is a Seifert manifold over the annulus $\tilde{N}/G'_N$ with action of G_N/G'_N orientation and fiber preserving. Let M_N be the orbit space of M'_N by G_N/G'_N. Note that the projection $\tilde{M}_N\to M'_N$ is a regular covering branched along a set of fibers. Let β be the image of the set of fibers in M_N. Then we see that M_N is a solid torus and the projection $M'_N\to M_N$ is a double covering branched along a Montesions link L_N and β consists of arcs (cf. Dunbar [2]). Let $\Gamma_N=\Gamma\cap M_N$ and $T=\partial M_N$. Since M'_N is a Seifert manifold over an annulus, meaning that it is irreducible and ∂-irreducible, with the lift of β a set of fibers and $T\cap\Gamma=\emptyset$, the torus T is incompressible in M_N-L_N and $M_N-\Gamma_N$. Let $M_E=\mathrm{cl}(M-M_N)$.

Using that $E(\Gamma, M)$ is hyperbolic, we see that T is compressible or ∂-parallel in $E(\Gamma, M)$, so that M_E is a solid torus with $M_E \cap \Gamma$ being $\emptyset$ or a core. This means that F is a disk and $\tilde{F}$ is an orientable surface. Let G_2 be the orientation-preserving index 2 subgroup of G on $\tilde{F}$. Then $F_2 = \tilde{F}/G_2$ is a 2-sphere and $\tilde{M}/G_2 = M_2$ is a Seifert manifold over F_2 and the projection $\tilde{M} \to M_2$ is a regular covering branched along a set of Seifert fibers. Note that the solid torus M_E lifts to two solid tori in M_2. Let $L = L_N$. Then we see that the projection $M_2 \to M$ is a double covering branched along L and $L^c = \mathrm{cl}(\Gamma - L)$ is a 1-manifold with at most one loop component whose lift to M_2 is a set of fibers (unless it is $\emptyset$). Further, since some meridian of the solid torus M_N lifts to a regular fiber of M_2, M is a lens space except $S^1 \times S^2$. This completes the proof.

For a good (3, 1)-manifold pair (M, L), we consider a finite regular covering space $\tilde{M}$ of M branched along L. Let G be the covering transformation group. Let $p_\wedge: \tilde{M}_\wedge \to M_\wedge$ be the G-equivariant extension map of the covering projeation $p: \tilde{M} \to M$ by spherical completion. Let $M^+ = p_\wedge(\tilde{M}_\wedge)$. Then the map $p_\wedge$ defines a covering $p^+: \tilde{M}_\wedge \to M^+$ with covering transformation group G and with branch set L^+ obtained from L by adjoining trivial Y-graphs (cf. Lemma 1.1).

Definition: For $n \geq 3$, a good 1-manifold L in a 3-manifold M is *n-prime* if there is no essential n-pointed spheres in (M, L).

A 3-manifold E is *semi-simple* if E is irreducible, ∂-irreducible and any proper annulus in E is inessential (that is, compressible or ∂-parallel), and *simple* if E is irreducible, ∂-irreducible and any torus in int E is inessential (that is, compressible or ∂-paralle). Thurston's hyperbolization theorem [23] means that a Haken 3-manifold is hyperbolic if and only if it is simple and semi-simple.

Lemma 1.3. *If a good* 1*-manifold L in a* 3*-manifold M is* 3*-prime and the exterior $E(L, M)$ is semi-simple, then $\tilde{M}_\wedge$ is irreducible and ∂-irreducible.*

Proof. Suppose $\tilde{M}_\wedge$ is reducible. Then by the equivariant sphere theorem (cf. Meeks/Yau [16], Plotnick [19]), $\tilde{M}_\wedge$ has a G-equivariant incompressible sphere S such that $F = p^+(S)$ is diffeomorphic to the 2-sphere S^2 or the projective plane P^2 or the disk D^2 and $\mathrm{int}\, F \cap v_3(L^+) = \emptyset$. Let $G_S = \{g \in G \mid gS = S\}$. Then $F \cong S/G_S$. For $F \cong S^2$ or P^2, we have $m = |F \cap L^+| < +\infty$. Since $E(L^+, M^+) \cong E(L, M)$ is irreducible, we have $m \neq 0$. For $F \cong S^2$, we have $m = 2$ or 3 by the Riemann/Hurwitz formula. By our assumption, F bounds a 3-ball B in M^+ with $B \cap L^+$ a trivial arc or a trivial Y-graph, so that S is compressible (cf. Lemma 1.1), a contradiction. For $F \cong P^2$, we have $m = 1$ by the Riemann/Hurwitz formula. Let N be a normal bundle of F in M^+, diffeomorphic to the projective 3-space P^3 with an open 3-ball removed. Since $\partial N - L^+ \cap \partial N$ is incompressible in $N - L^+ \cap N$ and N is not a 3-ball, $B' = M^+ - \mathrm{int}\, N$ is a 3-ball with $L^+ \cap B'$ a trivial arc by our assumption. Then we have $\tilde{M}_\wedge \cong S^3$ and S is compressible, a contradiction. When $F \cong D^2$, we have $F \cap L^+ \supset \partial F$ and by Riemann/Hurwitz formula, $|F \cap L^+ - \partial F| \leq 1$, contradicting that $E(L^+, M^+)$ is semi-simple. Hence $\tilde{M}_\wedge$ is irreducible. Next, suppose $\tilde{M}_\wedge$ is ∂-reducible. Then by the equivariant loop theorem (cf. Meeks/Yau [16]), $\tilde{M}_\wedge$ has a G-equivariant essential disk D. Since $E(L^+, M^+)$ is ∂-irreducible, $G_D = \{g \in G \mid gD = D\}$ is non-trivial. We have $F \cong D/G_D$ is a disk such that $F \cap L^+$ is a point in int F or an arc in ∂F, contradicting that $E(L^+, M^+)$ is semi-simple. Hence $\tilde{M}_\wedge$ is ∂-irreducible. This completes the proof.

DEFINITION: For a good (3, 1)-manifold pair (M, L) such that $\partial_1 M$ consists of 3-pointed spheres and $\partial_0 M \cap L = \emptyset$, L is *2-semi-prime* in M if there is no essential 2-pointed disk D in M with $\partial D \subset \partial_0 M$.

Lemma 1.4. *For a good* (3, 1)*-manifold pair* (M, L)*, assume that* $\partial_1 M$ *consists of* 3*-pointed spheres and* $\partial_0 M \cap L = \emptyset$*. If the exterior* $E(L, M)$ *is hyperbolic (i.e., simple and semi-simple) and* L *is* 3*-prime,* 4*-prime and* 2*-semi-prime in* M*, then we have the following* (1), (2) *or* (3) *for any non-trivial finite regular covering* $p^+ \colon \tilde{M}_\wedge \to M^+$ *branched along* L^+*:*

(1) *$\tilde{M}_\wedge$ is a simple, semi-simple and non-Seifert 3-manifold,*

(2) *$\tilde{M}_\wedge$ is a closed Seifert manifold having a spherical or Euclidean geometry,*

(3) *M^+ is a lens space except $S^1 \times S^2$ and there is a Montesinos link $L_0 \subset L^+$ with $L_0^c = \mathrm{cl}\,(L^+ - L_0)$ a 1-manifold with at most one loop component or $\emptyset$ and the covering $\tilde{M}_\wedge \to M^+$ is the composite of a double covering $M_2^+ \to M^+$ branched along L_0 and a regular covering $\tilde{M}_\wedge \to M_2^+$ branched along the lift $(L_0^c)_2$ of L_0^c to M_2^+ where M_2^+ is a Seifert manifold over S^2 with each component of $(L_0^c)_2$ a fiber. In particular, the exterior $E((L_0^c)_2, M_2^+)$ is a Seifert manifold.*

REMARK 1.5. In (1), $\tilde{M}_\wedge$ is hyperbolic by Thurston's hyperbolization theorem in [23] if it is a Haken manifold. Further, Thurston announces in [23, p. 379], [25] that a simple, semi-simple non-Seifert manifold with orientation-preserving non-free periodic map is hyperbolic.

Proof. By Lemma 1.3, $\tilde{M}_\wedge$ and, when $\partial_1 \tilde{M}_\wedge \neq \emptyset$, the double $D_1 \tilde{M}_\wedge$ of $\tilde{M}_\wedge$ pasting along $\partial_1 \tilde{M}_\wedge$ are irreducible and ∂-irreducible. Let G be the covering transformation group of $\tilde{M}_\wedge$. We prove the folloiwng later:

Assertion 1.4.1. *$\tilde{M}_\wedge$ has no G-equivariant essential torus or annulus.*

We proceed the proof by dividing into two cases.

Case(a): $\partial_1 \tilde{M}_\wedge = \emptyset$.

If $\tilde{M}_\wedge$ is neither Seifert nor simple, then $\tilde{M}_\wedge$ has a G-equivariant essential torus, contradicting Assertion 1.4.1, by the torus decomposition theorem due to Jaco/Shalen and Johannson theorem (cf. [5]) and the equivariant torus theorem [3]. This imples that $\tilde{M}_\wedge$ is either simple, semi-simple and non-Seifert or Seifert, since a simple non-semi-simple 3-manifold is Seifert ([5]). If $\tilde{M}_\wedge$ is a bounded Seifert manifold, then $\tilde{M}_\wedge \cong S^1\times S^1\times I$, $I=[-1, 1]$, for otherwise $\tilde{M}_\wedge$ would have a G-equivariant essential annulus, contradicting Assertion 1.4.1, by a result of Kobayashi [12]. We prove the following later:

Assertion 1.4.2. *$\tilde{M}_\wedge$ is not diffeomorphic to $S^1\times S^1\times I$.*

If $\tilde{M}_\wedge$ is a closed Seifert manifold and has no spherical or Euclidean geometry, then by Lemma 1.2 we have (3) for (M^+, L^+).

Case(b): $\partial_1 \tilde{M}_\wedge \neq \emptyset$.

Let Z_2 be the reflection group of $\mathrm{D}_1 \tilde{M}_\wedge$ along $\partial_1 \tilde{M}_\wedge$. If $\mathrm{D}_1 \tilde{M}_\wedge$ has a $G\times Z_2$-equivariant essential torus, then $\tilde{M}_\wedge$ has a G-equivariant essential torus or annulus, contradicting Assertion 1.4.1. Hence $\mathrm{D}_1 \tilde{M}_\wedge$ is either simple, semi-simple and non-Seifert or Seifert by the torus decomposition theorem [5] and the equivariant torus decomposition [3]. We show the folloiwng later:

Assertion 1.4.3. $\mathrm{D}_1 \tilde{M}_\wedge$ *is not a closed Seifert manifold.*

If $\mathrm{D}_1 \tilde{M}_\wedge$ is a bounded Seifert manifold, then $\mathrm{D}_1 \tilde{M}_\wedge$ has a $G\times Z_2$-equivariant essential annulus by [12]. Hence $\tilde{M}_\wedge$ has a G-equivariant essential annulus, contradicting Assertion 1.4.1, because $\mathrm{D}_1 \tilde{M}_\wedge$ is not diffeomorphic to $S^1\times S^1\times I$ by $\partial_1 \tilde{M}_\wedge \neq \emptyset$. This completes the proof of Lemma 1.4 except for the proofs of Assertions 1.4.1, 1.4.2 and 1.4.3.

PROOF OF ASSERTION 1.4.1. Suppose $\tilde{M}_\wedge$ has a G-equivariant essential torus T. Let $G_T=\{g\in G \mid gT=T\}$ and $F=p^+T$. Then $F\cong T/G_T$ and $\mathrm{int}\, F \cap \mathrm{v}_3(L^+)=\emptyset$. When F is a torus, Klein bottle, annulus or Möbius band, we have $\mathrm{int}\, F\cap L^+=\emptyset$ by the Riemann/Hurwitz formula. Since $E(L^+, M^+)$ is simple and semi-simple, such a case can not occur. When $F=S^2$, we let $m=|F\cap L^+|$. Then $m=3$ or 4 by the Riemann/Hurwitz formula. Since L is 3-prime and 4-prime in M, we see that T is compressible or ∂-parallel in $\tilde{M}_\wedge$, a contradiction. When $F=P^2$, we may consider that $F\subset M$. Let $m=|F\cap L^+|$. By the Riemann/Hurwitz formula, we have $m=2$. Let N be a normal bundle of F in M. Note that ∂N is a 4-pointed sphere for (M, L) and $\partial N - L\cap \partial N$ is incompressible in $N-L\cap N$ and each component of $(p^+)^{-1} N$ is diffeomorphic to $S^1\times S^1\times I$. $E=\mathrm{cl}\,(M^+-N)$ is a 3-ball or diffeomorphic to $S^2\times I$. When E is a 3-ball, $E\cap L^+$ is a trivial 2-string tangle or a trivial H-graph, so that each component of $(p^+)^{-1} E$ is a solid torus by Lemma 1.1. When E is diffeomorphic to $S^2\times I$, each component of $(p^+)^{-1}E$ is diffeomorphic to $S^1\times S^1\times I$. Thus, $\tilde{M}_\wedge$ must be a lens space or diffeomorphic to $S^1\times S^1\times I$ by Lemma 1.3 and T is compressible or ∂-paralle, a contradiction. When F is a disk, we have $\partial F\subset F\cap L^+$. Let $m=|F\cap L^+-\partial F|$, which is finite. We can see from the Riemann/Hurwitz formula that $m\leq 2$ and for $m=2$, $\partial F\cap v_3(L^+)=\emptyset$. The case $m\leq 1$ does not occur since $E(L, M)$ is semi-simple. If $m=2$, then we consider a 3-ball neighborhood N of F which is a bicollar of a disk F^+ with $F\subset \operatorname{int} F^+$. Then ∂N is a 4-pointed sphere for (M, L) and $\partial N-L\cap\partial N$ is incompressible in $N-L\cap N$ and each component of $(p^+)^{-1}N$ is diffeomorphic to $S^1\times S^1\times I$. By the same reason as that of the case $F=P^2$, $\tilde{M}_\wedge$ is a lens space or $S^1\times S^1\times I$ and T is compressible or ∂-parallel, a contradiction. Thus, we see that $\tilde{M}_\wedge$ has no G-equivariant essential torus. Next, suppose that $\tilde{M}_\wedge$ has a G-equivariant essential annulus A. Let $G_A=\{g\in G\,|\,gA=A\}$ and $F=p^+A$. Then $F\cong A/G_A$ and $(\operatorname{int} F\cup p^+(\partial A))\cap v_3(L^+)=\emptyset$. By the Riemann/Hurwitz theorem, when F is a disk with $p^+(\partial A)$ a union of two disjoint arcs, annulus or Möbius band, $F\cap L^+$ has no isolated point, and when F is a disk with $p^+(\partial A)$ an arc, $F\cap L^+$ has just one isolated point. These cases can not occur by the semi-simpleness of $E(L^+, M^+)\cong E(L, M)$. Thus, F is a disk with $p^+(\partial A)=\partial F$. Then $F\cap v_3(L^+)=\emptyset$ and $|F\cap L^+|=2$. Since L is 2-semi-prime in M, ∂F must be in a 3-pointed sphere component S of ∂M^+. Hence ∂F bounds an $n(\leq 1)$-pointed disk D in S. Since each component of $(p^+)^{-1}D$ must be a disk, we see that A is compressible in $\tilde{M}_\wedge$, a contradiction. This completes the proof of Assertion 1.4.1.

PROOF of ASSERTION 1.4.2. Suppose $\tilde{M}_\wedge\cong S^1\times S^1\times I$. If all elements of G preserve the components of $\partial\tilde{M}_\wedge$, we see from a result of Bonahon/Siebenmann in [1] that $(M^+, L^+)\cong(S^1\times S^1\times I, \emptyset)$ or $(S^2$, 3 or 4 points$)\times I$, which contradicts the semi-simpleness of $E(L, M)$. If an element of G changes the components of $\partial\tilde{M}_\wedge$, then M^+ is the orbit space of $S^1\times S^1\times I$ or $S^2\times I$ by an involution changing the boundary components, which is diffeomorphic to $S^1\times D^2$ or the 3-ball B^3, respectively. When $M^+\cong S^1\times D^2$, we see that $(M^+, L^+)=(M, L)$ and L is a link. Considering a minimal intersection of L with meridian disks for M, we see from the Z_2-equivariant loop theorem [16] that there is a meridian disk D for M with $|D\cap L|=2$. Let (M', L') be a (3, 1)-manifold pair obtained from (M, L) by splitting along D. Then M' is a 3-ball and since the double covering space of M branched along L is $S^1\times S^1\times I$, we see that the double covering space of M' branched along L' is a solid torus. This mens that $(M, L)\cong(D$, 2 points$)\times S^1$. This contradicts that $E(L, M)$ is semi-simple. When $M^+\cong B^3$, we note that L^+ is a union of a circle and the orbit space of $\{3$ or 4 points$\}\times I(\subset S^2\times I)$. Since $E(\{3$ or 4 points$\}\times I, S^2\times I)$ is a handlebody, we see from the Z_2-equivariant loop theorem [16] that $E(L^+, M^+)\cong E(L, M)$ has an essential disk or an annulus, contradicting the semi-simpleness. This completes the proof of Assertion 1.4.2.

PROOF OF ASSERTION 1.4.3. Suppose $D_1 \tilde{M}_\wedge$ is a closed Seifert manifold. We let (DM^+, DL^+) be the double of (M^+, L^+) pasting along $(\partial M^+, L^+ \cap \partial M^+)$. By Myers gluing lemma (cf. [8]), $E(DL^+, DM^+)$ is hyperbolic. Since $\partial_1 \tilde{M}_\wedge \neq \emptyset$, $D_1 \tilde{M}_\wedge$ can not have any spherical or Euclidean geometry. By Lemma 1.2, the base space of the Seifert manifold $D_1 \tilde{M}_\wedge$ is orientable, and by [5, VI.34] $\tilde{M}_\wedge$ is a trivial I-bundle $F \times I$ over a closed orientable connected surface F of genus ≥ 2. Moreover, G preserves this I-boundle structure, because $G \times Z_2$ preserves the fibers of the Seifert manifold $D_1 \tilde{M}_\wedge$ by [15]. By Lemma 1.2, DM^+ is a lens space except $S^1 \times S^2$, so that M^+ is a 3-ball. Further, there is an index 2 subgroup G_2 of G preserving each component of $\partial \tilde{M}_\wedge$ such that the orbit space $\tilde{M}_\wedge / G_2 = M_2^+$ is a trivial I-bundle over S^2 with a line-fiber preserving action of G/G_2 and the projeation $\tilde{M}_\wedge \to M_2^+$ is a covering branched along three or more line-fibers. Let E be a G/G_2-invariant compact exterior of these line-fibers in M_2^+, which is a handlebody. By the equivariant loop theorem [16], $E(L^+, M^+)$ has an essential disk or an annulus. This completes the proof of Assertion 1.4.3.

The following lemma is useful to construct a tangle with hyperbolic covering property:

Lemma 1.6. *An $r(\geq 3)$-string tangle t in a 3-ball B has the hyperbolic covering property if the exterior $E(t, B)$ and the double covering space $B(t)_2$ branched along t are hyperbolic.*

Proof. For any component union $t'(\neq \emptyset)$ of t and $t'' = t - t'$, let (M, L) be the double of $(E(t'', B), t')$. Note that $E(L, M)$ is hyperbolic by Myers gluing lemma. Since each component of L is a null-homologous loop in M, L is 3-prime in M. To see that L is 4-prime in M, suppose there is an essential 4-pointed sphere for (M, L). Then there is an essential 4-pointed sphere for (B, t) or an essential $n(\leq 2)$-pointed disk D for (B, t) with $\partial D \subset \partial B - \partial t$, contradicting that $B(t)_2$ is hyperbolic. If there is an essential 2-pointed disk D for (M, L) with ∂D a component of L, then there is an essential 4-pointed sphere for (M, L), contradicting the 4-primeness of L in M. Let $\tilde{M}$ be the double of any finite regular covering space of $E(t'', B)$ branched along t' which is a finite regular covering space of M branched along L. We apply Lemma 1.4 to $\tilde{M}(=\tilde{M}_\wedge)$. Since the surface $F = \partial E(t'', B)$ lifts to an incompressible surface in $\tilde{M}$ each component of which is of genus ≥ 2 by the Riemann/Hurwitz formula and $B(t)_2$ is hyperbolic, we see from Lemma 1.4 that $\tilde{M}$ is hyperbolic. Using that $E(t, B)$ is hyperbolic, we conclude that (B, t) has the hyperbolic covering property. This completes the proof.

Here is a criterion for a link in S^3 to have the hyperbolic covering property:

Lemma 1.7. *If the double covering space S_2^3 of S^3 branched along a link*

L is hyperbolic and there is a closed connected surface F in S^3, disjoint from or transverse to L such that a component, F_2 of the lift of F to S^3_2 is incompressible, then (S^3, L) has the hyperbolic covering property.

Proof. By [4, Corollary 2.1], the hyperbolicity of S^3_2 means that $E(L, S^3)$ is hyperbolic. Let L_0, L_1 be any component unions of L with $L_1=L-L_0$. It is an easy exercise that L_1 is 3-prime, 4-prime and 2-semiprime in $E_0=E(L_0, S^3)$. Then by Lemma 1.4 all finite regular covering spaces $\tilde{E}_0$ of E_0 branched along L_1 are hyperbolic unless $L_0=\emptyset$, i.e., $E_0=S^3$. Let $\tilde{S}^3$ be any finite regular covering space of S^3 branched along L. Let $\tilde{F}$ be a component of the lift of F to $\tilde{S}^3$. Since S^3_2 is hyperbolic, the genus of F_2 is ≥ 2, so that the genus of $\tilde{F}$ is ≥ 2 by the Riemann/Hurwitz formula. Suppose $\tilde{F}$ is compressible in $\tilde{S}^3$. By the equivariant loop theorem, there is a compression disk $\tilde{D}$ for $\tilde{F}$ in $\tilde{S}^3$, equivariant under the covering transformation group of $\tilde{S}^3$. Note that the image, D of $\tilde{D}$ under the covering $\tilde{S}^3 \to S^3$ is a disk such that $D \cap L$ is $\emptyset$ or one point in int D or an arc in ∂D. This means that the lift of D to S^3_2 gives a compression disk for F_2 in S^3_2, a contradiction. Thus, $\tilde{F}$ is incompressible in $\tilde{S}^3$. Using further that any $\tilde{E}_0$ with $L_0 \neq \emptyset$ is hyperbolic, we see from Lemma 1.4 and Thurston's hyperbolization theorem that $\tilde{S}^3$ is hyperbolic. This completes the proof.

2. A slight alteration of the notion of almost identical imitation. Let $I=[-1, 1]$. For a (3, 1)-manifold pair (M, L), a reflection α in $(M, L)\times I$ is *standard* if $\alpha(x, t)=(x, -t)$ for all $(x, t)\in M\times I$, and *normal* if $\alpha(x, t)=(x, -t)$ for all $(x, t)\in\partial(M\times I)\cup U_L\times I$ for a neighborhood U_L of L in M. The term '$\alpha(x, t)$' in [8, p.744 line 25] should be reas as '(x, t)', which is a typographical error. A reflection α in $(M, L)\times I$ is said to be *isotopically standard* if hah^{-1} is the standard reflection in $(M, L)\times I$ for an $h\in \mathrm{Diff}_0((M, L)\times I, \mathrm{rel}\,\partial(M\times I)\cap U_L\times I)$ for a neghborhood U_L of L in M. The term 'rel $\partial(M\times I)\cup U_L\times I$' stated here has been written as 'rel $\partial((M, L)\times I)$' in [8, p.744 line 27] and only this point is our alteration. For a good (3, 1)-manifold pair (M, L), a reflection α in $(M, L)\times I$ is said to be *isotopically almost standard* if α is isotopically standard in $(M, L-a)\times I$ for each connected component a of L. The letter 'ϕ' in [8, p.744 line 29] should be read as 'α', a typographical error. A smooth embedding ϕ from a (3,1)-manifold pair (M^*, L^*) to $(M, L)\times I$ with $\phi(M^*, L^*)=\mathrm{Fix}(\alpha, (M, L)\times I)$ is called a *reflector* of a reflection α in $(M, L)\times I$. (M^*, L^*) is an *imitation* (or a *normal imitation*, respectively) of (M, L) if there is a reflector $\phi\colon (M^*, L^*)\to(M, L)\times I$ of a reflection (or a normal reflection, respectively) α in $(M, L)\times I$, and the composite

$$q=p_1\phi\colon (M^*, L^*)\xrightarrow{\phi}(M, L)\times I\xrightarrow{p_1}(M, L)$$

is the *imitation map*, where p_1 denotes the projection to the first factor.

DEFINITION. A (3, 1)-manifold pair (M^*, L^*) is an *almost identical imitation* of a good (3, 1)-manifold pair (M, L) if there is a reflector ϕ: $(M^*, L^*)\to(M, L)\times I$ of an isotopically almost standard normal reflection α in $(M, L)\times I$, and the composite $q=p_1\phi$: $(M^*, L^*)\to(M, L)$ is the *imitation map*.

In this definition, (M^*, L^*) is also a good (3, 1)-manifold pair and q gives a diffeomorphism from a neighborhood U_{L^*} of L^* in M^* onto a neighborhood U_L of L in M. For any components a^*, a of L^*, L with $q(a^*)=a$, there are neighborhoods $U_{L^*-a^*}$, U_{L-a} of L^*-a^*, $L-a$ in M^*, M, respectively, such that the restriction of q to $(M^*, U_{L^*-a^*})\to(M, U_{L-a})$ is homotopic to a diffeomorphism by a homotopy relative to $\partial M^*\cup U_{L^*-a^*}$. By identifying M^* with M so that $q|\partial M$ is the identity on ∂M, we denote any almost identical imitation of (M, L) by (M, L^*). Note that if (M, L^*) is an almost identical imitation of (M, L) and (M, L^{**}) is an almost identical imitation of (M, L^*), then (M, L^{**}) is an almost identical imitation of (M, L) (cf. [7, Prop. 2.1]).

Proposition 2.1. *All results of* [8] *on almost identical imitations still hold under the above definition of almost identical imitation.*

Proof. It suffices to prove Lemma 5.5 of [8] when we use the term 'isotopically standard 'in the present sense. We show the assertion that the reflection $\alpha_1^\wedge$ in $(B^\wedge, T_0^\wedge)\times I$ extending α_1 defined in [8, p.755 line 24] is isotopically standard in the present sense. Then α_1 must be normal, and our proof will be completed because we can take this α_1 as α in [8, Lemma 5.5] with the term 'isotopically standard' used in the present sense. To show this assertion, note that g appearing in [8, p.755 line 7] is in $\mathrm{Diff}_0(B^\wedge\times I, \text{rel }\partial(B^\wedge\times I)\cup U_{F^\wedge\cup F'^\wedge})$ for a neighborhood $U_{F^\wedge\cup F'^\wedge}$ of $F^\wedge\cup F'^\wedge$ in $B^\wedge\times I$. This implies that

$$h_1^* = d^\wedge h^{-1}\bar{f}(d^\wedge)^{-1} = d^\wedge g\bar{f}^{-1}g^{-1}\bar{f}(d^\wedge)^{-1}$$

belongs to $\mathrm{Diff}_0(B^\wedge\times I, \text{rel }U_{T_0^\wedge\times I}\cup\partial(B^\wedge\times I))$ for a neighborhood $U_{T_0^\wedge\times I}$ of $T_0^\wedge\times I$ in $B^\wedge\times I$. Since $\alpha_1^\wedge=d^\wedge h^{-1}\alpha_0^\wedge h(d^\wedge)^{-1}$ and $\bar{f}$, $d^\wedge$ are $\alpha_0^\wedge$-invariant, we see that

$$(h_1^*)^{-1}\alpha_1^\wedge h_1^* = \alpha_0^\wedge .$$

Thus, $\alpha_1^\wedge$ is isotopically standard. This completes the proof.

For the remaineder of this paper, we will adopt the present definition of almost identical imitation.

3. A construction of an almost identical imitation with hyperbolic covering property of a trivial tangle. We consider an almost identical imitation q: $(B, t^*)\to(B, t)$ such that t is a trivial tangle in a 3-ball B with strings a_i, $i=1, \cdots, r$, and $q|\partial B=$the identity and $E(t^*, B)$ is hyperbolic (cf. [8]). Let $a_i^*=q^{-1}(a_i)$, $i=1, 2, \cdots, r$. We consider a smooth embeddeing f from the dis-

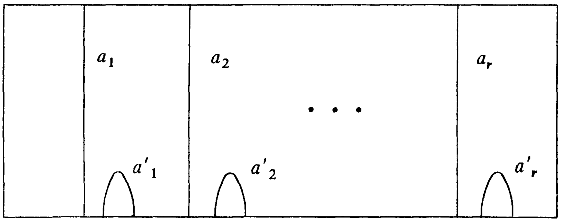

Fig. 1

joint union $\cup_{i=1}^{r} I\times I_i$ of r copies $I\times I_i$, $i=1, 2, \cdots, r$, of $I\times I$, $I=[-1, 1]$, to B such that $f(I\times 1_i)=a_i$ and $f(I\times I_i)\cap\partial B=f((\partial I)\times I_i)$. Then we call the tangle, t', in B with strings $a'_i=f(I\times(-1)_i)$, $i=1, 2, \cdots, r$, a *parallel tangle of t on the support* $\mathcal{P}=\cup_{i=1}^{r}\mathcal{P}_i$, $\mathcal{P}_i=f(I\times I_i)$. Let U^*, U be open neighborhoods of t^*, t in B such that $q^{-1}(U)=U^*$ and $q|U^*$: $U^*\to U$ is a diffeomorphism. We assume that $\mathcal{P}\subset U$. Let $\mathcal{P}^*=q^{-1}(\mathcal{P})$, $t'^*=q^{-1}(t')$, $a_i'^*=q^{-1}(a'_i)$, $i=1, 2, \cdots, r$. We illustrate a figure of the trivial tangle $t\cup t'$ in B in Fig. 1. Let F be a disk in ∂B containing $\partial a'_i$ and just one point of ∂a_i for all i, as it is indicated in Fig. 1. Let N, N' be disjoint tubular neighborhoods of t, t' in U, respectively, and $N^*=q^{-1}N$, $N'^*=q^{-1}N'$. Let $F_E=\mathrm{cl}\,(F-F\cap(N^*\cup N'^*))$, a disk with $3r$ open disks removed, and $E^*=E(t^*\cup t'^*, B)=\mathrm{cl}\,(B-(N^*\cup N'^*))$ and $F_E^c=\mathrm{cl}\,(\partial E^*-F_E)$, a disk with r open disks removed.

Lemma 3.1. *For $r(\geq 3)$, we have the following*:

(1) *E^* is irreducible and F_E, F_E^c are incompressible in E^*,*

(2) *E^* has no incompressible torus,*

(3) *There is no essential annulus A in E^* with $\partial A\cap\partial F_E=\emptyset$,*

(4) *There is no essential disk D in E^* with $\partial D\cap F_E$ one arc,*

(5) *There is no essential 4-pointed sphere for $(B, t^*\cup t'^*)$,*

(6) *There is no essential 2-pointed disk D for $(B, t^*\cup t'^*)$ with $\partial D\cap\partial F_E=\emptyset$,*

(7) *There is no essential 1-pointed disk D for $(B, t^*\cup t'^*)$.*

REMARK 3.2. The conditions (1)-(4) show that (E^*, F_E) has Property B' of [18], but the support $\mathcal{P}_i^*$ for the parallel string $a_i'^*$ of the string a_i^* gives a non-∂-parallel proper disk $D_i^*\subset E^*$ with $\partial D_i^*\cap F_E$ a union of two disjoint arcs and hence (E^*, F_E) does not have Property C$'$ of [18]. This makes more or less our argument complicated.

REMARK 3.3. Let E be a compact connected oriented 3-manifold and F, a compact surface in ∂E. In the arguments of [18], the following is a good exercise: (E, F) has Property C$'$ if and only if the double $D_F E$ of E pasting along F is simple and semi-simple (so that $D_F E$ is hyperbolic by Thurston's hyperbolization theorem).

PROOF OF LEMMA 3.1. We use that the manifold obtained from E^* by removing open collars of the proper disks $D_i^* \subset E^*$, $i=1, 2, \cdots, r$, in Remark 3.2 is diffeomorphic to the hyperbolic manifold $E(t^*, B)$. We can remove isotopically the interseations of the disks D_i^* with a sphere in int E^*, a disk $D \subset E^*$ such that $\partial D \subset F_E$ and a torus in int E^*. Hence we have (1) and (2) (The incompressibility of F_E^c is clear). For (3), suppose there is an essential annulus A in E^* with $\partial A \cap \partial F_E = \emptyset$. If $\partial A \cap F_E \neq \emptyset$ and $\partial A \cap F_E^c \neq \emptyset$, then we see from the hyperbolicity of $E(t^*, B)$ and $E(t'^*, B)$ that A splits B into two regions B_A, B'_A such that either $B_A \supset t^*$, $B'_A \supset t'^*$ or B'_A is a tubular neighborhood of a component a_i^* of t^* in B with $B'_A \supset t'^*$ and $B_A \supset t^* - a_i^*$. In this latter case, we obtain a new essential annulus A' in E^* with $\partial A' \subset \text{int } F_E$ such that $B_{A'} \supset t^*$, $B'_{A'} \supset t'^*$ by sliding the loop $\partial A \cap F_E^c$ along a tube in ∂E^* around a_i^*. If $\partial A \subset \text{int } F_E$, then A also splits B into two regions B_A, B'_A such that $B_A \supset t^*$, $B'_A \supset t'^*$ by the hyperbolicity of $E(t'^*, B)$. Suppose there is an essential annulus A in E^* with $\partial A \cap \partial F_E = \emptyset$, $B_A \supset t^*$ and $B'_A \supset t'^*$. Then since F_E and F_F^c are incompressible in E^*, it follows that after an isotopic deformation of A, the intersection $A \cap \mathcal{D}^*$ consists of proper arcs connecting the two loops in ∂A and each circle in ∂A intersects each arc of $\partial D_i^* \cap F_E$ with an odd number of points transversely. This means that $E(t^*, B)$ has an essential disk, a contradiction. This proves (3). For (4) suppose there is an essential disk $D \subset E^*$ with $\partial D \cap F_E$ one arc. ∂D can not meet any tube $\subset \partial E^*$ around any $a_i'^*$, since $E(t'^*, B)$ is hyperbolic. ∂D can not also meet any tube $\subset E^*$ around any a_i^* with an arc, since $E(t^*, B)$ is hyperbolic. If ∂D meets a tube $\subset E^*$ around some a_i^* with two disjoint arcs, D must be ∂-parallel by (3), a contradiction. This proves (4). If there is an essential 4-pointed sphere S in $(B, t^* \cup t'^*)$, then we consider the intersection $S \cap \mathcal{D}^*$. After an isotopic deformation of S in $(B, t^* \cup t'^*)$, the 3-ball B_S bounded by S in B meets $\mathcal{D}^*$ with one improper disk or two disjoint improper disks. If $B_S \cap \mathcal{D}^*$ has two disks, then S is not essential, a contradiction. If $B_S \cap \mathcal{D}^*$ has one disk and S meets only one component of $t^* \cup t'^*$, then S is not also essential. Thus, S must meet a_i^* and $a_i'^*$ for some i so that $B_S \cap \mathcal{D}_i^*$ is a disk. Since a_i^* is a trivial arc in B, we see that $B_S \cap (a_i^* \cup a_i'^*)$ is a trivial tangle in B_S, contradicting that S is essential. This proves (5). (6) is also proved by a similar method except a possibility of the existence of a 2-pointed essential disk D for $(B, t^* \cup t'^*)$ such that $\partial D \subset F$ and D meets two components a_i^*, $a_j^* (i \neq j)$ of t^*, and t'^* is contained in the 3-ball $B_D \subset B$, surrounded by D and a disk in F. Such a disk D does not also exist by the reason that for the complement $E_{i,j}$ of $t^* \cup t'^* - (a_i^* \cup a_j^*)$ in B, $F \cap E_{i,j}$ is still incompressible in $E_{i,j}$ and D would be a compressible disk in $E_{i,j}$ for $r \geq 3$. This proves (6). For (7), suppose there is an essential 1-pointed disk D for $(B, t^* \cup t'^*)$. Let a_D^* be the component of $t^* \cup t'^*$ meeting D. Since the tangle $t^* \cup t'^* - a_D^*$ is still a non-separable tangle in B, there is a 3-ball B_D, surrounded by D and a disk in ∂B, such that $B_D \cap (t^* \cup t'^*) = B_D \cap a_D^*$ and it is a 1-string tangle in B_D. Since $a_{\bar{D}}^*$ is a trivial arc in B, $B_D \cap a_{\bar{D}}^*$ is a trivial tangle in B_D and D is ∂-paralle, a contradiction. This proves (7). We complete the proof of Lemma 3.1.

Using the normal imitation $q: (B, t^* \cup t'^*) \to (B, t \cup t')$ and the disk $F \subset \partial B$, we prove the following:

Lemma 3.4. *For an $r(\geq 3)$-string trivial tangle t in a 3-ball B, there is an almost identical imitation q: $(B, t^*) \to (B, t)$ with (B, t^*) hyperbolic covering property.*

Proof. Let $\bar{q}$: $(\bar{B}, \bar{t}^* \cup \bar{t}'^*) \to (\bar{B}, \bar{t} \cup \bar{t}')$ be another copy of q: $(B, t^* \cup t'^*) \to (B, t \cup t')$. Let $\bar{F}$ be the copy of F in $\partial \bar{B}$. By identifying F with $\bar{F}$ as it is indicated in Fig. 2, we have an r-string trivial tangle t_b with strings $b_i = a_i \cup \bar{a}_i' \cup a_i' \cup \bar{a}_i$, $i=1, 2, \cdots, r$, in the 3-ball $B_b = B \cup \bar{B}$.

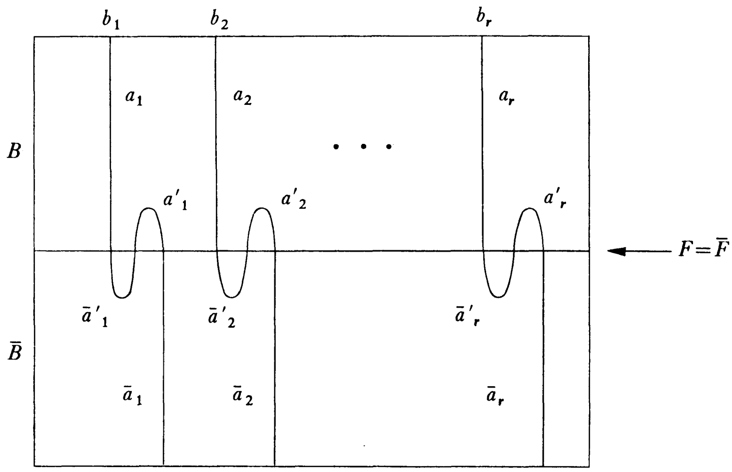

Fig. 2

Then q and $\bar{q}$ define an almost identical imitation q_b: $(B_b, t_b^*) \to (B_b, t_b)$. Let $b_i^* = q_b^{-1}(b_i) = a_i^* \cup \bar{a}_i'^* \cup a_i'^* \cup \bar{a}_i^*$, $i=1, 2, \cdots, r$. We denote the disk $\bar{F}^c (\subset \partial \bar{B})$ by F_b. Let q_d: $(B_d, t_d^*) \to (B_d, t_d)$ be an almost identical imitation obtained from two copies of q_b: $(B_b, t_b^*) \to (B_b, t_b)$ by taking the double pasting along the disk F_b. Clearly, t_d is an r-string trivial tangle. We show that (B_d, t_d^*) has the hyperbolic covering property. Let $E_b^* = E(t_b^*, B_b)$, $F_b^E = E(t_b^*, B_b) \cap F_b$. We may consider that $E_d^* = E(t_d^*, B_d)$ is the double of E_b^* pasing along F_b^E. Clearly, E_b^*, E_d^* are irreducible. If there is an essential disk $D \subset E_b^*$, then by Lemma 3.1 (1) the intersection $D \cap F_E$, where $F_E = F \cap E_b^*$, consists of proper arcs after an isotopic deformation of D, which contradicts Lemma 3.1 (4). Hence E_b^* is ∂-irreducible. Since F_b^E is incompressible in E_b^*, E_d^* is also ∂-irreducible. By Lemma 3.1 (1), (2), (3), E_b^* has no essential torus and no essential annulus A with $\partial A \cap \partial F_b^E = \emptyset$, so that E_d^* has no essential torus. By the same reason, E_d^* has no essential annulus A with $\partial A \cap \partial F_b^E = \emptyset$. Since E_b^* is ∂-irreducible, we see from this observation and an argument on the intersection of F_b^E and a proper annulus A in E_d^* that E_d^* has no essential annulus. Thus, E_d^* is hyperbolic by Thurston's hyperbolization theorem [23]. Next, we can see from Lemma 3.1 (4), (5), (6), (7) that (B_b, t_b^*) has no essential 4-pointed spheres and no essential 2-pointed disk and no essential 1-pointed disk. It is similar for (B_d, t_d^*). Then the double (M, L) of (B_d, t_d^*) is 4-prime. Since $M \cong S^3$, (M, L) is 3-prime. By Myers gluing lemma [8, Lemma 5.3], $E(L, M)$ is hyperbolic. Let M_2 be the double covering space of M branched along L. Since by Lemma 1.3 the 2-sphere ∂B_d lifts to a closed incompressible surface of genus $r-1(\geq 2)$ in M_2, we see that M_2 is not a Seifert manifold over S^2 (cf. [5, VI.3.4]). By Lemma 1.4, M_2 is hyperbolic. Hence the double covering space $(B_d)_2$ of B_d branched along t_d^* is hyperbolic. By Lemma 1.6, (B_d, t_d^*) has the hyperbolic covering property. This completes the proof.

4. The existence of an almost identical imitation with hyperbolic covering property of a good (3, 1)-manifold pair.

Lemma 4.1. *Let (M, L) be a good $(3, 1)$-manifold pair such that ∂M has no 3-pointed spheres. Then there is an almost identical imitation (M, L^*) with hyperbolic covering property of (M, L).*

Proof. We can obtain the 3-manifold M from two handlebodies H_i, $i=1, 2$, of the same genus g by pasting two compact connected surfaces $F_i \subset \partial H_i$ such that for each i,

(1) $F_i^c = \mathrm{cl}(\partial H_i - F_i)$ is a planar surface,
(2) $t_i = L \cap H_i$ is a trivial s_i-tangle in H_i with $g + s_i \geq 3$,
(3) Any component of L meets both H_1 and H_2,
(4) Any disk component of F_i^c necessarily meets at least two strings of t_i.

Our assumption that ∂M has no 3-pointed spheres needs for (4). Since H_i is the exterior of a trivial g-tangle in a 3-ball, we obtain from (2) and Lemma 3.4 an almost identical imitation (H_i, t_i^*) with hyperbolic covering property of (H_i, t_i). By (3), the imitation maps q_i: $(H_i, t_i^*) \to (H_i, t_i)$, $i=1, 2$, define an almost identical imitation map q: $(M, L^*) \to (M, L)$ with $L^* = t_1^* \cup t_2^*$. We show that (M, L^*) has the hyperbolic covering property. For any component unions L_0^*, L_1^* of L^* with $L_1^* = L^* - L_0^*$, let $E = E(L_0^*, M)$, $E_i = E \cap H_i$ and $F_i^E = E \cap F_i$. Let $\tilde{E}$ be a finite regular covering space of E branched along L_1^*, and $\tilde{E}_i$, $\tilde{F}_i^E$, the lifts of E_i, F_i^E, respectively. Each component of $\tilde{E}_i$ is hyperbolic by the hyperbolic covering property of (H_i, t_i^*). By (1), (2) and (3), $\tilde{F}_i^E$ has no disk, annulus, torus component. By (4), $(\tilde{F}_i^E)^c = \partial\tilde{E}$-int $\tilde{F}_i^E$ has no disk components. Then we see from Myers gluing lemma that $\tilde{E}$ is hyperbolic. This completes the proof.

Let an arc α be in S^2. Regarding S^2 as the 3-fold cyclic covering space of S^2 branched along $\partial\alpha$, we obtain three arcs α_i, $i=1, 2, 3$, in S^2 as the lift of α. These arcs divide S^2 into three disks D_i, $i=1, 2, 3$. Let $R=S^2\times I$, $R_i=D_i\times I$, $i=1, 2, 3$, and $I=[-1, 1]$. Let $b_i=p_i\times I$ for a point p_i in int D_i for each i and l be an $r(\geq 3)$-component proper 1-manifold in R without loop component and with $\partial l\subset S^2\times 1$ so that $l_i=l\cap R_i$ and b_i, $i=1, 2, 3$, are illustrated in Fig. 3. Let $t_b=\cup_{i=1}^3 b_i$.

Lemma 4.2. *There is a normal reflection α in $(R, l\cup t_b)\times I$ such that*

(1) *For each component a of l, the restriction of the reflection α to $(R, (l-a)\cup t_b)\times I$ is isotopically standard,*

(2) Fix $(\alpha, (R, l\cup t_b)\times I)\cong(R, l^*\cup t_b^*)$ *and the double $(W, l_W^*\cup t_W^*)$ of $(R, l^*\cup t_b^*)$ pasting along $S^2\times 1\cap(R, l^*\cup t_b^*)$ has the hyperbolic covering property.*

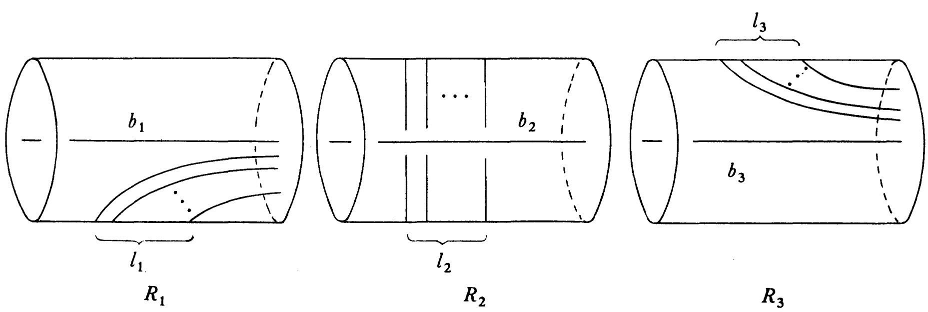

Fig. 3

Proof. First we take an isotopically almost standard reflection α_i' in $(R_i, l_i)\times I$ such that Fix $(\alpha_i', (R_i, l_i)\times I)\cong(R_i, l_i')$ has the hyperbolic covering property and the restriction of α_i' to a boundary collar of $R_i\times I$ is the standard reflection. By taking the point p_i close to ∂D_i, α_i' is also a normal reflection in $(R_i, l_i\cup b_i)\times I$ with Fix $(\alpha_i', (R_i, l_i\cup b_i)\times I)\cong(R_i, l_i'\cup b_i)$. Next taking an almost identical imitation $(R_i, l_i^*\cup b_i^*)$ with hyperbolic covering property of $(R_i, l_i'\cup b_i)$, we have a normal reflection α_i in $(R_i, l_i\cup b_i)\times I$ with Fix $(\alpha_i, (R_i, l_i'\cup b_i)\times I)\cong(R_i, l_i^*\cup b_i^*)$ such that the restriction of α_i to $(R_i, (l_i-a_i)\cup b_i)\times I$ for any component a_i in l_i is isotopically standard. The normal reflections α_i, $i=1, 2, 3$, constitute a normal reflection α in $(R, l\cup t_b)\times I$ with property (1). Let $l_+^*=l_W^*\cup t_W^*$. We show that (W, l_+^*) has the hyperbolic covering property. Let S be any 3-pointed sphere in (W, l_+^*). Since each component of l_W^* is a null-homologous loop in W and hence intersects S in even points, S must intersect a component of t_W^* in odd points. Then we see that S intersects each component of t^*_W in just one point and does not intersect l^*_W. Using that $(W_\wedge, l^*_W)$ has the hyperbolic covering property by Myers gluing lemma and $(W, t^*_W)\cong(S^2, 3\text{ points})\times I$, we conclude that S is ∂-parallel in $E(l^*_+, W)$ and (W, l^*_+) is 3-prime. Let S be any 4-pointed sphere in (W, l^*_+). Then some component b^*_W of t^*_W does not meet S. Note that the double covering space of $E(b^*_W, W)$ branched along $l^*_+ - b^*_W$ is hyperbolic by the hyperbolic covering property of $(R_i, l^*_i \cup b^*_i)$ and Myers gluing lemma. Hence S is not essential and (W, l^*_+) is 4-prime. Next, we show the following:

(#) *There is no disk D in W such that ∂D is a component of l^*_W and* $\operatorname{int} D$ *meets l^*_+ transversally with* 2 *points.*

To see (#), suppose there is such a disk D. We consider D in $W_\wedge = S^3$. Since l^*_W is an (almost identical) link imitation of a trivial link, the linking number of any two components of l^*_W in S^3 is 0. By Myers gluing lemma, note that (S^3, l^*_W) has the hyperbolic covering property. Then $\operatorname{int} D$ must intersect only one component of l^*_W with 2 points. The double covering space E_2 of the exterior $E=E(\partial D, S^3)$ branched along $l^*_W - \partial D$ is hyperbolic with boundary of two torus components. Since $D' = D \cap E$ lifts to an annulus A in E_2 spanning the two components of ∂E_2, which contradicts the hyperbolicity of E_2. This establishes (#).

Let l^*_0, l^*_1 be any component unions of l^*_+ with $l^*_1 = l^*_+ - l^*_0$. Let $E^*_0 = E(l^*_0, W)$. By the 3-primeness and 4-primeness of l^*_+ in W and (#), l^*_1 is 3-prime, 4-prime and 2-semi-prime in E^*_0. Note that $E(l^*_+, W)$ is hyperbolic by the hyperbolic covering property of $(R_i, l^*_i \cup b^*_i)$ and Myers gluing lemma. We show that for any finite regular covering space $\tilde{E}^*_0$ of E^*_0 branched along l^*_1, the spherical completion $(\tilde{E}^*_0)_\wedge$ is hyperbolic. It is obvious when $l^*_1 = \emptyset$. Let $l^*_1 \neq \emptyset$. Then we can apply Lemma 1.4 to $(\tilde{E}^*_0)_\wedge$. By Lemma 1.3, note that $(\tilde{E}^*_0)_\wedge$ is a Haken manifold with an incompressible surface lifting $S^2 \cap E^*_0$, whose component is not diffeomorphic to any sphere, disk, torus or annulus. By this reason the case (2) of Lemma 1.4 does not occur. If (3) of Lemma 1.4 occurs, then for some l_0 containing a component of t^*_W, the double covering space of E^*_0 branched along l^*_1 must be a Seifert manifold. But it is hyperbolic by the hyperbolic covering property of $(R_i, l^*_i \cup b^*_i)$ and Myers gluing lemma, which is a contradiction. Thus, $(\tilde{E}^*_0)_\wedge$ is hyperbolic and (W, l^*_+) has the hyperbolic covering property. This completes the proof.

Lemma 4.3. *For any good* (3, 1)*-manifold pair* (M, L) *there is an almost identical imitation* (M, L^*) *with hyperbolic covering property of* (M, L).

Proof. By Lemma 4.1 we may consider that ∂M has 3-pointed spheres. Let $S_j, j=1, 2, \cdots, k$, be the 3-pointed spheres in ∂M. For each j, we choose a boundary collar N_j of S_j in M so that (N_j, L_j) with $L_j = N_j \cap L$ is diffeomorphic to $(R, l \cup t_b)$ appearing in Lemma 4.2 with some $r \geq 3$ and each component of L contains a component of L_j not meeting S_j. Let $M' = \mathrm{cl}(M - \cup_{j=1}^{k} N_j)$ and $L' = L \cap M'$. By Lemma 4.1., we have an almost identical imitation (M', L'^*) with hyperbolic covering property of (M', L'). We also have a normal imitation (N_j, L_j^*) of (N_j, L_j) corresponding to $(R, l^* \cup t_b^*)$ in Lemma 4.2. Then we have an almost identical imitation (M, L^*) of (M, L) with $L^* = L'^* \cup (\cup_{j=1}^{k} L_j^*)$. Let $(L^*)_0, (L^*)_1$ be any component unions of L^* with $(L^*)_1 = L^* - (L^*)_0$. Let $(L'^*)_i = L'^* \cap (L^*)_i$, $(L_j^*)_i = L_j^* \cap (L^*)_i$, $i = 0, 1$. We denote by E, E', E_j the exteriors of $(L^*)_0, (L'^*)_0, (L_j^*)_0$ in M, M', N_j, respectively. Let $\bar{S}_j = \partial N_j - S_j$ and $F_j = \bar{S}_j \cap E_j$. Let $\tilde{E}$ be a finite regular covering space of E branched along $(L^*)_1$, and $\tilde{E}', \tilde{E}_j, \tilde{F}_j$ be the lifts of E', E_j, F_j to $\tilde{E}$, respectively. By Lemmas 4.1 and 4.2 (2), $(\tilde{E}', \cup_{j=1}^{k} \tilde{F}_j)$, $(\cup_{j=1}^{k} (\tilde{E}_j)_\wedge, \cup_{j=1}^{k} \tilde{F}_j)$ have the property C′ of [18] (cf. Remark 3.3). Hence by the original Myers gluing lemma in [18], the spherical completion $\tilde{E}_\wedge$ of $\tilde{E}$ is hyperbolic. This completes the proof.

5. Proof of Main Theorem. The following shows that for any given good (3, 1)-manifold pair, there exist infinitely many almost identical imitations of it with hyperbolic covering property and with mutually non-diffeomorphic exteriors.

Lemma 5.1. *Let (M, L) be a good $(3, 1)$-manifold pair. For any positive real number C, there are a positive number $C^+ > C$ and an infinite family $\mathfrak{F}$ of almost identical imitations (M, L^*) with hyperbolic covering property of (M, L) such that*

$$\mathrm{Vol}\, E(L^*, M) < C^+ \text{ and } \sup_{(M,L^*) \in \mathfrak{F}} \mathrm{Vol}\, E(L^*, M) = C^+ .$$

Proof. Let (B, t) be a basic tangle in (M, L) with complement (M', L'). Let O^n be an n-component trivial link in $\mathrm{int}\, B - L$. Let $(B, t^* \cup O^n)$ and (M', L'^*) be almost identical imitations with hyperbolic covering property of $(B, t \cup O^n)$ and (M', L'), respectively. Then these imitations define a normal imitation $(M, L^\# \cup O^n)$ of $(M, L \cup O^n)$, where $L^\# = t^* \cup L'^*$. By Myers gluing lemma, $(M, L^\# \cup O^n)$ has the hyperbolic covering property. By taking the $1/m$-Dehn surgery of B and M along each component of O^n, the imitations $(B, t^* \cup O^n) \to (B, t \cup O^n)$, $(M, L^\# \cup O^n) \to (M, L \cup O^n)$ induce almost identical imitations $(B, t_m^*) \to (B, t)$, $(M, L_m^*) \to (M, L)$, respectively. By an argument of [8, §5], there is an n with $\mathrm{Vol}\, E(L^\# \cup O^n, M) > C$ which we denote by C^+, and fixing such an n, we have a positive integer m_0 such that for all $m \geq m_0$, $E(L_m^*, M)$ is hyperbolic with $\mathrm{Vol}\, E(L_m^*, M) < C^+$ and $\sup_{m \geq m_0} \mathrm{Vol}\, E(L_m^*, M) = C^+$. If we take m_0 as a further large number, $E(t_m^*, B)$ and the double branched covering space $B(t_m^*)_2$ of B branched along t_m^* are hyperbolic for all $m \geq m_0$. By Lemma

1.6, (B, t_m^*) has the hyperbolic covering property for all such m. By Myers gluing lemma, (M, L_m^*) has the hyperbolic covering property for all such m. This completes the proof.

The following lemma is similar to Kojima's Lemma in [13, Lemma 5.2]:

Lemma 5.2. *Let E be a hyperbolic 3-manifold with a torus boundary componet T and $E_f = E \cup_f S^1 \times D^2$ be the adjunction 3-manifold by a diffeomorphism $f: S^1 \times \partial D^2 \to T$. Then E_f has no orientation-reversing diffeomorphism except f such that $f(p \times \partial D^2)$, $p \in S^1$, represents a finite number of homology classes of $H_1(T;Z)$.*

Proof. Since Isom E is finite, there are only finitely many (up to isotopies) orientation-reversing self-diffeomorphisms g_i of E, $i=1, 2, \cdots, r$, such that $g_i(T) = T$ and $g_{i*}(e_i) = \varepsilon_i e_i$, $\varepsilon_i = \pm 1$, for some indivisible element $e_i \in H_1(T; Z)$. Take an element e_i' of $H_1(T; Z)$ so that $\{e_i, e_i'\}$ forms a basis for $H_1(T; Z)$ with intersection number Int$(e_i, e_i') = 1$. Then we have $g_{i*}(e_i') = m_i e_i - \varepsilon_i e_i'$ for some integer m_i. By Thurston's hyperbolic Dehn surgery [23], [24] (cf. [13, Lemma 5.1]), E_f is hyperbolic with $S^1 \times 0$ the shortest geodesic except f such that $f(p \times \partial D^2)$ represents a finite number of homology classes of $H_1(T; Z)$. We consider any f such that $f(p \times \partial D^2)$ does not represent this exceptional homology classes and has $[f(p \times \partial D^2)] = b_i e_i + b_i' e_i'$ in $H_1(T; Z)$ with $b_i' \neq 0$ and $b_i/b_i' \neq -\varepsilon_i m_i/2$ for all i. Suppose such an E_f has an orientation-reversing diffeomorphism. Then by Mostow rigidity [23], [24], E_f has an orientation-reversing isometry τ. Since $\tau(S^1 \times 0) = S^1 \times 0$, τ is isotopic to a diffeomorphism g with $g(T) = T$ and $gf(p \times \partial D^2) = f(p \times \partial D^2)$. $g|E$ is isotopic to g_i for some i. Then $g_{i*}(b_i e_i + b_i' e_i') = \varepsilon_i'(b_i e_i + b_i' e_i')$ for some $\varepsilon_i' = \pm 1$, so that $\varepsilon_i b_i + b_i' m_i = \varepsilon_i' b_i$ and $\varepsilon_i' b_i' = -\varepsilon_i b_i'$. Then $b_i' = 0$ or $b_i/b_i' = -\varepsilon_i m_i/2$. This is a contradiction and completes the proof.

Lemma 5.3. *For a good (3, 1)-manifold pair (M, L), we assume the following (1), (2) and (3):*

(1) *L has no arc component and there is a double covering space M_2 of M branched along L,*

(2) *There is a family Σ of mutually disjoint 4-pointed spheres S_i, $i=1, 2, \cdots, m$, which split (M, L) into good (3, 1)-manifold pairs whose exteriors and whose double branched covering spaces associated with the covering $M_2 \to M$ are hyperbolic 3-manifolds,*

(3) *There are a subfamily Σ_0 of Σ and a finite group G acting on (M, L) such that each $S_i \in \Sigma_0$ splits (M, L) into mutually non-diffeomorphic two good (3, 1)-manifold pairs and gS_i is isotopic to S_i in (M, L) for all $g \in G$.*

Then there is an isotopy of (M, L) sending Σ_0 to a family Σ_0^* such that $gS_i^* = S_i^*$ for all $g \in G$ and $S_i^* \in \Sigma_0^*$.

Proof. Let $E = E(L, M)$ be a G-equivariant exterior and $F_i = S_i \cap E$ be a surface diffeomorphic to S^2 with 4 open disks removed. We apply a least area surface theory in [3] to the family Φ of surfaces F_i, $i=1, 2, \cdots, m$. For this purpose, we choose a G-equivariant Riemannian metric on E such that the mean curvature vector of ∂E is zero or inward pointing. By (2) note that F_i is incompressible and ∂-incompressible in E and does not split E into two components one of which is a twisted I-bundle of P^2 with two open disks removed. Then by [3] there is a family $\Phi^*=\{F_1^*, F_2^*, \cdots, F_m^*\}$ such that F_i^* is a least area (imbedded) surface in the isotopy class of F_i in E. For $i \neq j$, $F_i^* \cap F_j^* = \emptyset$ since F_i and F_j are disjoint and not isotopic in E by (2). By (1) and [1], any finite family of mutually disjoint essential 4-pointed spheres for (M, L) is isotopic, in (M, L), to a family whose members are disjoint from S_i for all $i=1, 2, \cdots, m$. This means that Φ^* is a G-equivariant family and isotopic to Φ in E. Then we have a G-equivariant family Σ^* of mutually disjoint 4-pointed spheres S_i^*, $i=1, 2, \cdots, m$, for (M, L) extending F_i^*, which is isotopic to Σ in (M, L). Let Σ_0^* be the subfamily of Σ^* sent to Σ_0 by the isotopy from Σ^* to Σ. For any $S_i^* \in \Sigma_0^*$ and any $g \in G$, $gS_i^* = S_i^*$ or $gS_i^* \cap S_i^* = \emptyset$. In the latter case, (3) means that gS_i^* is disjointedly parallel to S_i^* and S_i^* splits (M, L) into two non-diffeomorphic good (3, 1)-meanifold pairs. Since g is periodic, this is impossible. This completes the proof.

DEFINITION. Let (M, L) be a good (3, 1)-manifold pair. A 2-string tangle t in a 3-ball B is a *piece tangle* of a component a of L in (M, L) if $(B, t) \subset (\operatorname{int} M, \operatorname{int} a)$ and $B \cap (L-a) = \emptyset$ and there is an arc component e of a-int t such that $\partial e \subset \partial B$ and e is trivial in the complement of $\operatorname{int} B \cup (L - \operatorname{int} e)$ in M. This arc e is an *extra arc* of the piece tangle (B, t).

Lemma 5.4. *Let $(M, L)=(S^2$, 3 points$) \times I$. For a component b of L, let $B=E(b, M)$, a 3-ball and $L_b = L - b$. Then there is an almost identical imitation of $(M, L_b \cup b) = (M, L)$, written as $(M, L_b^* \cup b)$ such that (B, L_b^*) has the hyperbolic covering property and has no periodic map.*

Proof. Take two disjoint piece tangles (B_i, t_i), $i=1, 2$, with disjoint extra arcs of a component a of L_b in (B, L_b). By Lemma 5.1, we have two almost identical imitations of (M, L) with hyperbolic covering property and with non-diffeomorphic exteriors, written as $(M, L_b' \cup b)$, $(M, L_b'' \cup b)$. Consider (B, L_b'), (B, L_b'') as almost identical imitations with hyperbolic covering property of (B_1, t_1), (B_2, t_2), respectively. Let $M_0 = M - (\operatorname{int} B_1 \cup \operatorname{int} B_2)$ and $L_0 = M_0 \cap L$. Since (M_0, L_0) is a good (3, 1)-manifold pair, we take an almost identical imitation with hyperbolic covering property (M_0, L_0^*) of (M_0, L_0). Replacing (B_1, t_1), (B_2, t_2) and (M_0, L_0) with (B, L_b'), (B, L_b'') and (M_0, L_0^*), respectively, we obtain an almost identical imitation $(M, L_b^{\ddagger} \cup b)$ of (M, L). For a trivial knot O in $M - (L_b^{\ddagger} \cup b)$, let $(M, L_b^{\ddagger\ddagger} \cup b \cup O)$ be an almost identical imitation with hyperbolic covering property of $(M, L_b^{\ddagger} \cup b \cup O)$. By Thurston's hyperbolic Dehn surgery

[23], [24] and Lemmas 1.6, 5.2, there is a positive integer m_0 such that for all $m \geq m_0$, the $1/m$-Dehn surgery of M along O produces an almost identical imitation $(M, L_b^* \cup b)$ of $(M, L_b^\sharp \cup b)$ (and hence of (M, L)) such that (B, L_b^*) has the hyperbolic covering property and the core O' of the solid torus used for the Dehn surgery is the shortest geodesic in the complete hyperbolic manifold $B - L_b^*$ with $\partial(B - L_b^*)$ totally geodesic and $E(L_b^*, B)$ has no orientation-reversing diffeomorphism. Suppose (B, L_b^*) has a periodic map f, which must be orientation-preserving. By Mostow rigidity, the restriction of f to $B - L_b^*$ is isotopic to an isometry φ with the same period as f. Then we have a periodic map f' on (B, L_b^*) with the same period as f which coincides with φ outside a small tubular neighborhood of L_b^* in B. Since $\varphi(O') = O'$, we have $f'(O') = O'$. By the $(-1/m)$-Dehn surgery along O', we obtain from f', which is orientation-preserving, a periodic map f'' on $(B, L_b^\sharp) \cong (B, L_b^{\sharp\sharp})$ with the same period as f. Any two of (B, L_b'), (B, L_b'') or (M_0, L_0^*) are not diffeomorphic, so that by Lemma 5.3 we may have $f''(B, L_b') = (B, L_b')$ and $f''(B, L_b'') = (B, L_b'')$. This means that f'' preserves orientation-preservingly the component $a^\sharp$ of $L_b^\sharp$ corresponding to a in L_b. By Smith theory, we have $\mathrm{Fix}\,(f'', B) = a^\sharp$. Then f'' must act on the arc component $L_b^\sharp - a^\sharp$ freely, which is impossible. Thus, (B, L_b^*) has no periodic map. This completes the proof.

PROOF OF MAIN THEOREM. By Lemma 5.1, we may consider that (M, L) has the hyperbolic covering property and $\mathrm{Vol}\, E(L, M) \geq C$. Let (B, t) be a basic tangle for (M, L) with complement (M', L'). Let O be a trivial knot in $B - t$ and (B_0, t_0) be a piece tangle of O in $(B, t \cup O)$. Let $B' = B - \mathrm{int}\, B_0$ and $(t \cup O)' = B' \cap (t \cup O)$. We take the 2-string tangle (B, L_b^*) appearing in Lemma 5.4 as an almost identical imitation of (B_0, t_0). Replacing (B_0, t_0) by (B, L_b^*) and $(B', (t \cup O)')$ by an almost identical imitation with hyperbolic covering property $(B', (t \cup O)'^*)$ of it, we obtain an almost identical imitation $(B, t^\sharp \cup O)$ of $(B, t \cup O)$. Further, replacing $(B, t \cup O)$ by $(B, t^\sharp \cup O)$ and (M', L') by an almost identical imitation with hyperbolic covering property (M', L'^*) of it, we obtain a normal imitation $(M, L^\sharp \cup O)$ of $(M, L \cup O)$. By Myers gluing lemma, $E(L^\sharp \cup O, M)$ is hyperbolic. Let $C^+ = \mathrm{Vol}\, E(L^\sharp \cup O, M)$. By Lemma 1.6 and Myers gluing lemma and Thurston's hyperbolic Dehn surgery, there is a positive integer m_0 such that for all $m \geq m_0$ the $1/m$-Dehn surgery along O produces from the imitation map $(M, L^\sharp \cup O) \to (M, L \cup O)$ an almost identical imitation map $q_m\colon (M, L_m^*) \to (M, L)$ with (M, L_m^*) hyperbolic covering property. Then $C^+ > \mathrm{Vol}\, E(L_m^*, M) \geq \mathrm{Vol}\, E(L, M) \geq C$ (cf. [23], [24]), for there is a normal imitation map $E(L_m^*, M) \to E(L, M)$, which is a ∂-diffeomorphic degree one map. Note that given $N < +\infty$, we have only finitely many regular covering maps $p\colon \tilde{E}(L_0, M) \to E(L_0, M)$ branched along L_1 with covering transformation group of order $< N$ for all component unions L_0, L_1 of L with $L_1 = L - L_0$. Let $p_m^*\colon \tilde{E}_m^* \to E_m^*$ be the lift of this covering map $p\colon \tilde{E}(L_0, M) \to E(L_0, M)$ by the imitation map

q_m^E: $E_m^*=E((L_m^*)_0, M)\to E(L_0, M)$ induced from q_m. Let $O'\subset E_m^*$ be the core of the solid torus used for the Dehn surgery. By a property of imitation, p_m^* lifts O' to $\tilde{E}_m^*$ trivially and, in the spherical completion $(\tilde{E}_m^*)_\wedge$ of $\tilde{E}_m^*$, any component of the lift $\tilde{O}'$ of O' is null-homologous and any two components of $\tilde{O}'$ has the linking number zero. By the finiteness of the coverings p, we have an integer $m_1\geq m_0$ such that $\tilde{O}'$ consists of the shortest geodesics in the hyperbolic 3-manifold $(\tilde{E}_m^*)_\wedge$ for all such p and all $m\geq m_1$. By Lemma 5.2, we have an integer $m_2\geq m_1$ such that the exterior of $\tilde{O}'-O_1'$ in $(\tilde{E}_m^*)_\wedge$ has no orientation-reversing diffeomorphism for any component O_1' of $\tilde{O}'$ and any $m\geq m_2$. Let G be the covering transformation group of $\tilde{E}(L_0, M)$ and $G^*=\text{Isom}\,(\tilde{E}_m^*)_\wedge$. By Mostow rigidity, we have a monomorphism $G\to G^*$. Suppose $|G|<|G^*|$. First, we show that the action of G^* on $(\tilde{E}_m^*)_\wedge$ is orientation-preserving. To see this, note that G translates the components of $\tilde{O}'$ transitively and $g^*(\tilde{O}')=\tilde{O}'$ for all $g^*\in G^*$ and by Mostow rigidity each element of G is isotopic to an element of G^* in the exterior of $\tilde{O}'$ in $(\tilde{E}_m^*)_\wedge$. Then if G^* is not orientation-preserving, then we see that there is an orientation-reversing element g_1^* of G^* with $g_1^*(O_1')=O_1'$ for a component O_1' of $\tilde{O}'$, which contradicts our choice of m_2. Hence G^* acts on $(\tilde{E}_m^*)_\wedge$ orientation-preservingly. Then G^* acts on a pair $((\tilde{E}^\sharp)_\wedge, \tilde{O})$, obtained from the pair $((\tilde{E}_m^*)_\wedge, \tilde{O}')$ by the G-equivariant $(-1/m)$-Dhen surgery along all components of $\tilde{O}'$. Clearly, $(\tilde{E}^\sharp)_\wedge$ is obtained as the spherical completion of the covering space $\tilde{E}^\sharp$ over $E^\sharp$ whose covering map $p^\sharp$ is the lift of the covering map p: $\tilde{E}(L_0, M)\to E(L_0, M)$ by the imitation map $q_\sharp^E$: $E^\sharp=E((L^\sharp)_0, M)\to E(L_0, M)$ induced from the imitation map $(M, L^\sharp\cup O)\to(M, L\cup O)$. Further, $\tilde{O}$ is obtained as the lift of $O\subset E(L^\sharp, M)\subset E^\sharp$ by $p^\sharp$. Note that $((\tilde{E}^\sharp)_\wedge, \tilde{O})$ splits into $|G|$ copies $(B, L_b^*)_i (1\leq i\leq |G|)$ of (B, L_b^*) and one good (3, 1)-manifold pair (X, L_X), not diffeomorphic to (B, L_b^*). Since O is split from L in M, the covering monodromy $\pi_1(M-L)\to G$ extends to an epimorphism $\pi_1(M-(L\cup O))\to G\times Z_2$ sending a meridian of O to $1\in Z_2$. From the Myers gluing lemma and the hyperbolic covering property of $(B',(t\cup O)'^*)$, (M', L'^*) we see that $E(L_X, X)$ and the double covering space of X branched along L_X are hyperbolic. Since $|G|<|G^*|$, by [1] there are a non-trivial element $g^*\in G^*$ and an index i such that $g^*(B, L_b^*)_i$ is isotopic to $(B, L_b^*)_i$ in $((\tilde{E}^\sharp)_\wedge, \tilde{O})$. By Lemma 5.3, (B, L_b^*) has a periodic map, which contradicts Lemma 5.4. Hence $|G|=|G^*|$ and the monomorphism $G\to G^*$ is an isomorphism. Since $\sup_{m\geq m_2}\text{Vol}\,E(L_m^*, M)=C^+$, we complete the proof of the case when $N<+\infty$. When $L_1=\emptyset$, we have that $\tilde{O}'$ consists of the shortest geodesics in $(\tilde{E}_m^*)_\wedge$ for any finite regular covering map p: $\tilde{E}(L_0, M)\to E(L_0, M)$. Since we used N only for this assurance, we can take $N=+\infty$. This completes the proof of Main Theorem.

REMARK 5.5. In the above proof, the sphere $S=\partial B$ for the basic tangle (B, t) satisfies (3) of Main Theorem.

6. Applications. We call (M, L) a *good pair* if (M, L) is either a good $(3, 1)$-manifold pair or $L=\emptyset$ and M is a *good* 3-manifold (i.e., a compact connected oriented 3-manifold with $M_\wedge=M$). (M, L) is called a *good G-pair* if G is a finite group acting faithfully on a good pair (M, L) and orientation-preservingly on M and the G-orbit set, $(\bar{\mathrm{F}}(G, M)\cup L)/G$ of the G-set $\bar{\mathrm{F}}(G, M)\cup L$ is a good graph or $\emptyset$ in the G-orbit 3-manifold, M/G of M, where $\bar{\mathrm{F}}(G, M)$ denotes the union of the fixed point set $\mathrm{Fix}(g, M)$ for all non-trivial elements g of G. M is called a *good* 3-*manifold with G-action* if $(M, \emptyset)$ is a good G-pair (i.e., M is a good 3-manifold and G acts on M faithfully and orientation-preservingly).

Definition. A good G-pair (M^*, L^*) is a *normal* (or an *almost identical,* resp.) *G-imitation* of a good G-pair (M, L) with *G-imitation map* $q\colon (M^*, L^*)\to(M, L)$ if q is a G-map and the orbit map $\bar{q}/G\colon (M^*/G, (\bar{\mathrm{F}}(G, M^*)\cup L^*)/G)\to(M/G, (\bar{\mathrm{F}}(G, M)\cup L)/G)$ of the G-map $\bar{q}\colon (M^*, \bar{\mathrm{F}}(G, M^*)\cup L^*)\to(M, \bar{\mathrm{F}}(G, M)\cup L)$ defined by q is the spherical completion of a normal (or an almost identical, resp.) imitation map between good pairs.

When $(\bar{\mathrm{F}}(G, M^*)\cup L^*)/G$ is a graph, $\bar{q}/G$ is called a graph imitation in [8]. By a general property of imitation in [7], a normal G-imitation is a normal imitation. If $q\colon (M^*, L^*)\to(M, L)$ is an almost identical G-imitation, then the orbit map $(q|M^*)/G\colon M^*/G\to M/G$ is homotopic to a diffeomorphism. Further, if $L\not\subset\bar{\mathrm{F}}(G, M)$, then $q|M^*\colon M^*\to M$ is G-homotopic to a diffeomorphism and we can write (M^*, L^*) as (M, L^*). We first consider a good 3-minifold with free G-action.

Theorem 6.1. *For any good* 3-*manifold* M *and any positive number* C, *there are an infinite family* $\mathfrak{J}$ *of normal imitations* M^* *of* M *and a number* $C^+>C$ *such that*

(1) M^* *is a hyperbolic Haken manifold with*

$$\mathrm{Vol}\, M^*<C^+ \quad \textit{and} \quad \sup_{M^*\in\mathfrak{J}} \mathrm{Vol}\, M^* = C^+ ,$$

(2) *If* G *is the covering transformation group of any finite regular* (*unbranched*) *covering* $\tilde{M}\to M$, *then* G *is conjugate to* $\mathrm{Isom}\, \tilde{M}^*$ *in* $\mathrm{Diff}\, \tilde{M}^*$ *for the lift* $\tilde{q}\colon \tilde{M}^*\to\tilde{M}$ *of the imitation map* $q\colon M^*\to M$ *by the covering map* $\tilde{M}\to M$.

Proof. Let O be a trivial knot in int M. Take an almost identical imitation $q\colon (M, O^*)\to(M, O)$ such that (M, O^*) has the hyperbolic covering property with $\mathrm{Vol}\, E(O^*, M)>C$ and has the property (2) of Main Theorem. Let $C^+=\mathrm{Vol}\, E(O^*, M)$. Let $q_m\colon M^*_m\to M$ be a normal imitation map obtained from q by the $1/m$-Dehn surgery along O^* and O. By Thurstons' hyperbolic Dehn surgery argument, there is a positive integer m_0 such that M^*_m is hyperbolic with the core O_m^* of the solid torus used for the Dehn surgery the shortest geodesic and $\lim_{m\to\infty} \mathrm{Vol}\, M_m^* = C^+$ with $\mathrm{Vol}\, M_m^* < C^+$, for all $m \geq m_0$. Let $\tilde{q}_m: \tilde{M}_m^* \to \tilde{M}$ be the lift of q_m by any finite regular covering $\tilde{M} \to M$ with covering transformation group G. Let $G^* = \mathrm{Isom}\, \tilde{M}_m^*$. By Mostow rigidity, there is a monomorphism $G \to G^*$. Since the lift $\tilde{O}_m^*$ of O_m^* consists of shortest geodesics, G^* acts on $(\tilde{M}_m^*, \tilde{O}_m^*)$, so that G^* acts on $E(\tilde{O}_m^*, \tilde{M}_m^*)$. By (2) of Main Theorem, $\mathrm{Isom}\, E(\tilde{O}_m^*, \tilde{M}_m^*) \cong G$. By Mostow rigidity, there is a monomorphism $G^* \to G$. Hence the monomorphism $G \to G^*$ is an isomorphism. We can previosly assume that M is Haken, so that M_m^* is Haken for all $m \geq m_0$. This completes the proof.

By taking $G = \{1\}$ in Theorem 6.1, we obtain a hyperbolic version of a Haken manifold with no periodic map in [11]:

Corollary 6.2. *For any good 3-manifold M and any positive number C, there are an infinite family $\mathfrak{F}$ of normal imitations M^* of M and a number $C^+ > C$ such that M^* is a hyperbolic Haken manifold with no periodic map and*

$$\mathrm{Vol}\, M^* < C^+ \quad \textit{and} \quad \sup_{M^* \in \mathfrak{F}} \mathrm{Vol}\, M^* = C^+ .$$

Kojima showed in [14] that any finite group can be realized as the (full) isometry group of a hyperbolic 3-manifold. We can obtain a similar result:

Corollary 6.3. *For any finite group G and any positive number C, there are an infinite family $\mathfrak{F}$ of hyperbolic Haken manifolds $\tilde{M}^*$ and a number $C^+ > C$ such that*

$$\mathrm{Isom}\, \tilde{M}^* \cong G,\ \mathrm{Vol}\, \tilde{M}^* < C^+ \quad \textit{and} \quad \sup_{\tilde{M}^* \in \mathfrak{F}} \mathrm{Vol}\, \tilde{M}^* = C^+ .$$

Proof. For any finite group G, taking M to be a connected sum of some copies of $S^1 \times S^2$, we have an epimorphism $\pi_1(M) \to G$, so that G is the covering transformation group of a regular unbranched covering space $\tilde{M}$ over M. Then the proof is completed by Theorem 6.1, since $\mathrm{Vol}\, \tilde{M}^* = |G|\, \mathrm{Vol}\, M^*$ for the lift $\tilde{M}^* \to M^*$ of the covering map $\tilde{M} \to M$ by a normal imitation map $M^* \to M$ with M^* hyperbolic.

Corollary 6.4. *For any integer $N > 1$, there are N normal imitations of $S^1 \times S^1 \times S^1$ which are hyperbolic 3-manifolds with the same volume but with mutually non-isomorphic isometry groups.*

Proof. Let $G_n(p, q, r) = Z_{n^p} \oplus Z_{n^q} \oplus Z_{n^r}$ for integers $n(\geq 2), p(\geq 0), q(\geq 0), r(\geq 0)$. Let n be fixed. If an integer m is sufficiently large, then there are at least N mutually non-isomorphic groups among the groups $G_n(p, q, r)$ with $m = p + q + r$. Let $M = S^1 \times S^1 \times S^1$, and M^* a normal imitation of M in Theorem

6.1. Taking a regular covering $\tilde{M}\to M$ with covering transformation group $G_n(p, q, r)$, we obtain a normal $G_n(p, q, r)$-imitation $\tilde{M}^*$ of $\tilde{M}\cong S^1\times S^1\times S^1$ with $\mathrm{Isom}\,\tilde{M}^*\cong G_n(p, q, r)$ and $\mathrm{Vol}\,\tilde{M}^*=n^m\,\mathrm{Vol}\,M^*$. Since a normal $G_n(p, q, r)$-imitation is a normal imitation, we complete the proof.

Next, we consider a good 3-manifold with non-free G-action.

Theorem 6.5. *For any good 3-manifold M with non-free G-action and any positive number C, there are an infinite family $\mathfrak{F}$ of almost identical G-imitations M^* of M and a number $C^+>C$ such that*

(1) *M^* is a hyperbolic Haken manifold with*

$$\mathrm{Vol}\,M^*<C^+ \quad \textit{and} \quad \sup_{M^*\in\mathfrak{F}} \mathrm{Vol}\,M^* = C^+ , \quad \textit{and}$$

(2) *G is isomorphic to* $\mathrm{Isom}\,M^*$.

Proof. Since $(M/G, \bar{\mathrm{F}}(G, M)/G)$ is a spherical completion of a good (3, 1)-manifold pair (M', L'), we apply Main Theorem to (M', L') with N taking that $N>|G|$. Then we obtain an infinite family $\mathfrak{F}$ of almost identical G-imitations M^* of M with $G\cong\mathrm{Isom}\,M^*$. On volume, we can previously assmue that M is hyperbolic with $\mathrm{Vol}\,M\geq C$ by an argument of [8, §5] (cf. Lemma 5.1). Then the proof of Main Theorem assures that $\mathrm{Vol}\,M^*<\sup_{M^*\in\mathfrak{F}}\mathrm{Vol}\,M^*<+\infty$ and we can call this last number C^+. By (3) of Main Theorem, M^* is Haken. This completes the proof.

Riley [20] observed that for any hyperbolic knot k in S^3 the orintation-preserivng subgroup $\mathrm{Isom}^+\,E(k, S^3)$ of $\mathrm{Isom}\,E(k, S^3)$ is a dihedral group D_d of order $2d$ or a cyclic group Z_d of order d for some $d\geq 1$, according to whether k is invertible or not. As a consequence of Main Theorem, we obtain the following realization result of these groups:

Corollary 6.6. *For any positive integer d and any positive number C, there are two infinite families $\mathfrak{F}, \mathfrak{F}'$ of almost identical knot imitations O^* with hyperbolic covering property of a trivial knot O in S^3 and numbers $C^+, C'^+>C$ such that*

(1) *Each $O^*\in\mathfrak{F}$ is an invertible knot with*

$$\mathrm{Isom}^+\,E(O^*, S^3) = \mathrm{Isom}\,E(O^*, S^3) \cong D_d ,$$

and

$$\mathrm{Vol}\,E(O^*, S^3)<C^+ \quad \textit{and} \quad \sup_{O^*\in\mathfrak{F}} \mathrm{Vol}\,E(O^*, S^3) = C^+ ,$$

(2) *Each $O^*\in\mathfrak{F}'$ is a non-invertible knot with*

$$\mathrm{Isom}^+\,E(O^*, S^3) = \mathrm{Isom}\,E(O^*, S^3) \cong Z_d ,$$

and

$$\operatorname{Vol} E(O^*, S^3) < C'^+ \quad \textit{and} \quad \sup_{O^* \in \mathfrak{J}'} \operatorname{Vol} E(O^*, S^3) = C'^+ .$$

Proof. Let O be a great circle of S^3. Let D_d and Z_d act on (S^3, O) linearly so that $O \not\subset \bar{\mathrm{F}}(D_d, S^3)$ and $O \cap \bar{\mathrm{F}}(Z_d, S^3) = \emptyset$. Then note that if (S^3, O^*) is an almost identical D_d- or Z_d-imitation of (S^3, O), then O^* is an almost identical knot imitation of O. By Main Theorem and an argument of [8, §5], we have infinite families $\mathfrak{J}$, $\mathfrak{J}'$ of almost identical knot imitations O^* of O and numbers C^+, $C'^+ > C$ auch that $E(O^*, S^3)$ and the double covering space of S^3 branched along O^* are hyperbolic, and $\operatorname{Isom} E(O^*, S^3)$ and $\operatorname{Vol} E(O^*, S^3)$ have (1) or (2) stated above, according to $O^* \in \mathfrak{J}$ or $O^* \in \mathfrak{J}'$. Then each $O^* \in \mathfrak{J}$ is invertible and by Mostow rigidity, each $O^* \in \mathfrak{J}'$ is non-invertible. By (3) of Main Theorem and Lemma 1.7, each $O^* \in \mathfrak{J} \cup \mathfrak{J}'$ has the hyperbolic covering property. This completes the proof.

Wielenberg [26] constructed, for any integer $N > 1$, N hyperbolic links in S^3 whose exteriors have the same volume. We have a similar result regarded as a link version of Corollary 6.4.

Corollary 6.7. *For any integer $N > 1$, we have N links in S^3 with hyperbolic covering property which are normal link imitations of a fixed link in S^3, a split union of a Hopf link L_H and a trivial link, and whose exteriors have the same volume and mutually non-isomorphic isometry groups.*

Proof. Let L be a split link in S^3 of L_H and a trivial knot. Apply Main Theorem to (S^3, L). We obtain an almost identical imitation (S^3, L^*) with hyperbolic covering property of (S^3, L). Let $G_n(p, q) = Z_{n^p} \oplus Z_{n^q}$ for integers $n(\geq 2), p(\geq 0), q(\geq 0)$. For a fixed n, let m be a large positive integer such that there are at least N mutually non-isomorphic groups among the groups $G_n(p, q)$ with $m = p + q$. Let $(S^3, \tilde{L}) \to (S^3, L)$ be a regular covering branched along L_H with covering transformation group $G_n(p, q)$. Then $\tilde{L}$ is a split union of L_H and an n^m-component trivial link, whose link type is independnet of a choice of p, q with $m = p + q$. The amost identical $G_n(p, q)$-imitation $(S^3, \tilde{L}^*)$ of $(S^3, \tilde{L})$ lifting the imitation (S^3, L^*) of (S^3, L) has the property that $\tilde{L}^*$ is a hyperbolic link with $\operatorname{Isom} E(\tilde{L}^*, S^3) \cong G_n(p, q)$ and $\operatorname{Vol} E(\tilde{L}^*, S^3) = n^m \operatorname{Vol} E(L^*, S^3)$. Further, by the hyperbolic covering property of (S^3, L^*), the double covering space of S^3 branched along $\tilde{L}^*$ is hyperbolic, since it is a regular covering space of S^3 branched along L^* (with an abelian covering transformation group). By (3) of Main Theorem and Lemma 1.7, $(S^3, \tilde{L}^*)$ has the hyperbolic covering property. This completes the proof.

We remark here some results in [10] which may be interesting in comparison with Corollaries 6.4, 6.7. Namely, for any good G-pair (M, L) with $\bar{\mathrm{F}}(G, M) \cup L \neq \emptyset$ and any integer $N > 1$, we have N almost identical G-imitations (M^*, L^*)

(with $M^*=M$ if $L\neq\emptyset$) of (M, L) whose exteriors $E(L^*, M^*)$ are mutually non-diffeomorphic hyperbolic 3-manifolds with the same volume and with isometry group isomorphic to G. For any good 3-manifold M with free G-action and any integer $N>1$, we have N normal G-imitations M^* of M which are mutually non-diffeomorphic hyperbolic 3-manifolds with the same volume and with isometry group isomorphic to G.

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