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ON THE SPACE OF MINIMAL SURFACES WITH BOUNDARIES

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0. Introduction

In this paper we are concerned with the space of minimal surfaces with boundaries.

For each Jordan curve Γ in R^n the existence of minimal surfaces spanned by Γ is well known. We are interested in the behavior of such minimal surfaces as their boundary contours vary. In what case a minimal surface moves smoothly according as its boundary varies? And in what case a minimal surface vanishes or branches out into two or more minimal surfaces as a result of a perturbation of its boundary? Although we have not yet succeeded to answer such questions perfectly, we can prove a certain local property of the space of minimal surfaces, which is the first step of our attempt to solve the above mentioned problem. In this paper we shall derive it.

Let us restrict ourselves to minimal surfaces spanned by $C^{r,\alpha}$-curves ($r\geq 2$, $0<\alpha<1$). Consider a regular minimal surface f_0 spanned by a regular analytic Jordan curve. Then we shall show that in a neighborhood of f_0 the set of all minimal surfaces is isomorphic in a very natural way to an open set of an infinite dimensional Banach space (Corollary 4.6).

There exist some interesting results of other authors relating to our concerns. Using Sobolev spaces instead of $C^{r,\alpha}$, Tromba [9] proved the existence of an open and dense subset γ of all "fine embeddings" in $H^k(S^1, R^n)$ ($k\geq 7$), such that the set of minimal surfaces spanned by $\Gamma\in\gamma$ are "differentiable functions" of Γ. On the other hand Böhme [2] derived a more general result, but only in case $n=3$, employing the classical Weierstrass representation formula for minimal surfaces. More recently Böhme and Tromba [3] proved the existence of an open and dense subset $\hat{A}$ of all H^r embeddings with total curvature bounded by $\pi(s-2)$ (for integers r and s, $r\gg s\geq 7$) such that the number of all minimal surfaces spanned by $\Gamma\in\hat{A}$ is finite and these minimal surfaces are "stable" under perturbations of Γ.

If the boundary of our surface f_0 is contained in the Böhme's or Tromba's set of curves, our result may be almost trivial. But it should be noted that a regular analytic Jordan curve is not necessarily an element of the Böhme's or Tromba's set and that every minimal surface with least area spanned by an analytic curve is regular up to the boundary (Gulliver and Lesley [4]).

In proving our main result the infinite version of the inverse mapping theorem will play an essential rôle. Hence in order to make the set in question into a vector space we define a minimal surface with boundary as a more general surface than the ordinary one. Namely, the boundary of our minimal surface is not necessarily a Jordan curve but is allowed to intersect itself. But because of the locality of the assertion every resulting minimal surface of our main theorem is a solution of the usual Plateau problem, that is a minimal surface spanned by a Jordan curve.

This paper is constituted as follows: After preparations in section 1 we give in section 2 another characterization of $\mathfrak{M}$, a space of minimal surfaces. In section 3 we show some algebraic properties of Banach spaces of holomorphic mappings, one of which gives an example of infinite dimensional Banach space which is not a Hilbert space, and yet is splitted into two infinite dimensional subspaces. Finally in section 4 we prove the above mentioned result by means of our results obtained in section 3 and the inverse mapping theorem. In applying the results of section 3 it is essential that f_0 can be analytically extended across its boundary.

1. Preliminaries

In this section we give some fundamental notations and definitions.

Let Γ be the image of a continuous mapping γ of ∂B into R^n ($n\geq 2$), where

$$B=\{(u, v)\in R^2 \mid u^2+v^2<1\}$$

and ∂B is the boundary of B. Then we define a *minimal surface f spanned by* Γ as a mapping f which belongs to the class $C^0(\bar{B}, R^n)\cap C^2(B, R^n)$ and satisfies the differential equations

$$\Delta f=0\,, \tag{1.1}$$

$$|f_u|^2=|f_v|^2,\quad f_u\cdot f_v=0 \tag{1.2}$$

in B together with the following boundary condition:

There exists a topological mapping τ: $\partial B\rightarrow\partial B$ such that

$$f\mid\partial B=\gamma\circ\tau\,. \tag{1.3}$$

We call such f generically to be *minimal surface with boundary*.

Let r be an integer not smaller than 2 and let $0<\alpha<1$. The space $C^{r,\alpha}(\bar{B}, R^n)$ is a Banach space with the norm given by

$$\|f\|_{C^{r,\alpha}} = \max_{\substack{0\leq l\leq r \\ 0\leq \nu\leq l}} \sup_{w\in B} \left\| \frac{\partial^l f}{\partial u^\nu \partial v^{l-\nu}}(w) \right\| + \max_{\substack{0\leq l\leq r \\ 0\leq \nu\leq l}} \sup_{\substack{w_1, w_2\in B \\ w_1\neq w_2}} \frac{\left\| \frac{\partial^l f}{\partial u^\nu \partial v^{l-\nu}}(w_1) - \frac{\partial^l f}{\partial u^\nu \partial v^{l-\nu}}(w_2) \right\|}{\|w_1 - w_2\|^\alpha}$$

(Adams [1]).

We denote by $\mathfrak{H}$ the subset of $C^{r,\alpha}(\bar{B}, R^n)$ consisting of those mappings $f\in C^{r,\alpha}(\bar{B}, R^n)$ which satisfy only the condition (1.1) in B. Then $\mathfrak{H}$ is a closed subspace of $C^{r,\alpha}(\bar{B}, R^n)$, so $\mathfrak{H}$ itself is a Banach space.

Now let us define $\mathfrak{M}$ as a subset of $\mathfrak{H}$ which consists of all mappings $f\in\mathfrak{H}$ satisfying the condition (1.2) in B. It should be noted that if the contour Γ is of class $C^{r,\alpha}$ and regular, each minimal surface spanned by Γ belongs to $C^{r,\alpha}(\bar{B}, R^n)$ (Nitsche [7]). Therefore $\mathfrak{M}$ contains all minimal surfaces spanned by regular $C^{r,\alpha}$-boundaries.

2. Another characterization of $\mathfrak{M}$

We employ the complex variable $w=u+iv$ in place of the Cartesian coordinates (u, v) identifying R^2 with C. Let H be the space of all mappings $h\in C^{r-1,\alpha}(\bar{B}, C)$ which satisfy the condition

$$h_{\bar{w}} = 0 \tag{2.1}$$

in B. Then H becomes also a Banach space as a closed subspace of a Banach space $C^{r-1,\alpha}(\bar{B}, C)$.

For each $f=(f^1, \cdots, f^n)$ in $\mathfrak{H}$, set

$$\phi(f) = \sum_{j=1}^n (f^j{}_w{}^2) = \frac{1}{4}(|f_u|^2 - |f_v|^2) - \frac{i}{2} f_u \cdot f_v\,. \tag{2.2}$$

If g is a real-valued harmonic function in B, $g_w = \frac{1}{2}(g_u - ig_v)$ is holomorphic in B. So we can obtain the following

Lemma 2.1. *ϕ is a C^∞-mapping of $\mathfrak{H}$ into H and*

$$\mathfrak{M} = \phi^{-1}(0)\,. \tag{2.3}$$

Proof. Let $f=(f^1, \cdots, f^n)$ and $g=(g^1, \cdots, g^n)$ be arbitrary elements of $\mathfrak{H}$. Then

$$\begin{aligned}\phi(f+g) - \phi(f) &= \sum_{j=1}^n (f^j{}_w + g^j{}_w)^2 - \sum_{j=1}^n (f^j{}_w)^2 \\ &= 2\sum_{j=1}^n f^j{}_w g^j{}_w + \sum_{j=1}^n (g^j{}_w)^2\,,\end{aligned}$$

where

$$\sum_{j=1}^n (g^j{}_w)^2 = o(\|g\|_{C^{r,\alpha}})\,,$$

and $2\sum_{j=1}^{n} f^j{}_w g^j{}_w$ is linear with respect to g. Therefore ϕ is Fréchet differentiable at f and the Fréchet derivative of ϕ at f is given by the following formula:

$$D\phi(f)g = 2\sum_{j=1}^{n} f^j{}_w g^j{}_w \,. \tag{2.4}$$

From (2.4) we see the mapping $D\phi\colon \mathfrak{H} \to L(\mathfrak{H}, H)$ is linear, where $L(\mathfrak{H}, H)$ is the space of all linear mappings of $\mathfrak{H}$ into H. Hence $D\phi$ is infinitely differentiable in the sence of Fréchet, which means that ϕ is a C^∞-mapping.

(2.3) is trivial from the definitions of $\mathfrak{M}$ and ϕ. Q.E.D.

3. Some algebraic properties of spaces of holomorphic mappings

Although the following result can be derived using the Weierstrass' canonical product (Rudin [8], p. 329), we include an elementary alternative.

Theorem 3.1. *Let $R_1, \cdots, R_m$ $(m \geq 2)$ be polynomials with complex coefficients which have no common zeros to all R_j, $j=1, \cdots, m$. Then*

$$\sum_{j=1}^{m} R_j H = H \,, \tag{3.1}$$

where

$$\sum_{j=1}^{m} R_j H = \{\sum_{j=1}^{m} R_j h^j \,|\, h^j \in H,\ j = 1, \cdots, m\} \,.$$

Proof. In case that some R_j is identically zero, the other R_js vanish nowhere by the assumption, so that (3.1) is trivial. Therefore we may assume that no R_j is identically zero.

Obviously it suffices to prove the inclusion:

$$\sum_{j=1}^{m} R_j H \supset H \,. \tag{3.2}$$

To do this it is sufficient to show

$$\sum_{j=1}^{m} R_j H \ni 1 \,, \tag{3.3}$$

which will be obtained by induction with respect to the number $\nu=\nu_1+\nu_2+\cdots+\nu_m$, where ν_j is the degree of R_j $(j=1, \cdots, m)$.

The conclusion follows rather trivially for $0 \leq \nu \leq m$. Indeed if $0 \leq \nu \leq m-1$, some ν_{j_0} must vanish, which means R_{j_0} is non-zero constant. Therefore

$$\sum_{j=1}^{m} R_j H \supset R_{j_0} H \ni R_{j_0} \cdot \frac{1}{R_{j_0}} = 1 \,.$$

And if $\nu=m$, some ν_{j_0} equals 0, otherwise each ν_j equals 1 $(j=1, \cdots, m)$. In the former case, (3.3) is proved similarly as above. In the latter case, there exist some j_1 and j_2 $(1\leq j_1<j_2\leq m)$ such that the unique zero of R_{j_1} is different from that of R_{j_2} by the assumption. So we can write

$$R_{j_k}=\alpha_k(w-\beta_k)\,,\qquad k=1,2\,,$$

where none of α_1, α_2, and $\beta_1-\beta_2$ vanish. Hence

$$\begin{aligned}\sum_{j=1}^{m}R_jH&\supset\alpha_1(w-\beta_1)H+\alpha_2(w-\beta_2)H\\&\ni\alpha_1(w-\beta_1)\cdot\frac{-1}{\alpha_1(\beta_1-\beta_2)}+\alpha_2(w-\beta_2)\cdot\frac{1}{\alpha_2(\beta_1-\beta_2)}\\&=1\,.\end{aligned}$$

Next suppose that (3.3) is valid for $\nu=N\geq m$, and we prove it for $\nu=N+1$. Because $N+1>m$, at least one ν_j is not less than 2. We may assume $\nu_1\geq 2$ without loss of generality. Let β be one of zeros of R_1 and set

$$\tilde{R}_1=\frac{R_1}{w-\beta}\,.$$

From the assumption of the induction we know

$$(w-\beta)h^1+\sum_{j=2}^{m}R_jh^j=1$$

and

$$\tilde{R}_1k^1+\sum_{j=2}^{m}R_jk^j=1$$

for some $h^1,\cdots,h^m,k^1,\cdots,k^m\ni H$. Hence

$$\begin{aligned}1&=\{(w-\beta)h^1+\sum_{j=2}^{m}R_jh^j\}\{\tilde{R}_1k^1+\sum_{j=2}^{m}R_jk^j\}\\&=R_1h^1k^1+\sum_{j=2}^{m}R_j\{k^j(w-\beta)h^1+h^j(\tilde{R}_1k^1+\sum_{l=2}^{m}R_lk^l)\}\\&\in\sum_{j=1}^{m}R_jH\,.\end{aligned}$$

Q.E.D.

Corollary 3.2. *Let $h^1,\cdots,h^m$ $(m\geq 2)$ be elements of H. Assume that no point of $\bar{B}$ is a zero of every h^j and that each h^j can be extended to a holomorphic function on an open set containing $\bar{B}$. Then*

$$\sum_{j=1}^{m}h^jH=H\,.$$

Proof. If some h^j is identically zero, the result is trivial. Hence we may assume that none of h^j is identically zero.

On account of the assumption each h^j has only a finite number of zeros on $\bar{B}$. Let us denote the zeros of h^j by $w_1^j, \cdots, w_{\nu_j}^j$, where multiple zeros are repeated according to their multiplicities. And set

$$R_j(w) = \prod_{k=1}^{\nu_j} (w - w_k^j) .$$

Then these R_j satisfy the assumption of Theorem 3.1. Therefore

$$\sum_{j=1}^{m} R_j H = H .$$

So we can find some $k_0^1, \cdots, k_0^m \in H$ such that

$$\sum_{j=1}^{m} R_j k_0^j = 1 .$$

If we set

$$h_0^j = \frac{R_j}{h^j} k_0^j , \qquad j = 1, \cdots, m ,$$

then every h_0^j is contained in H and

$$\sum_{j=1}^{m} h^j h_0^j = \sum_{j=1}^{m} R_j k_0^j = 1 .$$

Consequently

$$\sum_{j=1}^{m} h^j H = H .$$

Q.E.D.

Corollary 3.3. *Let $h^1, \cdots, h^m$ $(m \geq 2)$ be elements of H. Assume that every h^j has no zeros on ∂B and that no point of B is a zero of all h^j. Then*

$$\sum_{j=1}^{m} h^j H = H .$$

Proof. Each h^j has only a finite number of zeros in B. Indeed, if h^j has an infinite number of zeros in B, the zeros of h^j has at least one accumulation point on ∂B, which contradicts the assumption.

Hence we can derive the conclusion similarly to the proof of Corollary 3.2.

The following theorem gives an example of Banach space which is splitted into two infinite dimensional subspaces. It should be noted that an infinite dimensional Banach space can not be splitted generally unless it is a Hilbert space.

Theorem 3.4. *Assume that $R_1, \cdots, R_m$ $(m \geq 2)$ satisfy the assumption of*

Theorem 3.1. *Let A be a subset of $H^m = \underbrace{H\times\cdots\times H}_{m}$ defined by*

$$A = \{h = (h^1, \cdots, h^m) \in H^m \mid \sum_{j=1}^{m} R_j h^j = 0\} .$$

Then A is a closed subspace of H^m and splits H^m. More presicely

$$H^m = A \oplus K ,$$

where

$$K = kH = \{(k^1 f, \cdots, k^m f) \mid f \in H\}$$

with some $k = (k^1, \cdots, k^m) \in H^m$ satisfying $\sum_{i=1}^{m} R_j k^j = 1$. Moreover the above k is unique up to modulo A.

Proof. By Theorem 3.1 there exists some $k = (k^1, \cdots, k^m) \in H^m$ such that $\sum_{j=1}^{m} R_j k^j = 1$. If another $\tilde{k} = (\tilde{k}^1, \cdots, \tilde{k}^m) \in H^m$ satisfies $\sum_{j=1}^{m} R_j \tilde{k}^j = 1$, it follows that

$$\sum_{j=1}^{m} R_j (k^j - \tilde{k}^j) = \sum_{j=1}^{m} R_j k^j - \sum_{j=1}^{m} R_j \tilde{k}^j = 1 - 1 = 0 ,$$

so $k - \tilde{k} \in A$. Thus the uniqueness part has been proved.

Obviously

$$H^m \supset A + K ,$$

hence we must show the opposite nclusion.

Let $h = (h^1, \cdots, h^m)$ be an arbitrary element of H^m and set

$$f = \sum_{j=1}^{m} R_j h^j .$$

If we define two functions h_1, h_2 in H^m as

$$h_1 = (h^1 - k^1 f, \cdots, h^m - k^m f)$$

and

$$h_2 = (k^1 f, \cdots, k^m f) ,$$

h_1 is contained in A and h_2 is contained in K. In fact

$$\sum_{j=1}^{m} R_j (h^j - k^j f) = \sum_{j=1}^{m} R_j h^j - f \sum_{j=1}^{m} R_j k^j = f - f = 0 .$$

Moreover

$$h = h_1 + h_2 .$$

Consequently we have proved the following relation:

$$H^m = A+K .$$

Finally assume $h \in A \cap K$. Then there exists some $f \in H$ such that $h=(k^1 f, \cdots, k^m f)$. Therefore

$$0 = \sum_{j=1}^{m} R_j(k^j f) = f \sum_{j=1}^{m} R_j k^j = f ,$$

which implies that $h=0$. Q.E.D.

4. Some local properties of $\mathfrak{M}$

In this section we return to our set $\mathfrak{M}$ of minimal surfaces, which was defined in section 1. An element of $\mathfrak{M}$ is called a regular minimal surface iff it has no branch points on $\bar{B}$. We shall show a local property of $\mathfrak{M}$ which is stated as follows:

Theorem 4.1. *Let $f_0 \in \mathfrak{M}$ be a regular minimal surface which can be analytically extended across its boundary. Then there exists an open neighborhood U of f_0 in $\mathfrak{H}$, such that $U \cap \mathfrak{M}$ is isomorphic to an open set of an infinite dimensional Banach space.*

In the course of proving Theorem 4.1 the following lemma plays a fundamental rôle.

Lemma 4.2 (Lang [5], p. 17 Corollary 2s.). *Let U be an open subset of a Banach space E and let ψ: $U \rightarrow F$ a C^k-mapping ($k \geq 1$) into a Banach space F. For an arbitrary point $x_0 \in U$ assume that the Fréchet derivative $D\psi(x_0)$ of ψ at x_0 is surjective and that its kernel splits E. Then there exists an open subset U' of U containing x_0 and an open subset V of $Ker\, D\psi(x_0)$ such that $U' \cap \psi^{-1}(0)$ and V are isomorphic.*

This is a corollary to the infinite dimensional version of the so-called inverse mapping theorem.

Now let us suppose that $f_0=(f_0^1, \cdots, f_0^n) \in \mathfrak{M}$ satisfies the assumption of Theorem 4.1. Then each f_{0w}^j is holomorphic on an open set containing $\bar{B}$, and therefore it has a finite number of zeros on $\bar{B}$, otherwise is a constant 0 on $\bar{B}$. Since f_0 is a regular surface, at least 2 functions f_0^j are non-constant. We may assume that $f_0^1, f_0^2, \cdots, f_0^m$ ($2 \leq m \leq n$) are non-constant without loss of generality. Therefore each f_{0w}^j ($j=1, 2, \cdots, m$) has a finite number of zeros on $\bar{B}$.

Let us denote the zeros of f_{0w}^j by $w_1^j, \cdots, w_{\nu_j}^j$ ($j=1, 2, \cdots, m$), where multiple zeros are repeated as many times as their order indicates. And set

$$R_j(w) = \prod_{k=1}^{\nu_j} (w-w_k^j) .$$

Now recall the mapping ϕ which was defined in section 2. Let us concern ourselves with the Fréchet derivative of ϕ at f_0, which is denoted by $D\phi(f_0)$. First we prove the following

Lemma 4.3. *$D\phi(f_0)\colon \mathfrak{H}\to H$ is surjective.*

Proof. At first we shall show the identity

$$D\phi(f_0)\mathfrak{H}=\sum_{j=1}^{m}R_jH\,. \tag{4.1}$$

Indeed, (2.4) implies trivially

$$D\phi\,(f_0)\mathfrak{H}\subset\sum_{j=1}^{m}R_jH\,.$$

So we have only to prove the opposite inclusion. Let h be an arbitrary element of $\sum_{j=1}^{m}R_jH$. Then

$$h=\sum_{j=1}^{m}R_jh^j\,,$$

with some $h^j\in H$ $(j=1, 2, \cdots, m)$. Note that the function

$$G^j(w)=\frac{R_j(w)h^j(w)}{f_{0w}^j(w)}\,,\qquad j=1, 2, \cdots, m\,,$$

is well-defined and belongs to H. The path-independent indefinite integral

$$g^j(w)=Re\int^w G^j(w)dw$$

with the normalization $g^j(0)=0$ determines a function belonging to $C^{r,\alpha}(\bar{B}, R)$ which induces a n-vector

$$g=(g^1, g^2, \cdots, g^m, \underbrace{0, \cdots, 0}_{n-m})\in\mathfrak{H}\,.$$

Since

$$D\phi(f_0)g=2\sum_{j=1}^{n}f_{0w}^j g^j{}_w=\sum_{j=1}^{m}f_{0w}^jG^j=h\,,$$

we have obtained our desired inclusion:

$$\sum_{j=1}^{m}R_jH\subset D\phi(f_0)\mathfrak{H}\,,$$

which proves (4.1).

By virtue of the relation (4.1) and Theorem 3.1 we can deduce

$$D\phi(f_0)\mathfrak{H}=H\,.$$

Q.E.D.

In the next place we shall investigate *Ker* $D\phi(f_0)$. To this end it is convenient to introduce a new function space

$$\mathfrak{H}_1 = \{p \in C^{r,\alpha}(\bar{B}, R) \mid \Delta p = 0 \text{ in } B\}$$

and to define a mapping σ: $\mathfrak{H} \to H^m \times R^m \times \mathfrak{H}_1^{n-m}$ by

$$\sigma(g) = \left(\frac{f_{0w}^1 g^1{}_w}{R_1}, \cdots, \frac{f_{0w}^m g^m{}_w}{R_m}, g^1(0), \cdots, g^m(0), g^{m+1}, \cdots, g^n\right)$$

for each $g=(g^1, g^2, \cdots, g^n) \in \mathfrak{H}$. Then σ has the following property:

Lemma 4.4. *σ is a continuous linear isomorphism of $\mathfrak{H}$ onto $H^m \times R^m \times \mathfrak{H}_1^{n-m}$. And the restriction of σ to Ker $D\phi(f_0)$ induces a linear isomorphism from Ker $D\phi(f_0)$ onto $A \times R^m \times \mathfrak{H}_1^{n-m}$, where A is a closed subspace of H^m defined by*

$$A = \{h = (h^1, \cdots, h^m) \in H^m \mid \sum_{j=1}^m R_j h^j = 0\} .$$

Proof. Obviously σ is linear. Assume $\sigma(g)=0$ for some $g=(g^1, \cdots, g^n) \in \mathfrak{H}$. Then $g^1{}_u = g^1{}_v = \cdots = g^m{}_u = g^m{}_v = g^1(0) = \cdots = g^m(0) = g^{m+1} = \cdots = g^n = 0$. Hence $g=0$, and therefore σ is injective. Let $h=(h^1, \cdots, h^m, c^1, \cdots, c^m, g^{m+1}, \cdots, g_n)$ be an element of $H^m \times R^m \times \mathfrak{H}_1^{n-m}$. Then we can define a unique harmonic function $g^j \in \mathfrak{H}_1$ which satisfies relations

$$g^j(w) = 2\, Re \int^w \frac{R_j(w) h^j(w)}{f_{0w}^j(w)} dw$$

$$g^j(0) = c^j$$

for each $j=1, 2, \cdots, m$. Put $g=(g^1, g^2, \cdots, g^n)$. Then g belongs to $\mathfrak{H}$ and $\sigma(g) = h$, which means that σ is surjective. And we have proved the first half of the lemma.

If $g=(g^1, \cdots, g^n) \in \mathfrak{H}$,

$$D\phi(f_0)g = 2\sum_{j=1}^n f_{0w}^j g^j{}_w = 2\sum_{j=1}^m R_j \frac{f_{0w}^j g^j{}_w}{R_j} .$$

So obviously g belongs to *Ker* $D\phi(f_0)$ if and only if $\sigma(g)$ belongs to $A \times R^m \times \mathfrak{H}_1^{n-m}$, and the latter half of the lemma has been proved.

Using the above Lemma 4.4 and Theorem 3.4, we see the splitting property of *Ker* $D\phi(f_0)$.

Lemma 4.5. *Ker $D\phi(f_0)$ splits $\mathfrak{H}$.*

From Theorem 3.4, A is infinite dimensional, and therefore *Ker* $D\phi(f_0)$ is also infinite dimensional. By virtue of Lemma 4.3 and Lemma 4.5 we can use Lemma 4.2 and obtain Theorem 4.1.

Corollary 4.6. *Let $f_0 \in \mathfrak{M}$ be a regular minimal surface spanned by a regular analytic Jordan curve. Then there exists an open neighborhood U of f_0 in $\mathfrak{H}$, such that $U \cap \mathfrak{M}$ is isomorphic to an open set of an infinite dimensional Banach space.*

Proof. Since $f_0 \in \mathfrak{M}$ has a regular analytic boundary, it can be extended as a minimal surface beyond ∂B (Lewy [6]). Consequently f_0 satisfies the assumption of Theorem 4.1. Q.E.D.

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