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A HYPERSURFACE OF THE IRREDUCIBLE HERMITIAN SYMMETRIC SPACE OF TYPE EIII

YOSHIO KIMURA

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Introduction

Let M be the compact irreducible Hermitian symmetric space of type *EIII*. Then M can be imbedded holomorphically and isometrically into the 26 dimensional complex projective space $P_{26}(\boldsymbol{C})$ (Nakagawa and Takagi [5]). In this note we prove the following theorem.

Theorem. *There exists a hyperplane W of $P_{26}(\boldsymbol{C})$ such that $M \cap W$ is a hypersurface of M and a Kähler C-space. Further $M \cap W = G/U$, where G is the simply connected complex simple Lie group of type F_4 and U is a parabolic Lie subgroup of G.*

It has been proved that there is no non-zero holomorphic vector field on the hypersurfaces of M with degree>1 (Kimura [3]). The theorem shows that the above result does not hold for a hypersurface of M with degree 1.

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1. The exceptional Lie algebras of type F_4 and E_6

First we shall recall Chevalley-Schafer's models of the complex simple Lie algebras of type F_4 and E_6. Denote by Q the quaternion algebra over $\boldsymbol{C}$ with the usual base $\{1, i, j, k\}$ subject to the multiplication rules:

$$i^2 = j^2 = k^2 = -1, ij = k = -ji, jk = i = -kj, ki = j = -ik .$$

Then the Cayley algebra $\mathfrak{C}$ over $\boldsymbol{C}$ can be defined as $\mathfrak{C} = Q + Q \cdot e$ (direct sum) with the following multiplication rule:

$$(a+be)(c+de) = (ac-\bar{d}b)+(da+b\bar{c})e$$

for $a,b,c,d \in Q$. Here $a \rightarrow \bar{a}$ is the usual involution in Q.

We define a 27 dimensional Jordan algebra $\mathfrak{J}$ by

$$\mathfrak{J} = \left\{ \begin{pmatrix} \xi_1 & c & \bar{d} \\ \bar{c} & \xi_2 & a \\ b & \bar{a} & \xi_3 \end{pmatrix} ; \ \xi_i \in \boldsymbol{C} (i = 1, 2, 3), a, b, c \in \mathfrak{C} \right\}$$

with the Jordan product $x \cdot y = \frac{1}{2}(xy + yx)$ for x, $y \in \mathfrak{J}$. Here xy means the usual matrix-product under the multication rule in $\mathfrak{C}$. Define elements e_1, e_2 and e_3 of $\mathfrak{J}$ by

$$e_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad e_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

For $a \in \mathfrak{C}$, we define elements a_1, a_2 and a_3 of $\mathfrak{J}$ by

$$a_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a \\ 0 & \bar{a} & 0 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 0 & 0 & \bar{a} \\ 0 & 0 & 0 \\ a & 0 & 0 \end{pmatrix}, \quad a_3 = \begin{pmatrix} 0 & a & 0 \\ \bar{a} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Then we see the following identities.

$$(1) \quad \begin{cases} e_i \cdot e_i = e_i, \quad i = 1, 2, 3, \\ e_i \cdot e_j = 0, \quad i \neq j, \quad i, j = 1, 2, 3, \\ e_i \cdot a_i = 0, \quad a \in \mathfrak{C}, \quad i = 1, 2, 3, \\ e_i \cdot a_j = \frac{1}{2} a_j, \quad a \in \mathfrak{C}, \quad i \neq j, \quad i, j = 1, 2, 3, \\ a_i \cdot b_i = (a, b)(e_j + e_k), \quad a, b \in \mathfrak{C}, \quad \{i, j, k\} \text{ a permutation of } \{1, 2, 3\}, \\ a_i \cdot b_j = \frac{1}{2}(\bar{b}\bar{a})_k, \quad a, b \in \mathfrak{C} \{i, j, k\} \text{ a cyclic permutation of } \{1, 2, 3\}, \end{cases}$$

where (a, b) is the symmetric form on $\mathfrak{C}$ defined by

$$a\bar{b} + b\bar{a} = 2(a, b)1.$$

Put $\mathfrak{J}_i = \{a_i; a \in \mathfrak{C}\}$, $i = 1, 2, 3$. Then

$$\mathfrak{J} = Ce_1 + Ce_2 + Ce_3 + \mathfrak{J}_1 + \mathfrak{J}_2 + \mathfrak{J}_3 \text{ (direct sum)}.$$

Hence every element x of $\mathfrak{J}$ can be written as

$$x = \xi_1 e_1 + \xi_2 e_2 + \xi_3 e_3 + a_1 + b_2 + c_3, \quad \xi_i \in \boldsymbol{C}, \quad a, b, c \in \mathfrak{C}.$$

We define the trace $T(x)$ of this element x by

$$T(x) = \xi_1 + \xi_2 + \xi_3.$$

Also let R_x be the right multiplication by x;

$$R_x(y) = y \cdot x .$$

We need in the later discussion the subalgebra $\mathfrak{J}_0$ of 26 dimensions:

$$\mathfrak{J}_0 = \{x \in \mathfrak{J};\ T(x) = 0\} .$$

A derivation of $\mathfrak{J}$ is a linear endomorphism D of $\mathfrak{J}$ satisfying

$$(2) \quad D(x \cdot y) = (Dx) \cdot y + x \cdot (Dy) .$$

The condition (2) for a derivation D may be written as

$$(3) \quad [D, R_x] = R_{Dx} \qquad \text{for all } x \in \mathfrak{J} .$$

Denote by $\mathfrak{D}(\mathfrak{J})$ the Lie algebra of all derivations of $\mathfrak{J}$. Then the following theorem is known.

Theorem (Chevalley and Schafer [1]). *$\mathfrak{D}(\mathfrak{J})$ (resp. $\mathfrak{D}(\mathfrak{J}) + R_0(\mathfrak{J})$) is the complex simple Lie algebra of type F_4 (resp. E_6), where $R_0(\mathfrak{J}) = \{R_x;\ x \in \mathfrak{J}_0\}$.*

Let us denote $\mathfrak{D}(\mathfrak{J}) + R_0(\mathfrak{J})$ by $\mathfrak{E}_6$ for simplicity. It is known that $\mathfrak{E}_6$ acts irreducibly on $\mathfrak{J}$ and $\mathfrak{J}$ is decomposed into two irreducible components as $\mathfrak{D}(\mathfrak{J})$-module:

$$(4) \quad \mathfrak{J} = \boldsymbol{C}1 + \mathfrak{J}_0 \text{ (direct sum)}$$

(Sechafter [6]).

Let

$$\mathfrak{D}_0 = \{\mathfrak{D}(\mathfrak{J});\ De_1 = De_2 = De_3 = 0\} ,$$

and

$$\mathfrak{D}_i = \{[R_{a_i}, R_{e_j - e_k}];\ a_i \in \mathfrak{J}_i\} ,$$

where $\{i, j, k\}$ is a permutation of $\{1, 2, 3\}$.

Then

$$\mathfrak{D}(\mathfrak{J}) = \mathfrak{D}_0 + \mathfrak{D}_1 + \mathfrak{D}_2 + \mathfrak{D}_3 \text{ (direct sum)}$$

(Schafer [6]).

It is known that $\mathfrak{D}_0$ is isomorphic to $\mathfrak{o}(8, \boldsymbol{C})$, the Lie algebra of 8 dimensional complex orthogonal group, as Lie algebra (Schafer [6]).

Proposition 1 (Jacobson [2]). *$\mathfrak{D}_0 \mathfrak{J}_i \subset \mathfrak{J}_i$, $i = 1, 2, 3$, and the representations $\mathfrak{D}_0$ on $\mathfrak{J}_1$, $\mathfrak{J}_2$ and $\mathfrak{J}_3$ are respectively equivalent to the natural representation on $\boldsymbol{C}^8$, the even half-spin representation and the odd half-spin representation of $\mathfrak{o}(8, \boldsymbol{C})$.*

Proposition 2. *For each* $i=1, 2, 3$, $\mathfrak{D}_i$ *and* $\mathfrak{J}_i$ *are isomorphic* $\mathfrak{D}_0$*-modules.*

Proof. Let $D\in\mathfrak{D}_0$. Since D satisfies the condition (3),

$$\begin{aligned}[D, [R_{a_i}, R_{e_j-e_k}]] &= [[D, R_{a_i}], R_{e_j-e_k}]+[R_{a_i}, [D, R_{e_j-e_k}]] \\ &= [R_{Da_i}, R_{e_j-e_k}]+[R_{a_i}, R_{De_l-De_k}] = [R_{Da_i}, R_{e_j-e_k}]\,,\end{aligned}$$

where $\{i,j,k\}$ is a permutation of $\{1, 2, 3\}$. q.e.d.

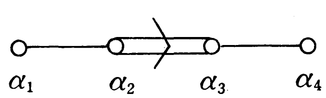
We take a Cartan subalgebra $\mathfrak{h}'$ of $\mathfrak{D}_0$ and a basis $\{H_1, H_2, H_3, H_4\}$ of $\mathfrak{h}'$. Define linear forms λ_i, $i=1, 2, 3, 4$, by

$$\lambda_i\colon \sum_{j=1}^{4}\lambda_j H_j \to \lambda_i\,.$$

We may assume that $\pm\lambda_i\pm\lambda_j$, $i<j$, are roots of $\mathfrak{D}_0$. By Propositions 1 and 2, $\mathfrak{h}'$ is a Cartan subalgebra of $\mathfrak{D}(\mathfrak{J})$ and its roots are as follows:

$$\begin{aligned}&\pm\lambda_i \pm\lambda_j\,, \quad i<j\,, \quad i,j=1,2,3,4\,,\\ &\pm\lambda_i\,, \quad i=1,2,3,4\,,\\ &\pm\Lambda'_i\,, \quad \text{where } \Lambda'_i=\tfrac{1}{2}(\lambda_1+\lambda_2+\lambda_3+\lambda_4)-\lambda_i,\ i=1,2,3,4\,,\\ &\pm\Lambda^*_i\,, \quad \text{where } \Lambda^*_1=\tfrac{1}{2}(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\,,\\ &\quad \Lambda^*_2=\tfrac{1}{2}(\lambda_1+\lambda_2-\lambda_3-\lambda_4)\,, \quad \Lambda^*_3=\tfrac{1}{2}(\lambda_1-\lambda_2+\lambda_3-\lambda_4)\,,\\ &\quad \Lambda^*_4=\tfrac{1}{2}(\lambda_1-\lambda_2-\lambda_3+\lambda_4)\,.\end{aligned}$$

Put $\alpha_1=\lambda_2-\lambda_3$, $\alpha_2=\lambda_3-\lambda_4$, $\alpha_3=\lambda_4$ $\alpha_4=-\Lambda'_1$. Then $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ is a fundamental root system and its Dynkin diagram is:



Let $\{\omega_1, \omega_2, \omega_3, \omega_4\}$ be the fundamental weights with respect to $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$. Then $\omega_4=\lambda_1$.

Now we give a Cartan subalgebra and roots of $\mathfrak{E}_6$. Set $H_5=R_{e_1}$, $H_6=R_{e_2}$, $H_7=R_{e_3}$. Then (3) and the following lemma imply that $\mathfrak{h}=\{\sum_{i=1}^{7}\lambda_i H_i;\ \lambda_i\in C,\ \lambda_5+\lambda_6+\lambda_7=0\}$ is a commutative subalgebra of $\mathfrak{E}_6$.

Lemma 1. $[R_{e_i}, R_{e_j}]=0$ *for* $1\leqq i,j\leqq 3$.

Proof. Obviously we may assume that i is not j. We have the following identities from (1).

$$[R_{e_i}, R_{e_j}]e_k = (e_k\cdot e_j)\cdot e_i-(e_k\cdot e_i)\cdot e_j = 0\,, \quad k\neq i,j\,.$$
$$[R_{e_i}, R_{e_j}]e_i = (e_i\cdot e_j)\cdot e_i-(e_i\cdot e_i)\cdot e_j = 0\,.$$

Rimilarly we get $[R_{e_i}, R_{e_j}]e_j=0$. On the other hand

$$[R_{e_i}, R_{e_j}]a_k = (a_k\cdot e_j)\cdot e_i-(a_k\cdot e_i)\cdot e_j$$
$$= \tfrac{1}{2}a_k\cdot e_i-\tfrac{1}{2}a_k\cdot e_j = \tfrac{1}{4}a_k-\tfrac{1}{4}a_k = 0\,, \quad a\in\mathfrak{C},\, k\neq i,j\,.$$
$$[R_{e_i}, R_{e_j}]a_i = (a_i\cdot e_j)\cdot e_i-(a_i\cdot e_i)\cdot e_j = \tfrac{1}{2}a_i\cdot e_i = 0\,, \quad a\in\mathfrak{C}\,.$$

Similarly we get $[R_{e_i}, R_{e_j}]a_j=0$. q.e.d.

We now claim that $ad\,\mathfrak{h}$ acts diagonally on $\mathfrak{E}_6$, which will prove that $\mathfrak{h}$ is a Cartan subalgebra of $\mathfrak{E}_6$. We shall also determine the root system of $\mathfrak{E}_6$ with respect to $\mathfrak{h}$. We define linear forms $\tilde{\lambda}_i$, $1\leqq i\leqq 7$, on $\mathfrak{h}$ by

$$\tilde{\lambda}_i\colon \sum_{i=1}^{7}\lambda_j H_i\to\lambda_i\,.$$

The definition of $\mathfrak{h}$ implies $\tilde{\lambda}_5+\tilde{\lambda}_6+\tilde{\lambda}_7=0$. Since $\tilde{\lambda}_i$, $i=1, 2, 3, 4$, are trivial extensions on $\mathfrak{h}$ of λ_i, we denote $\tilde{\lambda}_i$ by λ_i, $1\leqq i\leqq 7$, for simplicity. And we regard Λ_1' and Λ_i^*, $1\leqq i\leqq 4$, as linear forms on $\mathfrak{h}$.

We first note that the root vectors of $\mathfrak{D}_0$ with respect to $\mathfrak{h}'$ are root vectors for $\mathfrak{E}_6$ with respect to $\mathfrak{h}$, since such a root vector is a derivation D mapping e_i into 0, and so $[R_{e_i}, D]=0$, $i=1, 2, 3$. In this way we obtained the roots $\pm\lambda_i\pm\lambda_j$, $1\leqq i<j\leqq 4$, for $\mathfrak{E}_6$. Next let

$$\mathfrak{r}_{ij} = \{S_{ij} = R_{a_k}+2[R_{a_k}, R_{e_i}];\, a\in\mathfrak{C}\}\,,$$

where $\{i, j, k\}$ is a permutation of $\{1, 2, 3\}$. Then we have

$$\mathfrak{E}_6 = \{\sum_{i=5}^{7}\lambda_i H_i;\, \lambda_5+\lambda_6+\lambda_7 = 0\} +\mathfrak{D}_0+\sum_{i\neq j}\mathfrak{r}_{ij} \quad \text{(direct sum)}$$

by the following lemma.

Lemma 2. $[R_{a_i}, R_{e_i}]=0$ *and* $[R_{a_i}, R_{e_j}]=-[R_{a_i}, R_{e_k}]$ *for* $a\in\mathfrak{C}$ *and* $\{i, j, k\}$ *a permutation of* $\{1, 2, 3\}$.

Proof. By (1) we have the following identities.

$$[R_{a_i}, R_{e_i}]e_i = (e_i\cdot e_i)\cdot a_i-(e_i\cdot a_i)\cdot e_i = e_i\cdot a_i = 0\,,$$
$$[R_{a_i}, R_{e_i}]e_j = (e_j\cdot e_i)\cdot a_i-(e_j\cdot a_i)\cdot e_i = -\tfrac{1}{2}a_i\cdot e_i = 0\,,$$
$$[R_{a_i}, R_{e_i}]b_i = (b_i\cdot e_i)\cdot a_i-(b_i\cdot a_i)\cdot e_i = -(b, a)(e_j+e_k)\cdot e_i = 0\,, \qquad b\in\mathfrak{C}\,,$$
$$[R_{a_i}, R_{e_i}]b_j = (b_j\cdot e_i)\cdot a_i-(b_j\cdot a_i)\cdot e_i = \tfrac{1}{2}b_j\cdot a_i-\tfrac{1}{2}b_j\cdot a_i = 0\,, \qquad b\in\mathfrak{C}\,.$$

Therefore $[R_{a_i}, R_{e_i}]=0$. Since $R_{e_i}+R_{e_2}+R_{e_3}=1_{\mathfrak{J}}$, we have $[R_{a_i}, R_{e_j}]+[R_{ai}, R_{e_k}]=0$. q.e.d.

Lemma 3. $[H, S_{a_{il}}]=-\frac{1}{2}(\lambda_{i+4}-\lambda_{j+4})S_{a_{ij}}$ *for* $H=\sum_{k=5}^{7}\lambda_k H_k$.

Proof. Since $\mathfrak{J}$ is a Jordan algebra, we have

$$[[R_x, R_y], R_z] = R_{[R_x, R_y]z} \qquad \text{for } x, y, z \in \mathfrak{J}$$

(Schafer [6]). By this fact and Lemma 2, we have

$$[R_{e_k}, S_{a_{ij}}] = [R_{e_k}, R_{a_k} + 2[R_{a_k}, R_{e_i}]] = -2R_{[R_{a_k}, R_{e_i}]e_k} = 0\,,$$
$$[R_{e_i}, S_{a_{ij}}] = [R_{e_i}, R_{a_k} + 2[R_{a_k}, R_{e_i}]] = -[R_{a_k}, R_{e_i}] - 2R_{[R_{a_k}, R_{e_i}]e_i}\,,$$

where $k \neq i, j$. On the other hand,

$$[R_{a_k}, R_{e_i}]e_i = (e_i \cdot e_i) \cdot a_k - (e_i \cdot a_k) \cdot e_i = e_i \cdot a_k - \tfrac{1}{2} a_k \cdot e_i = \tfrac{1}{2} a_k - \tfrac{1}{4} a_k = \tfrac{1}{4} a_k\,.$$

Hence $[R_{e_i}, S_{a_{ij}}] = -\frac{1}{2} S_{a_{ij}}$. Since $R_{e_1} + R_{e_2} + R_{e_3} = 1_{\mathfrak{J}}$ and $[R_{e_k}, S_{a_{ij}}] = 0$, we get $[R_{e_i} + R_{e_j}, S_{a_{ij}}] = 0$.
Therefore $[R_{e_j}, S_{a_{ij}}] = \frac{1}{2} S_{a_{ij}}$. q.e.d.

Let $H \in \mathfrak{h}' \subset \mathfrak{D}_0$. Then,

$$[H, S_{a_{ij}}] = [H, R_{a_k}] + 2[H, [R_{a_k}, R_{e_i}]] = R_{Ha_k} + 2[R_{Ha_k}, R_{e_i}]\,, \quad k \neq i, j\,.$$

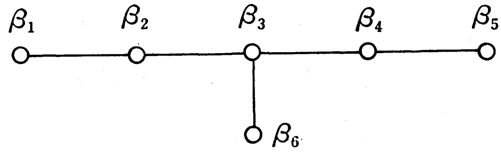
It follows that if $a_k \in \mathfrak{J}_k$ is a weight vector for the representation of $\mathfrak{D}_0$ on $\mathfrak{J}_k$, then the corresponding $S_{a_{ij}}$ will be a root vector for $\mathfrak{h}$. In this way we obtain the following roots:

$$\pm\lambda_i \pm \tfrac{1}{2}(\lambda_6 - \lambda_7),\ \pm\Lambda_i' \pm \tfrac{1}{2}(\lambda_5 - \lambda_7),\ \pm\Lambda_i^* \pm \tfrac{1}{2}(\lambda_5 - \lambda_6)\,,$$

where $i = 1, 2, 3, 4$. Thus we have shown that *ad* $\mathfrak{h}$ acts diagonally on $\mathfrak{E}_6$, and obtained all roots of $\mathfrak{E}_6$ with respect to $\mathfrak{h}$. We may take a fundamental root system $\{\beta_1, \cdots, \beta_6\}$ as follows:

$$\beta_1 = -\Lambda_1' + \tfrac{1}{2}(\lambda_7 - \lambda_5)\,, \quad \beta_2 = \lambda_4 + \tfrac{1}{2}(\lambda_6 - \lambda_7)\,,$$
$$\beta_3 = \lambda_3 - \lambda_4\,, \quad \beta_4 = \lambda_4 - \tfrac{1}{2}(\lambda_6 - \lambda_7)\,,$$
$$\beta_5 = -\Lambda_1' - \tfrac{1}{2}(\lambda_7 - \lambda_5)\,, \quad \beta_6 = \lambda_2 - \lambda_3\,.$$

Then the Dynkin diagram of $\{\beta_1, \cdots, \beta_6\}$ is:



Let $\{\tilde{\omega}_1, \cdots, \tilde{\omega}_6\}$ be the fundamental weights with respect to $\{\beta_1, \cdots, \beta_6\}$. Then $\tilde{\omega}_1 = \lambda_1 + \frac{1}{2}(\lambda_6 + \lambda_7)$.

2. Proof of the theorem

By (1) and Proposition 1, we have the following propositions.

Proposition 3. *The weights of the irreducible representation of* $\mathfrak{E}_6$ *on* $\mathfrak{J}$ *are the followings*:

$$\lambda_5, \lambda_6, \lambda_7, \pm\lambda_i+\tfrac{1}{2}(\lambda_6+\lambda_7), \pm\Lambda_i'+\tfrac{1}{2}(\lambda_5+\lambda_7), \pm\Lambda_i^*+\tfrac{1}{2}(\lambda_5+\lambda_6)\,,$$

where $i=1, 2, 3, 4$. *Further the highest weight among these is* $\tilde{\omega}_1=\lambda_1+\frac{1}{2}(\lambda_6+\lambda_7)$.

Proposition 4. *The weights of the irreducible representation of* $\mathfrak{D}(\mathfrak{J})$ *on* $\mathfrak{J}_0$ *are the followings*:

$$0, \pm\lambda_i, \pm\Lambda_i', \pm\Lambda_i^*\,, \qquad i=1, 2, 3, 4\,.$$

Further the highest weight among these is $\omega_4=\lambda_1$.

Let $v\in\mathfrak{J}_1$ be an eigen vector belonging to the highest weight ω_4 of the representation of $\mathfrak{D}(\mathfrak{J})$ on $\mathfrak{J}_0$. By Propositions 3 and 4, v is also a highest weight vector of the representation of $\mathfrak{E}_6$ on $\mathfrak{J}$. Therefore v is a common highest weight vector of the above two representations.

Let E_6 be a simply connected complex Lie group with Lie algebra $\mathfrak{E}_6$ and let F_4 be a connected Lie subgroup of E_6 with Lie algebra $\mathfrak{D}(\mathfrak{J})$. Then there exists the irreducible representation $(f_{\omega_1}, \mathfrak{J})$ of $\mathfrak{E}_6$ in $\mathfrak{J}$ which induces the representation of $\mathfrak{E}_6$ on $\mathfrak{J}$. Denote by $P(\mathfrak{J})$ the complex projective space consisting of all 1-dimensional subspaces of $\mathfrak{J}$. Then E_6 acts canonically on $P(\mathfrak{J})$ via the representation $(f_{\omega_1}, \mathfrak{J})$. The weight space $\boldsymbol{C}v$ in $\mathfrak{J}$ for the highest weight $\tilde{\omega}_1$ being of dimension 1, it is an element of $P(\mathfrak{J})$. It is known that the isotropy subgroup U of E_6 at $\boldsymbol{C}v$ is a parabolic subgroup of E_6 and the quotient manifold E_6/U is fully imbedded in $P(\mathfrak{J})$ as the orbit of $\boldsymbol{C}v$ (Nakagawa and Takagi [5]). And E_6/U is compact irreducible Hermitian symmetric space of type *EIII*.

The restriction to F_4 of f_{ω_1} leaves $\mathfrak{J}_0$ invariant. By Proposition 4, the representation of F_4 on $\mathfrak{J}_0$ induced by f_{ω_1} is irreducible (with highest weight ω_4). Let $P(\mathfrak{J}_0)$ be the complex projective space consisting of all 1-dimensional subspaces of $\mathfrak{J}_0$. Then F_4 acts canonically on $P(\mathfrak{J}_0)$. Similarly as for the above case, the isotropy subgroup U' of F_4 at $\boldsymbol{C}v\in P(\mathfrak{J}_0)$ is a parabolic subgroup of F_4 and the quotient manifold F_4/U' is a Kähler C-space imbedded in $P(\mathfrak{J}_0)$ as the orbit of $\boldsymbol{C}v$. Therefore F_4/U' is contained in $E_6/U\cap P(\mathfrak{J}_0)$. It is known that dim $E_6/U=16$ and dim $F_4/U'=15$ (Nakagawa and Takagi [5]). Since E_6/U is fully imbedded in $P(\mathfrak{J})$, E_6/U is not contained in $P(\mathfrak{J}_0)$, namely, $E_6/U\cap P(\mathfrak{J}_0)\neq \mathbf{E}_6/U$. Since E_6/U is connected, it follows that dim $E_6/U\cap P(\mathfrak{J}_0)$ $=15=$dim F_4/U'. The fact that E_6/U is connected implies easily that $E_6/U\cap$

$P(\mathfrak{J}_0)$ is connected (Milnor [4]). Therefore $F_4/U'=E_6/U \cap P(\mathfrak{J}_0)$. Thus we have proved our theorem.

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