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ON THE HYPERSURFACES OF HERMITIAN SYMMETRIC SPACES OF COMPACT TYPE

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Introduction

Let M be an irreducible Hermitian symmetric space of compact type and let L be a holomorphic line bundle over M. We denote by $\Omega^p(L)$ the sheaf of germs of L-valued holomorphic p-forms on M. In this paper we study the cohomology groups $H^q(M, \Omega^p(L))$. Further, applying the results so far obtained, we shall consider the hypersurfaces of M.

The paper divided into three parts. § 1 is devoted to recalling basic notions and results which are necessary in the following. In § 2, for the cases that M is an irreducible Hermitian symmetric space of compact type BDI, EIII or EVII, we obtain the theorems analogous to the following theorem of Bott [3] for $M=P_n(\boldsymbol{C})$.

Theorem. *Let E be the hyperplane bundle over an n-dimensional complex projective space $P_n(\boldsymbol{C})$. Then the group $H^q(P_n(\boldsymbol{C}), \Omega^p(E^k))$ vanishes except for the following cases:*

(i) $p=q$ *and* $k=0$, (ii) $q=0$ *and* $k>p$, (iii) $q=n$ *and* $k<p-n$,
where $E^k=E\otimes\cdots\otimes E$ *(k factors).*

Further we shall discuss when the group $H^q(M, \Omega^p(L))$ vanishes for any irreducible Hermitian symmetric space of compact type for $p=0, 1$. These results are obtained by analyzing in detail structure of Lie algebras and their Weyl groups and applying the generalized Borel-Weil theorem.

Let V be a hypersurface of M. Denote by Θ (resp. Ω) the sheaf of germs of holomorphic vector fields (resp. holomorphic functions) on V. In § 3 we study the cohomology groups $H^q(V, \Theta)$ and $H^q(V, \Omega)$ using the results in § 2. And we find that if M is BDI, EIII or EVII, one has

$$H^0(V, \Theta) = 0$$

for the hypersurfaces V of M except for a certain special case (Theorem 8).

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1. Preparations

1.1. The generalized Borel-Weil theorem. In this section we recall the generalized Borel-Weil theorem in a form convenient for our purpose.

Let G be a simply connected complex semi-simple Lie group and let U be a parabolic Lie subgroup of G. Then the quotient manifold $M=G/U$ is a Kähler C-space, that is, a simply connected compact complex homogeneous manifold admitting a Kähler metric. And every Kähler C-space is obtained in this way (Wang [10]). Let $\mathfrak{g}$ be the Lie algebra of G and $\mathfrak{h}$ a Cartan subalgebra of $\mathfrak{g}$. We denote by Δ the root system of $\mathfrak{g}$ with respect to $\mathfrak{h}$. We shall identify a linear form λ on $\mathfrak{h}$ with the element H_λ of $\mathfrak{h}$ defined by

$$\lambda(H)=(H_\lambda, H) \qquad \text{for} \quad H\in\mathfrak{h},$$

where $(\ ,\ )$ is the Killing form of $\mathfrak{g}$. We fix a linear order on the real form $\mathfrak{h}_0=\{\alpha\in\Delta\}_R$ of $\mathfrak{h}$. Let Δ^+ (resp. Δ^-) be the set of all positive (resp. negative) roots. Let $\Pi=\{\alpha_1, \cdots, \alpha_l\}$ be the fundamental root system and Π_1 be a subsystem of Π, where l is the rank of $\mathfrak{g}$. We put

$$\Delta_1=\{\alpha\in\Delta;\ \alpha=\sum_{i=1}^{l} m_i\alpha_i,\ m_j=0 \quad \text{for any} \quad \alpha_j\notin\Pi_1\}$$

$$\Delta(\mathfrak{n}^+)=\{\beta\in\Delta;\ \beta=\sum_{i=1}^{l} m_i\alpha_i,\ m_j>0 \quad \text{for some} \quad \alpha_j\notin\Pi_1\}$$

$$\Delta(\mathfrak{u})=\Delta_1\cup\Delta(\mathfrak{n}^+).$$

Define Lie subalgebras $\mathfrak{g}_1$, $\mathfrak{n}^+$ and $\mathfrak{u}$ of $\mathfrak{g}$ by

$$\mathfrak{g}_1=\mathfrak{h}+\sum_{\alpha\in\Delta_1}\mathfrak{g}_\alpha$$

$$\mathfrak{n}^+=\sum_{\beta\in\Delta(\mathfrak{n}^+)}\mathfrak{g}_\beta$$

$$\mathfrak{u}=\mathfrak{h}+\sum_{\alpha\in\Delta(\mathfrak{u})}\mathfrak{g}_\alpha,$$

where $\mathfrak{g}_\alpha$ is the root space corresponding to $\alpha\in\Delta$. Then $\mathfrak{g}_1$ (resp. $\mathfrak{n}^+$) is a reductive (resp. nilpotent) subalgebra and $\mathfrak{u}=\mathfrak{g}_1+\mathfrak{n}^+$ (semi-direct). We denote by U the connected Lie subgroup of G with Lie algebra $\mathfrak{u}$. Then U is a parabolic subgroup of G, and $M=G/U$ is a Kähler C-space.

We denote by D (resp. D_1) the set of dominant integral forms of $\mathfrak{g}$ (resp. $\mathfrak{g}_1$). Let $\xi\in D_1$ and choose an irreducible representation $(\rho^1_{-\xi}, W_{-\xi})$ of $\mathfrak{g}_1$ with the lowest weight $-\xi$. We may extend it to a representation of $\mathfrak{u}$ so that its restriction to $\mathfrak{n}^+$ is trivial, which will be denoted by $(\rho_{-\xi}, W_{-\xi})$. Since any irreducible representation of $\mathfrak{u}$ is trivial on $\mathfrak{n}^+$, we may call $(\rho_{-\xi}, W_{-\xi})$ the irreducible representation of $\mathfrak{u}$ with the lowest weight $-\xi$. Moreover there exists a representation of U which induces the representation $(\rho_{-\xi}, W_{-\xi})$ of $\mathfrak{u}$, and we denote it by $(\tilde{\rho}_{-\xi}, W_{-\xi})$. This representation $(\tilde{\rho}_{-\xi}, W_{-\xi})$ defines the holomorphic vector bundle $E_{-\xi}$ over M associated to the principal bundle $G \rightarrow M$ by the representation $\tilde{\rho}_{-\xi}$ of U.

For a holomorphic vector bundle E over a complex manifold, we denote by $\Omega(E)$ the sheaf of germs of local holomorphic sections of E. Let W be the Weyl group of $\mathfrak{g}$ and Δ_1^+ the set of all positive roots of Δ_1. We define a subset W^1 of W by

$$W^1 = \{\sigma \in W;\ \sigma^{-1}(\Delta_1^+) \subset \Delta^+\} ,$$

For any set S, we denote by $\sharp S$ the cardinality of S. The index $n(\sigma)$ of $\sigma \in W$ is then defined by

$$n(\sigma) = \sharp(\sigma(\Delta^+) \cap \Delta^-) .$$

We denote by δ the half of sum of all positive roots of $\mathfrak{g}$.

Theorem of Bott [3](cf. Kostant [7]). *Under the notations defined above let* $\xi \in D_1$. *Then if* $\xi + \delta$ *is not regular,*

$$H^j(M, \Omega E_{-\xi}) = 0 \qquad \text{for all} \quad j = 0, 1, \cdots .$$

If $\xi + \delta$ *is regular,* $\xi + \delta$ *is expressed uniquely as* $\xi + \delta = \sigma(\lambda + \delta)$, *where* $\lambda \in D$ *and* $\sigma \in W^1$, *and*

$$H^j(M, \Omega E_{-\xi}) = 0 \qquad \text{for all} \quad j \neq n(\sigma) ,$$
$$\dim H^{n(\sigma)}(M, E_{-\xi}) = \dim V_{-\lambda} ,$$

where $(\rho_{-\lambda}, V_{-\lambda})$ *is the irreducible representation of* G *with the lowest weight* $-\lambda$.

We prove the following lemmas to restate this theorem in a form suitable for our purpose.

Lemma 1. *Let* $\xi \in D_1$. *If*

$$(\xi + \delta, \beta) \neq 0 \qquad \text{for all} \quad \beta \in \Delta(\mathfrak{n}^+) ,$$

then $\xi + \delta$ *is regular.*

Proof. Let α be any root of Δ_1^+. Then we have $(\xi, \alpha) \geqq 0$ and $(\delta, \alpha) > 0$, so that $(\xi + \delta, \alpha) > 0$. Since $\Delta^+ = \Delta_1^+ \cup \Delta(\mathfrak{n}^+)$, we get

$$(\xi + \delta, \gamma) \neq 0 \qquad \text{for all} \quad \gamma \in \Delta^+ .$$

q.e.d.

Lemma 2. *Let* $\xi \in D_1$. *Assume that there are* $\lambda \in D$ *and* $\sigma \in W^1$ *such that* $\xi + \delta = \sigma(\lambda + \delta)$. *Then*

$$n(\sigma) = \sharp\{\beta \in \Delta(\mathfrak{n}^+);\ (\xi + \delta, \beta) < 0\} .$$

Proof. Since $\sigma^{-1}(\Delta_1^+)\subset\Delta^+$, we have

$$n(\sigma)=\#\{\beta\in\Delta(\mathfrak{n}^+);\ \sigma^{-1}(\beta)<0\}\ .$$

By the assumption

$$(\xi+\delta,\alpha)=(\lambda+\delta,\sigma^{-1}(\alpha))\qquad \text{for}\quad \alpha\in\Delta\ .$$

Since $\lambda+\delta$ is dominant and regular, $\sigma^{-1}(\alpha)$ is negative if and only if $(\lambda+\delta,\sigma^{-1}(\alpha))$ is negative. The conclusion now follows from these observations.

Theorem of Bott may be restated as follows by these lemmas.

Theorem 1. *Let $\xi\in D_1$. If there exists a root α of $\Delta(\mathfrak{n}^+)$ such that $(\xi+\delta,\alpha)=0$, then*

$$H^j(M,\Omega E_{-\xi})=0\qquad \textit{for}\quad j=0,1,\cdots\ .$$

If there exists no root β of $\Delta(\mathfrak{n}^+)$ such that $(\xi+\delta,\beta)=0$, then

$$H^j(M,\Omega E_{-\xi})=0\qquad \textit{for}\quad j\neq q\ ,$$

and

$$H^q(M,\Omega E_{-\xi})\neq 0\ ,$$

where $q=\#\{\beta\in\Delta(\mathfrak{n}^+);\ (\xi+\delta,\beta)<0\}$.

1.2. Kostant s results. We denote by $T(M)$ the holomorphic tangent bundle of M and denote by $T(M)^*$ its dual bundle. Let L be a holomorphic line bundle over M. Then it is easy to see that $\Omega^p(L)$ coincides with $\Omega(\overset{p}{\Lambda}T(M)^*\otimes L)$, where $\overset{p}{\Lambda}T(M)^*$ is p-th exterior product of $T(M)^*$. Since any holomorphic line bundle over a Kähler C-space M is associated to the principal bundle $G\to M$ by a representation of U (Murakami [8]), we may put $L=E_{-\xi}$ for $\xi\in D_1$. It is known $\mathfrak{n}^+$ is invariant by the adjoint representation of U on $\mathfrak{g}$. Hence p-th exterior product of $\mathfrak{n}^+$ has a U-module structure. Since $\mathfrak{n}^+$ may be identified with the cotangent space of M at U, $\overset{p}{\Lambda}T(M)^*\to M$ coincides with the holomorphic vector bundle associated to the principal bundle $G\to M$ by the representation of U on $\overset{p}{\Lambda}\mathfrak{n}^+$.

From now on we assume that $M=G/U$ is a Hermitian symmetric space of compact type. Then $\mathfrak{n}^+$ is abelian. For any integer $p\geqq 0$, put

$$W^1(p)=\{\sigma\in W^1;\ n(\sigma)=p\}\ .$$

Kostant [7] has proved that $\overset{p}{\Lambda}\mathfrak{n}^+$ is decomposed into direct sum:

$$\overset{p}{\Lambda}\mathfrak{n}^+=\sum_{\sigma\in W^1(p)}(\overset{p}{\Lambda}\mathfrak{n}^+)_{-(\sigma\delta-\delta)}\qquad (\text{as } U\text{-module}),\tag{1}$$

where $(\Lambda\mathfrak{n}^+)_{-(\sigma\delta-\delta)}$ denotes an irreducible U-module with the lowest weight

$-(\sigma\delta-\delta)$.

Let S be a holomorphic U-module represented as follows:

$$S=W_{-\xi_1}+\cdots+W_{-\xi_l} \qquad \text{for} \quad \xi_i\in D_1\,.$$

Denote by E_S the holomorphic vector bundle over M associated to the principal bundle $G\rightarrow M$ by the representation of U on S.

Proposition 1. *Under the notations introduced above we have*

$$\dim H^j(M,\,\Omega E_S)=\sum_{i=1}^{l}\dim H^j(M,\,\Omega E_{-\xi_i}) \qquad \textit{for} \quad j=0,\,1,\,\cdots.$$

We recall the results of Bott [3] which are necessary to prove the above proposition.

Theorem A. *Let S be a holomorphic U-module, and let V be a holomorphic G-module. If E_S is the holomorphic vector bundle over M associated to the principal bundle $G\rightarrow M$ by the representation of U on S, then*
multiplicity of V in $H^j(M,\,\Omega E_S)=\dim H^j(\mathfrak{u},\,\mathfrak{g}_1,\,\mathrm{Hom}\,(V,\,S))$ *for* $j=0,\,1,\,\cdots$, *where* $H^j(\mathfrak{u},\,\mathfrak{g}_1,\,\mathrm{Hom}\,(V,\,S))$ *denotes the j-th relative cohomology group of Lie algebras* $\mathfrak{u}$, $\mathfrak{g}_1$ *with coefficients in the u-module* $\mathrm{Hom}\,(V,\,S)$.

For $\mathfrak{g}_1$-module T, $T^{\mathfrak{g}_1}$ denotes the subspace of T annihilated by all $X\in\mathfrak{g}_1$.

Theorem B. *Let F be a* $\mathfrak{u}$*-module which, considered as* $\mathfrak{g}_1$*-module, is completely reducible. Then*

$$\dim H^j(\mathfrak{u},\,\mathfrak{g}_1,\,F)=\dim H^j(\mathfrak{n}^+,\,F)^{\mathfrak{g}_1}\,.$$

Proof of Proposition 1. Let V be a holomorphic G-module. Then by Theorems A and B, we have
multiplicity of V in $H^j(M,\,\Omega E_S)=\dim H^j(\mathfrak{n}^+,\,\mathrm{Hom}\,(V,\,S))^{\mathfrak{g}_1}$ for $j=0,\,1,\,\cdots$. Since $W_{-\xi_i}$, $1\leqq i\leqq l$, are irreducible $\mathfrak{u}$-modules, the restrictions to $\mathfrak{n}^+$ of the representations of $\mathfrak{u}$ on $W_{-\xi_i}$ and W are both trivial. Hence

$$\begin{aligned}
&\dim H^j(\mathfrak{n}^+,\,\mathrm{Hom}\,(V,\,S))^{\mathfrak{g}_1}\\
&\quad=\dim\,(H^j(\mathfrak{n}^+,\,\mathrm{Hom}\,(V,\,\boldsymbol{C}))\otimes S)^{\mathfrak{g}_1}\\
&\quad=\sum_{i=1}^{l}\dim\,(H^j(\mathfrak{n}^+,\,\mathrm{Hom}\,(V,\,\boldsymbol{C}))\otimes W_{-\xi_i})^{\mathfrak{g}_1}\\
&\quad=\sum_{i=1}^{l}\dim H^j(\mathfrak{n}^+,\,\mathrm{Hom}(V,\,W_{-\xi_i}))^{\mathfrak{g}_1}\,.
\end{aligned}$$

By Theorems A and B

$\dim H^j(\mathfrak{n}^+,\,\mathrm{Hom}\,(V,\,W_{-\xi_i}))^{\mathfrak{g}_1}=\sum_{i=1}^{l}$ multiplicity of V in $H^j(M,\,E_{-\xi_i})$

for $j=0,\,1,\,\cdots$. q.e.d.

The following theorem follows immediately from (1) and the above proposition.

Theorem 2. *Let M be a Hermitian symmetric space of compact type. Assume that $E_{-\xi}$, $\xi\in D_1$, is a line bundle over M. Then*

$$\dim H^q(M, \Omega^p(E_{-\xi})) = \sum_{\sigma\in W^1(p)} \dim H^q(M, \Omega(E_{-(\sigma\delta-\beta+\xi)}))$$

for $q=0, 1, \cdots$.

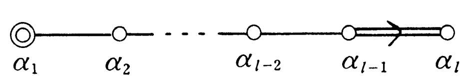
Theorem 2 shows us the importance of the study of the structure of W^1 for our purpose.

2. Vanishing of $H^q(M, \Omega^p(L))$

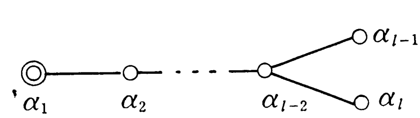
We retain the notations and assumptions introduced in the previous section.

Assume that M is an irreducible Hermitian symmetric space of compact type. Then G is simple and there exists $\alpha_j\in\Pi$ such that $\Pi_1=\Pi-\{\alpha_j\}$. Let $\{\omega_1, \cdots, \omega_l\}$ be fundamental weights with respect to $\Pi=\{\alpha_1, \cdots, \alpha_l\}$. Then any holomorphic line bundle L over M is isomorphic to $E_{-k\omega_j}$ for some integer k, since any 1-dimensional representation of $\mathfrak{g}_1$ is induced by a representation of the center of $\mathfrak{g}_1$.

2.1. The case that M is of type BDI i.e. a complex quadric. Put $\dim M=n$. The Dynkin diagram of Π is as follows:



if $n = 2l-1$ ($l\geqq 2$),



if $n = 2l-2$ ($l\geqq 3$),

where α_1◎ shows that $\alpha_j=\alpha_1$. Let $\{\varepsilon_i;\ i=1, \cdots, l\}$ be a basis of $\mathfrak{h}_0$ which satisfies $(\varepsilon_i, \varepsilon_j)=\delta_{ij}$. Then, we have:

$$\Delta = \begin{cases} \{\pm(\varepsilon_i\pm\varepsilon_j);\ 1\leqq i<j\leqq l,\ \ \varepsilon_i;\ 1\leqq i\leqq l\}, & \text{if } n=2l-1, \\ \{\pm(\varepsilon_i\pm\varepsilon_j);\ 1\leqq i<j\leqq l\}, & \text{if } n=2l-2, \end{cases}$$

$$\Pi = \begin{cases} \{\alpha_i=\varepsilon_i-\varepsilon_{i+1};\ 1\leqq i\leqq l-1,\ \ \alpha_l=\varepsilon_l\}, & \text{if } n=2l-1, \\ \{\alpha_i=\varepsilon_i-\varepsilon_{i+1};\ 1\leqq i\leqq l-1,\ \ \alpha_l=\varepsilon_{l-1}+\varepsilon_l\}, & \text{if } n=2l-2, \end{cases}$$

$$\Delta(\mathfrak{n}^+) = \begin{cases} \{\varepsilon_1\pm\varepsilon_j;\ 2\leqq j\leqq l,\ \varepsilon_1\}, & \text{if } n=2l-1, \\ \{\varepsilon_1\pm\varepsilon_j;\ 2\leqq j\leqq l\}, & \text{if } n=2l-2, \end{cases}$$

$$\omega_1 = \varepsilon_1,$$

$$2\delta = \begin{cases} (2l-1)\varepsilon_1+(2l-3)\varepsilon_2+\cdots+\varepsilon_l & \text{if } n=2l-1, \\ 2(l-1)\varepsilon_1+2(l-2)\varepsilon_2+\cdots+2\varepsilon_{l-1} & \text{if } n=2l-2. \end{cases}$$

An element $\sigma\in W$ acts in $\mathfrak{h}_0$ by $\sigma\varepsilon_i=\pm\varepsilon_{\sigma(i)}$ for $1\leqq i\leqq l$, where σ in the index is a permutation of $\{1, 2, \cdots, l\}$. We represent this element $\sigma\in W$ by

$$\begin{pmatrix} 1 & 2 & \cdots & l \\ \pm\sigma(1) & \pm\sigma(2) & \cdots & \pm\sigma(l)\end{pmatrix}.$$

Then

$$W^1=\begin{cases}\left\{\sigma\in W;\ \sigma^{-1}=\begin{pmatrix}1 & 2\cdots l\\ s(\sigma)i & i_2\cdots i_l\end{pmatrix},\ \begin{matrix}0<i_2<\cdots<i_l\leqq l\\ s(\sigma)=\pm 1\end{matrix}\right\} & \text{if}\quad n=2l-1,\\ \left\{\sigma\in W;\ \sigma^{-1}=\begin{pmatrix}1 & 2\cdots l-1 & l\\ s(\sigma)i & i_2\cdots i_{l-1} & s(\sigma)i_l\end{pmatrix},\ \begin{matrix}0<i_2<\cdots<i_l\leqq l\\ s(\sigma)=\pm 1\end{matrix}\right\} & \text{if}\quad n=2l-2.\end{cases}$$

An element $\sigma\in W^1$ is determined by i and $s(\sigma)$, and its index $n(\sigma)$ of σ is given by

$$n(\sigma)=\begin{cases} i-1 & \text{if}\quad s(\sigma)=1,\\ n-(i-1) & \text{if}\quad s(\sigma)=-1.\end{cases}$$

Furthermore for $\sigma\in W^1$, the values of $\left(\sigma\delta, \frac{2\beta}{(\beta), (\beta)}\right)$ $(\beta\in\Delta(\mathfrak{n}^+))$ are as follows.

If $n=2l-1$,

$s(\sigma)=1$

$\Delta(\mathfrak{n}^+)$	$\left(\sigma\delta, \frac{2\beta}{(\beta,\beta)}\right)$
$\varepsilon_1-\varepsilon_2$	$-(i-1)$
$\varepsilon_1-\varepsilon_3$	$-(i-2)$
$\cdots$	$\cdots$
$\varepsilon_1-\varepsilon_i$	-1
$\varepsilon_1-\varepsilon_{i+1}$	1
$\varepsilon_1-\varepsilon_{i+2}$	2
$\cdots$	$\cdots$
$\varepsilon_1-\varepsilon_l$	$l-i$
$\varepsilon_1+\varepsilon_l$	$l-i+1$
$\varepsilon_1+\varepsilon_{l-1}$	$l-i+2$
$\cdots$	$\cdots$
$\varepsilon_1+\varepsilon_{i+1}$	$2l-2i$
ε_1	$2l-2i+1$
$\varepsilon_1+\varepsilon_i$	$2l-2i+2$
$\cdots$	$\cdots$
$\varepsilon_1+\varepsilon_2$	$2l-i$

For $\sigma\in W^1$, $s(\sigma)=1$, we put

$$\sigma_- = \begin{pmatrix} 1 & 2 & \cdots & l \\ -\sigma(1) & \sigma(2) & \cdots & \sigma(l) \end{pmatrix} \in W^1 .$$

Then we see easily the following:

$$(\sigma-\delta,\ \varepsilon_1+\varepsilon\varepsilon_k) = -(\sigma\delta,\ \varepsilon_1-\varepsilon\varepsilon_k)\,,$$

where $\varepsilon=\pm 1$ and $2\leqq k\leqq l$. Therefore the case: $s(\sigma)=-1$ is omitted. If $n=2l-2$,

$s(\sigma)=1$

$(\Delta\mathfrak{n}^+)$	$\left(\sigma\delta,\ \frac{2\beta}{(\beta,\beta)}\right)$
$\varepsilon_1-\varepsilon_2$	$-(i-1)$
$\varepsilon_1-\varepsilon_3$	$-(i-2)$
$\cdots$	$\cdots$
$\varepsilon_1-\varepsilon_i$	-1
$\varepsilon_1-\varepsilon_{i+1}$	1
$\varepsilon_1-\varepsilon_{i+2}$	2
$\cdots$	$\cdots$
$\varepsilon_1-\varepsilon_l$	$l-i$
$\varepsilon_1+\varepsilon_l$	$l-i$
$\varepsilon_1\varepsilon+_{l-1}$	$l-i+1$
$\cdots$	$\cdots$
$\varepsilon_1+\varepsilon_{i+1}$	$2l-2i-1$
$\varepsilon_1+\varepsilon_i$	$2l-2i+1$
$\varepsilon_1+\varepsilon_{i-1}$	$2l-2i+2$
$\cdots$	$\cdots$
$\varepsilon_1+\varepsilon_2$	$2l-i-1$

$s(\sigma)=-1$

$1\leqq i<l$

$(\Delta\mathfrak{n}^+)$	$\left(\sigma\delta,\ \frac{2\beta}{(\beta,\beta)}\right)$
$\varepsilon_1-\varepsilon_2$	$-2l+i+1$
$\varepsilon_1-\varepsilon_3$	$-2l+i+2$
$\cdots$	$\cdots$
$\varepsilon_1-\varepsilon_i$	$-2l+2i-1$
$\varepsilon_1-\varepsilon_{i+1}$	$-2l+2i+1$
$\varepsilon_1-\varepsilon_{i+2}$	$-2l+2i+2$
$\cdots$	$\cdots$
$\varepsilon_1-\varepsilon_l$	$-(l-i)$
$\varepsilon_1+\varepsilon_l$	$-(l-i)$
$\varepsilon_1+\varepsilon_{l-1}$	$-(l-i-1)$
$\cdots$	$\cdots$
$\varepsilon_1+\varepsilon_{i+1}$	-1
$\varepsilon_1+\varepsilon_i$	1
$\varepsilon_1+\varepsilon_{i-1}$	2
$\cdots$	$\cdots$
$\varepsilon_1+\varepsilon_2$	$i-1$

$i=l$

$(\Delta\mathfrak{n}^+)$	$\left(\sigma\delta,\ \frac{2\beta}{(\beta,\beta)}\right)$
$\varepsilon_1-\varepsilon_2$	$-(l-1)$
$\varepsilon_1-\varepsilon_3$	$-(l-2)$
$\cdots$	$\cdots$
$\varepsilon_1-\varepsilon_{l-1}$	-2
$\varepsilon_1-\varepsilon_l$	1
$\varepsilon_1+\varepsilon_l$	-1
$\varepsilon_1+\varepsilon_{l-1}$	2
$\varepsilon_1+\varepsilon_{l-2}$	3
$\cdots$	$\cdots$
$\varepsilon_1+\varepsilon_2$	$l-1$

Furthermore we have

$$\left(k\omega_1,\ \frac{2\beta}{(\beta,\beta)}\right) = \begin{cases} k & \text{if} \quad \beta=\varepsilon_1\pm\varepsilon_j,\ 2\leqq j\leqq l\,, \\ 2k & \text{if} \quad \beta=\varepsilon_1\,. \end{cases}$$

Let $\sigma\in W^1$. Take an integer k which is not equal to $-\dfrac{(\omega_1,\beta)}{(\sigma\delta,\beta)}$ for all $\beta\in\Delta(\mathfrak{n}^+)$. Put $q=\#\{\beta\in\Delta(\mathfrak{n}^+);\ (\sigma\delta+k\omega_1,\beta)<0\}$. Then by Theorem 1 we have

$$H^j(M,\ \Omega E_{-(\sigma\delta-\delta+k\omega_1)}) = 0\,, \qquad \text{for all} \quad j\neq q\,,$$
$$H^q(M,\ \Omega E_{-(\sigma\delta-\delta+k\omega_1)}) \neq 0\,.$$

Therefore we obtain the following theorem by Theorem 2.

Theorem 3. *Let M be a complex quadric of dimension n, $n\geqq 3$. Then the*

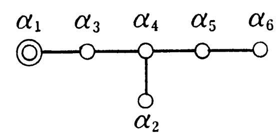
Table 1

Values $\left(\sigma\delta, \frac{2\beta}{(\beta, \beta)}\right)$ for $\sigma \in W^1$ and $\beta \in \Delta(\mathfrak{n}^+)$ (type EIII)

β / $\sigma\delta$	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	the numbers which do not appear in the sequence
$\sigma_0\delta$	1	2	3	4	4	5	5	6	6	7	7	8	8	9	10	11	
$\sigma_1\delta$	−1	1	2	3	3	4	4	5	5	6	7	8	7	9	10	11	0
$\sigma_2\delta$	−2	−1	1	3	2	4	3	4	5	6	6	7	7	8	10	11	0, 9
$\sigma_3\delta$	−3	−2	−1	1	1	2	3	4	4	5	5	6	7	8	9	11	0, 10
$\sigma_4\delta$	−4	−3	−2	−1	1	1	2	4	3	5	4	6	6	7	8	11	0, 9, 10
$\sigma_4'\delta$	−4	−3	−2	1	−1	2	2	3	3	4	4	5	7	8	9	10	0, 6
$\sigma_5\delta$	−5	−4	−3	−2	1	−1	2	3	3	4	4	5	5	6	7	11	0, 8, 9, 10
$\sigma_5'\delta$	−5	−4	−3	−1	−1	1	1	3	2	4	3	5	6	7	8	10	—2, 0, 9
$\sigma_6\delta$	−6	−5	−4	−2	−1	−1	1	2	2	3	3	4	5	6	7	10	—3, 0. 8, 9
$\sigma_6'\delta$	−6	−5	−3	−2	−2	1	−1	2	1	4	2	5	5	6	8	9	—4, 0, 3, 7
$\sigma_7\delta$	−7	−6	−4	−3	−2	−1	−1	1	1	3	2	4	4	5	7	9	—5, 0, 6, 8
$\sigma_7'\delta$	−7	−5	−4	−3	−3	1	−2	2	−1	3	1	5	4	6	7	8	—6, 0
$\sigma_8\delta$	−8	−7	−4	−3	−3	−2	−2	−1	1	2	2	3	3	4	7	8	—6, —5, 0, 5, 6
$\sigma_8'\delta$	−8	−6	−5	−4	−3	−1	−2	1	−1	2	1	4	3	5	6	8	—7, 0, 7
$\sigma_8''\delta$	−7	−6	−5	−4	−4	1	−3	2	−2	3	−1	4	4	5	6	7	0
$\sigma_9\delta$	−9	−7	−5	−4	−4	−2	−3	−1	−1	1	1	3	2	4	6	7	—8, —6, 0, 5
$\sigma_9'\delta$	−8	−7	−6	−5	−4	−1	−3	1	−2	2	−1	3	3	4	5	7	0, 6
$\sigma_{10}\delta$	−9	−8	−6	−5	−5	−2	−4	−1	−2	1	−1	2	2	3	5	6	—7, —3, 0, 4
$\sigma_{10}'\delta$	−10	−7	−6	−4	−5	−3	−3	−2	−2	−1	1	2	1	4	5	6	—9, —8, 0, 3
$\sigma_{11}\delta$	−11	−7	−6	−5	−5	−4	−4	−3	−3	−2	1	2	−1	3	4	5	—10, —9, —8, 0
$\sigma_{11}'\delta$	−10	−8	−7	−5	−6	−3	−4	−2	−3	−1	−1	1	1	3	4	5	—9, 0, 2
$\sigma_{12}\delta$	−11	−8	−7	−6	−6	−4	−5	−3	−4	−2	−1	1	−1	2	3	4	—9, —10, 0
$\sigma_{12}'\delta$	−10	−9	−8	−5	−7	−4	−4	−3	−3	−2	−2	−1	1	2	3	4	—6, 0
$\sigma_{13}\delta$	−11	−9	−8	−6	−7	−5	−5	−4	−4	−3	−2	−1	−1	1	2	3	—10, 0
$\sigma_{14}\delta$	−11	−10	−8	−7	−7	−6	−6	−5	−4	−3	−3	−2	−2	−1	1	2	—9, 0
$\sigma_{15}\delta$	−11	−10	−9	−8	−7	−7	−6	−5	−5	−4	−4	−3	−3	−2	−1	1	0
$\sigma_{16}'\delta$	−11	−10	−9	−8	−8	−7	−7	−6	−5	−5	−4	−4	−3	−3	−2	−1	

group $H^q(M, \Omega^p(E_{-k\omega_1}))=0$ *except for the following cases*: (i) $q=0$ *and* $k>p$, (ii) $p=q$ *and* $k=0$, (iii) $p+q=n$ *and* $k=2p-n$, (iv) $q=n$ *and* $k<p-n$.

2.2. The case M is of type EIII. The Dynkin diagram is:



where α_1◎ shows that $\alpha_j=\alpha_1$ in this case. We have $\sharp(\mathfrak{n}^+)=16$ and $\sharp W^1=27$. We express $\beta=\sum_{i=1}^{6} m_i\alpha_i\in\Delta(\mathfrak{n}^+)$ by $(m_1m_2m_3m_4m_5m_6)$. For σ of W^1, we put $\sigma\delta=(n_1n_2n_3n_4n_5n_6)$ if $\sigma\delta=\sum_{i=1}^{6} n_i\omega_i$. Then we give the values $\left(\sigma\delta, \frac{(\beta, \beta)}{2\beta}\right)$ for $\sigma\in W^1$ and $\beta\in\Delta(\mathfrak{n}^+)$ by Table 1. From Table 1, Theorems 1 and 2, we obtain the following theorem.

Theorem 4. *Let* M *be of type EIII. Then the group* $H^q(M, \Omega^p(E_{-k\omega_j}))$ *vanishes except for* (p, q, k) *listed in Table* 2.

Table 2

p	$q=0$	$1\leqq q\leqq 15$, (a, b) shows $q=a$ and $k=b$	$q=16$
0	$k>-1$		$k<-11$
1	$k>1$	(1, 0)	$k<-11$
2	$k>2$	(2, 0), (14, −9)	$k<-11$
3	$k>3$	(3, 0), (15, −10)	$k<-11$
4	$k>4$	(4, 0), (12, −6), (15, −9), (15, −10)	$k<-11$
5	$k>5$	(5, 0), (3, 2), (15, −8), (15, −9), (15, −10)	$k<-11$
6	$k>6$	(6, 0), (3, 3), (2, 4), (10, −3), (14, −7), (15, −8), (15, −9)	$k<-10$
7	$k>7$	(7, 0), (1, 6), (2, 5), (14, −6), (15, −8)	$k<-9$
8	$k>8$	(8, 0), (1, 7), (2, 5), (2, 6), (14, −5), (14, −6), (14, −7)	$k<-8$
9	$k>9$	(9, 0), (1, 8), (2,6), (14, −5), (15, −6)	$k<-7$
10	$k>10$	(10, 0), (1, 8), (1, 9), (2, 7), (6, 3), (13, −3), (14, −4)	$k<-6$
11	$k>11$	(11, 0), (1, 8), (1, 9), (1, 10), (13, −2)	$k<-5$
12	$k>11$	(12, 0), (1, 9), (1, 10), (4, 6)	$k<-4$
13	$k>11$	(13, 0), (1, 10)	$k<-3$
14	$k>11$	(14, 0), (2, 9)	$k<-2$
15	$k>11$	(15, 0)	$k<-1$
16	$k>11$		$k<1$

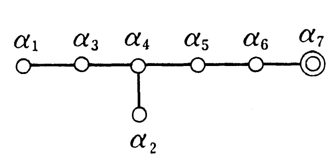
where $\sigma\delta$, $\sigma\in W^1$, and $\beta\in\Delta(\mathfrak{n}^+)$ are expressed as follows:

$\sigma_0\delta$	(1 1 1 1 1 1)
$\sigma_1\delta$	(−1 1 2 1 1 1)
$\sigma_2\delta$	(−2 1 1 2 1 1)
$\sigma_3\delta$	(−3 2 1 1 2 1)
$\sigma_4\delta$	(−4 3 1 1 1 2)
$\sigma_4'\delta$	(−4 1 1 1 3 1)
$\sigma_5\delta$	(−5 4 1 1 1 1)
$\sigma_5'\delta$	(−5 2 1 1 2 2)
$\sigma_6\delta$	(−6 3 1 1 2 1)
$\sigma_6'\delta$	(−6 1 1 2 1 3)
$\sigma_7\delta$	(−7 2 1 2 1 2)
$\sigma_7'\delta$	(−7 1 2 1 1 4)
$\sigma_8\delta$	(−8 1 1 3 1 1)
$\sigma_8'\delta$	(−8 2 2 1 1 3)
$\sigma_8''\delta$	(−7 1 1 1 1 5)
$\sigma_9\delta$	(−9 1 2 2 1 2)
$\sigma_9'\delta$	(−8 2 1 1 1 4)
$\sigma_{10}\delta$	(−9 1 1 2 1 3)
$\sigma_{10}'\delta$	(−10 1 3 1 2 1)
$\sigma_{11}\delta$	(−11 1 4 1 1 1)
$\sigma_{11}'\delta$	(−10 1 2 1 2 2)
$\sigma_{12}\delta$	−(11 1 3 1 1 2)
$\sigma_{12}'\delta$	(−10 1 1 1 3 1)
$\sigma_{13}\delta$	(−11 1 2 1 2 1)
$\sigma_{14}\delta$	(−11 1 1 2 1 1)
$\sigma_{15}\delta$	(−11 2 1 1 1 1)
$\sigma_{16}\delta$	(−11 1 1 1 1 1)

β_1	(1 0 0 0 0 0)
β_2	(1 0 1 0 0 0)
β_3	(1 0 1 1 0 0)
β_4	(1 0 1 1 1 0)
β_6	(1 1 1 1 0 0)
β_6	(1 0 1 1 1 1)
β_7	(1 1 1 1 1 0)
β_8	(1 1 1 1 1 1)
β_9	(1 1 1 2 1 0)
β_{10}	(1 1 1 2 1 1)
β_{11}	(1 1 2 2 1 0)
β_{12}	(1 1 2 2 1 1)
β_{12}	(1 1 1 2 2 1)
β_{14}	(1 1 2 2 2 1)
β_{15}	(1 1 2 3 2 1)
β_{16}	(1 2 2 3 2 1)

REMARK. In this case the structure of W^1 is given by N. Iwahori (Takeuchi [9]).

2.3. The case M is of type EVII. The Dynkin diagram of Π is:



where α_7◎ shows that $\alpha_j=\alpha_7$ in this case. We have $\sharp\Delta(\mathfrak{n}^+)=27$ and $\sharp W^1=56$. We express β of $\Delta(\mathfrak{n}^+)$ and $\sigma\delta$ for $\sigma\in W^1$ in a similar way as in 2.2. Then the values $\left(\sigma\delta, \frac{2\beta}{(\beta, \beta)}\right)$ for $\sigma\in W^1$ and $\beta\in\Delta(\mathfrak{n}^+)$ are as in Table 3. From Table 3, Theorems 1 and 2, we obtain the following theorem.

Table 3

Values $\left(\sigma\delta, \frac{2\beta}{(\beta, \beta)}\right)$ for $\sigma \in W^1$ and $\beta \in (\Delta n^+)$ (type EVII)

$\sigma\delta$ \ β	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}
$\sigma_0\delta$	1	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10
$\sigma_1\delta$	−1	1	2	3	4	4	5	5	6	6	7	7	9	9	8	10
$\sigma_2\delta$	−2	−1	1	2	3	3	4	4	5	5	7	6	8	8	7	9
$\sigma_3\delta$	−3	−2	−1	1	2	2	3	3	5	4	7	6	8	7	7	8
$\sigma_4\delta$	−4	−3	−2	−1	1	1	3	2	3	4	4	5	5	6	7	7
$\sigma_5\delta$	−5	−4	−3	−2	1	−1	2	1	2	4	3	5	4	6	6	7
$\sigma_5'\delta$	−5	−4	−3	−2	−1	1	2	2	3	3	4	4	5	5	7	6
$\sigma_6\delta$	−6	−5	−4	−3	−1	−1	1	1	2	3	3	4	4	5	6	6
$\sigma_6'\delta$	−6	−5	−4	−3	1	−2	2	−1	3	3	4	4	5	5	5	6
$\sigma_7\delta$	−7	−6	−5	−4	−1	−2	1	−1	2	2	3	3	4	4	5	5
$\sigma_7'\delta$	−7	−6	−5	−3	−2	−2	−1	1	1	2	2	4	3	5	5	6
$\sigma_8\delta$	−8	−7	−6	−4	−2	−3	−1	−1	1	1	2	3	3	4	4	5
$\sigma_8'\delta$	−8	−7	−5	−4	−3	−3	−2	1	−1	2	1	3	2	5	4	6
$\sigma_9\delta$	−9	−8	−7	−4	−3	−3	−2	−2	1	−1	2	2	3	3	3	4
$\sigma_9'\delta$	−9	−8	−6	−5	−3	−4	−2	−1	−1	1	1	2	2	4	3	5
$\sigma_9''\delta$	−9	−7	−6	−5	−4	−4	−3	1	−2	2	−1	3	1	4	4	6
$\sigma_{10}\delta$	−10	−9	−7	−5	−4	−4	−3	−2	−1	−1	1	1	2	3	2	4
$\sigma_{10}'\delta$	−10	−8	−7	−6	−4	−5	−3	−1	−2	1	−1	2	1	3	3	5
$\sigma_{10}''\delta$	−9	−8	−7	−6	−5	−5	−4	1	−3	2	−2	3	−1	4	4	5
$\sigma_{11}\delta$	−11	−10	−7	−6	−5	−4	−3	−3	2	−2	11	−1	2	2	1	3
$\sigma_{11}'\delta$	−11	−9	−8	−6	−5	−5	−4	−2	−2	−1	−1	1	1	2	2	4
$\sigma_{11}''\delta$	−10	−9	−8	−7	−5	−6	−4	−1	−3	1	−2	2	−1	3	3	4
$\sigma_{12}\delta$	−12	−11	−7	−6	−5	−5	−4	−4	−3	−3	1	−2	2	2	−1	3
$\sigma_{12}'\delta$	−12	−10	−8	−7	−6	−5	−4	−3	−3	−2	−1	−1	1	1	1	3
$\sigma_{12}''\delta$	−11	−10	−9	−7	−6	−6	−5	−2	−3	−1	−2	1	−1	2	2	3
$\sigma_{13}\delta$	−13	−11	−8	−7	−6	−6	−5	−4	−4	−3	−1	−2	1	1	−1	3
$\sigma_{13}'\delta$	−13	−10	−9	−8	−7	−5	−4	−4	−3	−3	−2	−2	1	−1	1	2
$\sigma_{13}''\delta$	−12	−11	−9	−8	−7	−6	−5	−3	−4	−2	−2	−1	−1	1	1	2
$\sigma_{14}\delta$	−14	−11	−9	−8	−7	−6	−5	−5	−4	−4	−2	−3	1	−1	−1	2
$\sigma_{14}'\delta$	−13	−12	−9	−8	−7	−7	−6	−4	−5	−3	−2	−2	−1	1	−1	1
$\sigma_{14}''\delta$	−13	−11	−10	−9	−8	−6	−5	−4	−4	−3	−3	−2	−1	−1	1	1
$\sigma_{15}\delta$	−13	−12	−11	−10	−9	−6	−5	−5	−4	−4	−3	−3	−2	−2	1	−1
$\sigma_{15}'\delta$	−14	−12	−10	−9	−8	−7	−6	−5	−5	−4	−3	−3	−1	−1	−1	1
$\sigma_{15}''\delta$	−15	−11	−10	−8	−7	−7	−6	−6	−4	−5	−3	−3	1	−2	−2	2
$\sigma_{16}\delta$	−14	−13	−11	−10	−9	−7	−6	−6	−5	−5	−3	−4	−2	−2	−1	−1
$\sigma_{16}'\delta$	−15	−12	−11	−9	−8	−8	−7	−6	−5	−5	−4	−3	−1	−2	−2	1
$\sigma_{16}''\delta$	−16	−11	−10	−9	−7	−8	−6	−7	−5	−5	−4	−4	1	−3	−3	2
$\sigma_{17}\delta$	−15	−13	−12	−10	−9	−8	−7	−7	−5	−6	−4	−4	−2	−3	−2	−1
$\sigma_{17}'\delta$	−16	−12	−11	−10	−8	−9	−7	−7	−6	−5	−5	−4	−1	−3	−3	1
$\sigma_{17}''\delta$	−17	−11	−10	−9	−8	−8	−7	−7	−6	−6	−5	−5	1	−4	−4	2
$\sigma_{18}\delta$	−15	−14	−13	−10	−9	−9	−8	−8	−5	−7	−4	−4	−3	−3	−3	−2
$\sigma_{18}'\delta$	−16	−13	−12	−11	−9	−9	−7	−8	−6	−6	−5	−5	−2	−4	−3	−1
$\sigma_{18}''\delta$	−17	−12	−11	−10	−9	−9	−8	−7	−7	−6	−6	−5	−1	−4	−4	1
$\sigma_{19}\delta$	−16	−14	−13	−11	−9	−10	−8	−9	−6	−7	−5	−5	−3	−4	−4	−2
$\sigma_{19}'\delta$	−17	−13	−12	−11	−10	−9	−8	−8	−7	−7	−6	−6	−2	−5	−4	−1
$\sigma_{20}\delta$	−16	−15	−13	−12	−9	−11	−8	−10	−7	−7	−5	−6	−4	−4	−5	−3
$\sigma_{20}'\delta$	−17	−14	−13	−11	−10	−10	−9	−9	−7	−8	−6	−6	−3	−5	−5	−2
$\sigma_{21}\delta$	−16	−15	−14	−13	−9	−12	−8	−11	−7	−7	−6	−6	−5	−5	−5	−4
$\sigma_{21}'\delta$	−17	−15	−13	−12	−10	−11	−9	−10	−8	−8	−6	−7	−4	−5	−6	−3
$\sigma_{22}\delta$	−17	−15	−14	−13	−10	−12	−9	−11	−8	−8	−7	−7	5	−6	−6	−4
$\sigma_{22}'\delta$	−17	−16	−13	−12	−11	−11	−10	−10	−9	−9	−6	−8	−5	−5	−7	−4
$\sigma_{23}\delta$	−17	−16	−14	−13	−11	−12	−10	−11	−9	−9	−7	−8	−6	−6	−7	−5
$\sigma_{24}\delta$	−17	−16	−15	−13	−12	−12	−11	−11	−9	−10	−8	−8	−7	−7	−7	−6
$\sigma_{25}\delta$	−17	−16	−15	−14	−13	−12	−11	−11	−10	−10	−9	−9	−8	−8	−7	−7
$\sigma_{26}\delta$	−17	−16	−15	−14	−13	−13	−12	−11	−11	−10	−10	−9	−9	−8	−8	−7
$\sigma_{27}\delta$	−17	−16	−15	−14	−13	−13	−12	−12	−11	−11	−10	−10	−9	−9	−8	−8

Table 3—continued

$\sigma\delta$ \ β	β_{17}	β_{18}	β_{19}	β_{20}	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}	β_{26}	β_{27}	the numbers which do not appear in the sequence
$\sigma_0\delta$	10	11	11	12	12	13	13	14	15	16	17	
$\sigma_1\delta$	9	11	10	12	11	11	13	14	15	16	17	0
$\sigma_2\delta$	9	10	10	11	11	13	12	14	15	16	17	0
$\sigma_3\delta$	8	9	10	11	11	12	12	13	15	16	17	0, 14
$\sigma_4\delta$	8	9	9	10	11	11	12	13	14	16	17	0, 15
$\sigma_5\delta$	7	8	8	9	10	11	12	13	14	15	17	0, 16
$\sigma_5'\delta$	8	9	9	10	10	11	11	12	13	16	17	0, 14, 15
$\sigma_6\delta$	7	8	8	9	10	10	11	12	13	15	17	—2, 0, 14, 16
$\sigma_6'\delta$	6	7	7	8	11	9	21	13	14	15	16	0, 10
$\sigma_7\delta$	6	7	7	8	10	9	21	12	13	15	16	—3, 0, 14
$\sigma_7'\delta$	6	7	8	9	9	10	11	11	13	14	17	—4, 0, 12, 15, 16
$\sigma_8\delta$	5	6	7	8	9	9	10	11	13	14	16	—5, 0, 12, 15
$\sigma_8'\delta$	6	7	7	8	8	10	10	11	12	13	17	—6, 0, 14, 15, 16
$\sigma_9\delta$	4	5	7	8	8	9	9	10	13	14	15	—5, —6, 0, 6, 11, 12
$\sigma_9'\delta$	5	6	6	7	8	9	9	11	12	13	16	—7, 0,1 0, 14, 15
$\sigma_9''\delta$	5	7	6	8	7	9	9	10	11	12	17	—8, 0, 13, 14, 15, 16
$\sigma_{10}\delta$	4	5	6	7	7	9	9	10	12	13	15	—8, —6, 0, 11, 14
$\sigma_{10}'\delta$	4	6	5	7	7	8	8	10	11	12	16	—9, 0, 13, 14, 15
$\sigma_{10}''\delta$	5	6	6	7	7	8	9	9	10	11	17	0, 12, 13, 14, 15, 16
$\sigma_{11}\delta$	4	5	5	6	6	9	8	10	11	13	14	—8, —9, 0, 8, 12
$\sigma_{11}'\delta$	3	5	5	7	6	8	7	9	11	12	15	—10, —7, —3, 0, 10, 13, 14
$\sigma_{11}''\delta$	4	5	5	6	7	7	8	9	10	11	16	0, 12, 13, 14, 15
$\sigma_{12}\delta$	3	4	4	5	5	9	8	10	11	12	13	—10, —9, —8, 0, 7, 8
$\sigma_{12}'\delta$	3	5	4	6	5	8	6	9	10	12	14	—11, —9, 0, 2, 11, 13
$\sigma_{12}''\delta$	3	4	5	6	6	7	7	8	10	11	15	—8, —4, 0, 9, 12, 13, 14
$\sigma_{13}\delta$	2	4	3	5	4	8	7	9	10	11	13	—12, —10, —9, 0, 7, 12
$\sigma_{13}'\delta$	2	5	3	6	4	7	6	8	9	12	13	—12, —11, —6, 0, 10, 11
$\sigma_{13}''\delta$	3	4	4	5	5	7	7	8	9	11	14	—10, 0, 10, 12, 13
$\sigma_{14}\delta$	1	4	2	5	3	7	6	8	9	11	12	—13, —12, —10, 0, 10
$\sigma_{14}'\delta$	2	3	3	4	4	7	6	8	9	10	14	—11, —10, 0, 6, 11, 12
$\sigma_{14}''\delta$	2	4	3	5	4	6	5	7	8	11	14	—12, —7, 0, 9, 10, 12
$\sigma_{15}\delta$	2	3	3	4	4	5	6	6	7	11	12	—8, —7, 0, 8, 9, 10
$\sigma_{15}'\delta$	1	3	2	4	3	6	5	7	8	10	12	—13, —11, —2, 0, 9, 11
$\sigma_{15}''\delta$	—1	3	1	5	2	6	5	7	9	10	11	—14, —13, —12, —9, 0, 4, 9
$\sigma_{16}\delta$	1	2	2	3	3	5	6	6	7	10	11	—12, —8, 0, 8, 9
$\sigma_{16}'\delta$	—1	2	1	4	2	5	4	6	8	9	11	—14, —13, —10, 0, 3, 7, 10
$\sigma_{16}''\delta$	—2	3	—1	4	1	5	5	7	8	9	10	—15, —14, —13, —12, 0
$\sigma_{17}\delta$	—1	1	1	3	2	4	6	5	7	9	10	—14, —11, 0, 6, 8
$\sigma_{17}'\delta$	—2	2	—1	3	1	4	4	6	7	8	10	—15, —14, —13, 0, 9
$\sigma_{17}''\delta$	—3	3	—2	4	—1	5	5	6	7	8	9	—16, —15, —14, —13, —12, 0
$\sigma_{18}\delta$	—2	—1	1	2	2	3	5	4	7	8	9	—12, —11, —6, 0, 5, 6
$\sigma_{18}'\delta$	—2	1	—1	2	1	3	3	5	6	8	9	—15, —14, —10, 0, 7
$\sigma_{18}''\delta$	—3	2	—2	3	—1	4	4	5	6	7	9	—16, —15, —14, —13, 0, 8
$\sigma_{19}\delta$	—3	—1	—1	1	1	2	4	4	6	7	8	—15, —12, 0, 5
$\sigma_{19}'\delta$	—3	1	—2	2	—1	3	3	4	5	7	8	—16, —15, —14, 0, 6
$\sigma_{20}\delta$	—3	—2	—2	—1	1	1	3	4	5	6	7	—14, 0, 3
$\sigma_{20}'\delta$	—4	—1	—2	1	—1	2	2	3	5	6	7	—16, —15, —12, 0, 4
$\sigma_{21}\delta$	—4	—3	—3	—2	1	—1	2	3	4	5	6	—10, 0
$\sigma_{21}'\delta$	—4	—2	—3	—1	—1	1	1	3	4	5	6	—16, —14, 0, 2
$\sigma_{22}\delta$	—5	—3	—4	—2	—1	—1	1	2	3	4	5	—16, 0
$\sigma_{22}'\delta$	—4	—3	—3	—2	—2	1	—1	2	3	4	5	—15, —14, 0
$\sigma_{23}\delta$	—5	—4	—4	—3	—2	—1	—1	1	2	3	4	—15, 0
$\sigma_{24}\delta$	—6	—5	—4	—3	—3	—2	—2	—1	1	2	3	—14, 0
$\sigma_{25}\delta$	—6	—5	—5	—4	—4	—3	—3	—2	—1	1	2	0
$\sigma_{26}\delta$	—7	—6	—6	—5	—5	—4	—4	—3	—2	—1	1	0
$\sigma_{27}\delta$	—8	—7	—7	—6	—6	—5	—5	—4	—3	—2	—1	

Theorem 5. *Let M be of type EVII. Then the group $H^q(M, \Omega^p(E_{-k\omega_j}))=0$ except for (p, q, k) listed in Table 4.*

Table 4

p	$q=0$	$1\leqq q\leqq 26$, (a, b) shows $q=a$ and $k=b$	$q=27$
0	$k>-1$		$k<-17$
1	$k>1$	(1, 0)	$k<-17$
2	$k>2$	(2, 0)	$k<-17$
3	$k>3$	(3, 0), (24, —14)	$k<-17$
4	$k>4$	(4, 0), (25, —15)	$k<-17$
5	$k>5$	(5, 0), (25, —14~—15)	$k<-17$
6	$k>6$	(6, 0), (4, 2), (21, —10), (25, —14), (26, —16)	$k<-17$
7	$k>7$	(7, 0), (3, 4), (4, 3), (24, —12), (25, —14), (26, —15~—16)	$k<-17$
8	$k>8$	(8, 0), (2, 6), (3, 5), (24, —12), (26, —14~—16)	$k<-17$
9	$k>9$	(9, 0), (1, 8), (2, 7), (3, 56), (18, —6), (23, —10), (24, —11~—12), (26, —13~16)	$k<-17$
10	$k>10$	(10, 0), (1, 9), (2, 8), (3, 6), (24, —11), (26, —12~—16)	$k<-17$
11	$k>11$	(11, 0), (1, 10), (2, 8~9), (3, 7), (7, 3), (22, —8), (24, —10), (25, —12), (26, —12~—15)	$k<-16$
12	$k>12$	(12, 0), (1, 11), (2, 8~10), (3, 8), (7, 4), (15, —2), (22, —7~—8), (24, —9), (25, —11), (26, —12~—14)	$k<-15$
13	$k>13$	(13, 0), (1, 11~12), (2, 9~10), (5, 6), (22, —7), (25, —10~—11), (26, —12~—13)	$k<-14$
14	$k>14$	(14, 0), (1, 12~13), (2, 10~11), (5, 7), (22, —6), (25, —9~—10), (26, —11~—12)	$k<-13$
15	$k>15$	(15, 0), (1, 12~14), (2, 11), (5, 7~8), (3, 9), (12, 2), (20, —4), (24, —8), (25, —8~—10), (26, —11)	$k<-12$
16	$k>16$	(16, 0), (1, 12~15), (2, 12), (3, 10), (5, 8), (20, —3), (24, —7), (25, —8~—9), (26, —10)	$k<-11$
17	$k>17$	(17, 0), (1, 12~16), (3, 11), (24, —6), (25, —8), (26, —9)	$k<-10$
18	$k>17$	(18, 0), (1, 13~16), (3, 11~12), (4, 10), (9, 6)	$k<-9$
19	$k>17$	(19, 0), (1, 14~16), (3, 12), (24, —5), (25, —6)	$k<-8$
20	$k>17$	(20, 0), (1, 15~16), (2, 14), (3, 12), (23, —3), (24, —4)	$k<-7$
21	$k>17$	(21, 0), (1, 16), (2, 14)(6, 10), (23, —2)	$k<-6$
22	$k>17$	(22, 0), (1, 16), (2, 14~15)	$k<-5$
23	$k>17$	(23, 0), (2, 15)	$k<-4$
24	$k>17$	(24, 0), (3, 14)	$k<-3$
25	$k>17$	(25, 0)	$k<-2$
26	$k>17$	(26, 0)	$k<-1$
27	$k>17$		$k<1$

where $\sigma\delta$, $\sigma\in W^1$, and $\beta\in\Delta(\mathfrak{n}^+)$ are expressed as follows:

$\sigma_0\delta$	(1 1 1 1 1 1 1)
$\sigma_1\delta$	(1 1 1 1 1 2 —1)
$\sigma_2\delta$	(1 1 1 1 2 1 —2)
$\sigma_3\delta$	(1 1 1 2 1 1 —3)
$\sigma_4\delta$	(1 2 2 1 1 1 —4)
$\sigma_5\delta$	(2 3 1 1 1 1 —5)
$\sigma_5'\delta$	(1 1 3 1 1 1 —5)
$\sigma_6\delta$	(2 2 2 1 1 1 —6)
$\sigma_6'\delta$	(1 4 1 1 1 1 —6)
$\sigma_7\delta$	(1 3 2 1 1 1 —7)
$\sigma_7'\delta$	(3 1 1 2 1 1 —7)
$\sigma_8\delta$	(2 2 1 2 1 1 —8)
$\sigma_8'\delta$	(4 1 1 1 2 1 —8)
$\sigma_9\delta$	(1 1 1 3 1 1 —9)
$\sigma_9'\delta$	(3 2 1 1 2 1 —9)
$\sigma_9''\delta$	(5 1 1 1 1 2 —9)
$\sigma_{10}\delta$	(2 1 1 2 2 1 —10)
$\sigma_{10}'\delta$	(4 2 1 1 1 2 —10)
$\sigma_{10}''\delta$	(6 1 1 1 1 1 —9)
$\sigma_{11}\delta$	(1 1 2 1 3 1 —11)
$\sigma_{11}'\delta$	(3 1 1 2 1 2 —11)
$\sigma_{11}''\delta$	(5 2 1 1 1 1 —10)
$\sigma_{12}\delta$	(1 1 1 1 4 1 —12)
$\sigma_{12}'\delta$	(2 1 2 1 2 2 —12)
$\sigma_{12}''\delta$	(4 1 1 2 1 1 —11)
$\sigma_{13}\delta$	(2 1 1 1 3 2 —13)
$\sigma_{13}'\delta$	(1 1 3 1 1 3 —13)
$\sigma_{13}''\delta$	(3 1 2 1 2 1 —12)
$\sigma_{14}\delta$	(1 1 2 1 2 3 —14)
$\sigma_{14}'\delta$	(3 1 1 1 3 1 —13)
$\sigma_{14}''\delta$	(2 1 3 1 1 2 —13)
$\sigma_{15}\delta$	(1 1 4 1 1 1 —13)
$\sigma_{15}'\delta$	(2 1 2 1 2 2 —14)
$\sigma_{15}''\delta$	(1 1 1 2 1 4 —15)
$\sigma_{16}\delta$	(1 1 3 1 2 1 —14)
$\sigma_{16}'\delta$	(2 1 1 2 1 3 —15)
$\sigma_{16}''\delta$	(1 2 1 1 1 5 —16)
$\sigma_{17}\delta$	(2 1 2 2 1 2 —15)
$\sigma_{17}'\delta$	(2 2 1 1 1 4 —16)
$\sigma_{17}''\delta$	(1 1 1 1 1 6 —17)
$\sigma_{18}\delta$	(1 1 1 3 1 1 —15)
$\sigma_{18}'\delta$	(1 2 2 1 1 3 —16)
$\sigma^{1}{}_{8}''\delta$	(2 1 1 1 1 5 —17)
$\sigma_{19}\delta$	(1 2 1 2 1 2 —16)
$\sigma_{19}'\delta$	(1 1 2 1 1 4 —17)
$\sigma_{20}\delta$	(1 3 1 1 2 1 —16)
$\sigma_{20}'\delta$	(1 1 1 2 1 3 —17)
$\sigma_{21}\delta$	(1 4 1 1 1 1 —16)
$\sigma_{21}'\delta$	(1 2 1 1 2 2 —17)
$\sigma_{22}\delta$	(1 3 1 1 1 2 —17)
$\sigma_{22}'\delta$	(1 1 1 1 3 1 —17)
$\sigma_{23}\delta$	(1 2 1 1 2 1 —17)
$\sigma_{24}\delta$	(1 1 1 2 1 1 —17)
$\sigma_{25}\delta$	(1 1 2 1 1 1 —17)
$\sigma_{26}\delta$	(2 1 1 1 1 1 —17)
$\sigma_{27}\delta$	(1 1 1 1 1 1 —17)

β_1	(0 0 0 0 0 0 1)
β_2	(0 0 0 0 0 1 1)
β_3	(0 0 0 0 1 1 1)
β_4	(0 0 0 1 1 1 1)
β_5	(0 1 0 1 1 1 1)
β_6	(0 0 1 1 1 1 1)
β_7	(0 1 1 1 1 1 1)
β_8	(1 0 1 1 1 1 1)
β_9	(0 1 1 2 1 1 1)
β_{10}	(1 1 1 1 1 1 1)
β_{11}	(0 1 1 2 2 1 1)
β_{12}	(1 1 1 2 1 1 1)
β_{13}	(0 1 1 2 2 2 1)
β_{14}	(1 1 1 2 2 1 1)
β_{15}	(1 1 2 2 1 1 1)
β_{16}	(1 1 1 2 2 2 1)
β_{17}	(1 1 2 2 2 1 1)
β_{18}	(1 1 2 2 2 2 1)
β_{19}	(1 1 2 3 2 1 1)
β_{20}	(1 1 2 3 2 2 1)
β_{21}	(1 2 2 3 2 1 1)
β_{22}	(1 1 2 3 3 2 1)
β_{23}	(1 2 2 3 2 2 1)
β_{24}	(1 2 2 3 3 2 1)
β_{25}	(1 2 2 4 3 2 1)
β_{26}	(1 2 3 4 3 2 1)
β_{27}	(2 2 3 4 3 2 1)

REMARK. In this case the structure of W^1 is given by T. Yokonuma (Takeuchi [9]).

2.4. Other cases. If M is of type AIII, DIII or CI, it is not known completely when the groups $H^q(M, \Omega^p(E))$ vanish. In this section we consider the case when p is equal to 0 or 1.

We denote by K_N the canonical line bundle of a complex manifold N. If N is an irreducible Hermitian symmetric space M, there exists an integer λ such that $K_M=E_{\lambda\omega_j}$. Further we know

$$\lambda = 2 \sum_{\beta\in\Delta(\mathfrak{n}^+)} (\beta, \alpha_j)/(\alpha_j, \alpha_j)$$

(Borel-Hirzebruch [2]). Applying this formula, we may calculate λ for each type and get the following table.

AIII	$SU(m+n)/S(U(m)\times U(n))$,	$\lambda = m+n$,
DIII	$SO(2n)/U(n)$,	$\lambda = 2n-2$,
CI	$Sp(n)/U(n)$,	$\lambda = n+1$,
BDI	$SO(n+2)/SO(2)\times SO(n)$,	$\lambda = n$,
EIII	$E_6/\mathrm{Spin}(10)\times T^1$,	$\lambda = 12$,
EVII	$E_7/E_6\times T^1$,	$\lambda = 18$.

Theorem 6. *Let M be an n-dimensional irreducible Hermitian symmetric space of compact type. Then the group $H^q(M, \Omega E_{-k\omega_j})=0$ except for the following cases: (i) $q=1$ and $k\geqq 0$, (ii) $q=n$ and $k\leqq -\lambda$.*

Proof. By the theorem of Bott, we get

$$H^0(M, \Omega E_{-k\omega_j}) \neq 0 \qquad \text{if } k\geqq 0\,, \tag{2.1}$$

$$H^j(M, \Omega E_{-k\omega_j}) = 0 \qquad \text{for } j>0\,, \text{ if } k\geqq 0\,, \tag{2.2}$$

$$H^0(M, \Omega E_{-k\omega_j}) = 0 \qquad \text{if } k<0\,. \tag{2.3}$$

By Serre's duality theorem, we have

$$\dim H^q(M, \Omega E_{-k\omega_j}) = \dim H^{n-q}(M, \Omega(K_M\otimes E_{k\omega_j}))\,.$$

Hence we obtain, from (2.1) and (2.2)

$$H^n(M, \Omega E_{-k\omega_j}) \neq 0 \qquad \text{if } k\leqq -\lambda\,, \tag{2.4}$$

$$H^j(M, \Omega E_{-k\omega_j}) = 0 \qquad \text{for } j<n\,, \text{ if } k\leqq -\lambda\,. \tag{2.5}$$

We note that $E_{-k\omega_j}$ is positive if $k>0$. Then by Kodaira's vanishing theorem, we see

$$H^j(M, \Omega E_{-k\omega_j}) = 0 \qquad \text{for } j>0\,, \text{ if } k>-\lambda\,. \tag{2.6}$$

The conclusion follows from (2.1), (2.3), (2.4), (2.5) and (2.6).

REMARK. If M is a Kähler C-space whose 2nd Betti number is 1, we get the same conclusion in the same way as above.

Theorem 7. *Let M be an irreducible Hermitian symmetric space of compact type. Assume that M is not $P_n(\boldsymbol{C})$, $Sp(2)/U(2)$, $SO(6)/U(3)$ or $SO(8)/U(4)$. Then the group $H^q(M, (\Omega^1 E_{-k\omega_j}))=0$ except for the following cases:* (i) $q=0$ *and* $k>1$, (ii) $q=1$ *and* $k=0$, (iii) $q=n$ *and* $k<-\lambda+1$.

Proof. We may assume that M is of type AIII, CI or DIII by Theorems 3, 4 and 5.

It is known

$$n(\sigma)=\min\{k;\ \sigma=\tau_{\alpha_{i_1}}\cdots\tau_{\alpha_{i_k}},\ \alpha_{i_s}\in\Pi\} \qquad \text{for} \quad \sigma\in W,$$

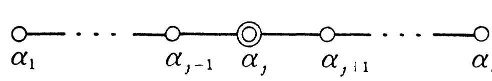
where τ_α denotes the symmetry with respect to $\alpha\in\Delta$. Therefore by the definition of $W^1(1)$, we have

$$W^1(1)=\{\tau_{\alpha_j}\}.$$

Since $\tau_{\alpha_j}\delta=\delta-\alpha_j$, we have by Theorem 2

$$(2.7) \qquad \dim H^q(M, \Omega^1(E_{-k\omega_j}))=\dim H^q(M, \Omega(E_{-(k\omega_j-\alpha_j)})) \quad \text{for } q=0, 1, \cdots.$$

1. The case $M=SU(l+1)/S(U(j)\times U(l+1-j))$ for $1<j<l$. The Dinkin diagram of Π is:



We may assume that $\mathfrak{h}_0$ is the set of points $(x_i)\in R^{l+1}$ such that $\sum_{i=1}^{l+1} x_i=0$. Let $\{\varepsilon_i\}_{i=1}^{l+1}$ be the natural basis of R^{l+1}. Then

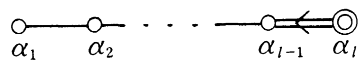
$$\begin{aligned}
&\Delta^+=\{\varepsilon_i-\varepsilon_j;\ 1\leqq i<j\leqq l+1\}\,,\\
&\Pi=\{\alpha_i=\varepsilon_i-\varepsilon_{i+1};\ i=1, 2, \cdots, l\}\,,\\
&\omega_j=\varepsilon_1+\cdots+\varepsilon_j\,,\\
&\delta=l\varepsilon_1+(l-1)\varepsilon_2+\cdots+2\varepsilon_{l-1}+\varepsilon_l\,,\\
&\Delta(\mathfrak{n}^+)=\{\varepsilon_s-\varepsilon_t;\ 1\leqq s\leqq j<t\leqq l+1\}\,.
\end{aligned}$$

It follows that

$$\left\{\left(\delta-\alpha_j, \frac{2\beta}{(\beta, \beta)}\right);\ \beta\in\Delta(\mathfrak{n}^+)\right\}=\{-1, 1, 2, \cdots, l\}\,.$$

Further if $\beta\in\Delta(\mathfrak{n}^+)$ satisfies $\left(\delta-\alpha_j, \frac{2\beta}{(\beta, \beta)}\right)=-1$, then $\beta=\alpha_j$. Therefore the conclusion follows from Theorem 1.

2. The case $M=Sp(l)/U(l)$ for $l\geqq 3$. The Dinkin diagram of Π is:



where α_l◎ means $\alpha_j=\alpha_l$. Let $\{\varepsilon_i\}_{i=1}^{l+1}$ be the basis of $\mathfrak{h}_0$ which satisfies $(\varepsilon_i, \varepsilon_j)=\delta_{ij}$. Then

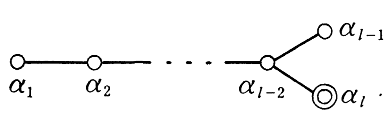
$$\Delta^+ = \{\varepsilon_i\pm\varepsilon_j;\ 1\leqq i<j\leqq l,\ 2\varepsilon_i;\ 1\leqq i\leqq l\}\ ,$$
$$\Pi = \{\alpha_1 = \varepsilon_1-\varepsilon_2, \cdots, \alpha_{l-1} = \varepsilon_{l-1}-\varepsilon_l, \alpha_l = 2\varepsilon_l\}\ ,$$
$$\omega_l = \varepsilon_1+\varepsilon_2+\cdots+\varepsilon_l\ ,$$
$$\delta = l\varepsilon_1+(l-1)\varepsilon_2+\cdots+2\varepsilon_{l-1}+\varepsilon_l\ ,$$
$$\Delta(\mathfrak{n}^+) = \{\varepsilon_i+\varepsilon_j;\ 1\leqq i<j\leqq l,\ 2\varepsilon_i;\ 1\leqq i\leqq l\}\ .$$

Hence we have

$$\{(\delta-\alpha_l, \beta);\ \beta\in\Delta(\mathfrak{n}^+)\} = \begin{cases} \{-2, 1, 2, \cdots, 2l-1, 2l\} & \text{if } l>3\ , \\ \{-2, 1, 2, 4, 5, 6\} & \text{if } l=3\ . \end{cases}$$

Further if $\beta\in\Delta(\mathfrak{n}^+)$ satisfies $(\delta-\alpha_l, \beta)=-2$, then $\beta=\alpha_l$. Since $(k\omega_l, \beta)=2k$, $\beta\in\Delta(\mathfrak{n}^+)$, the conclusion follows from Theorem 1.

3. $M=SO(2l)/U(l)$ for $l\geqq 5$. The Dynkin diagram of Π is:



where α_l◎ means $\alpha_j=\alpha_l$. Let $\{\varepsilon_i\}_{i=1}^{l}$ be a basis of $\mathfrak{h}_0$ such that $(\varepsilon_i, \varepsilon_j)=\delta_{ij}$. Then

$$\Delta^+ = \{\varepsilon_i\pm\varepsilon_j;\ 1\leqq i<j\leqq l\}$$
$$\Pi = \{\alpha_1 = \varepsilon_1-\varepsilon_2, \cdots, \alpha_{l-1} = \varepsilon_{l-1}-\varepsilon_l, \alpha_l = \varepsilon_{l-1}+\varepsilon_l\}$$
$$\omega_l = \tfrac{1}{2}(\varepsilon_1+\varepsilon_2+\cdots+\varepsilon_l)\ ,$$
$$\delta = (l-1)\varepsilon_1+(l-2)\varepsilon_2+\cdots+\varepsilon_{l-1}\ ,$$
$$\Delta(\mathfrak{n}^+) = \{\varepsilon_i+\varepsilon_j;\ 1\leqq i<j\leqq l\}\ .$$

Hence we have

$$\{(\delta-\alpha_l, \beta);\ \beta\in\Delta(\mathfrak{n}^+)\} = \{-1, 1, 2, \cdots, 2l-3\}\ ,$$
$$(k\omega_l, \beta) = k \qquad \text{for any}\quad \beta\in\Delta(\mathfrak{n}^+)\ .$$

Further, if $\beta\in\Delta(\mathfrak{n}^+)$ satisfies $(\delta-\alpha_l, \beta)=-1$, then $\beta=\alpha_l$. The conclusion follows from Theorem 1. q.e.d.

REMARK. Assume that M is one of the following Hermitian symmetric spaces of compact type. Then the group $H^q(M, \Omega^p(E_{-k\omega_j}))=0$ except for the following cases:

$Sp(2)/U(2)$ (i) $q=0$ and $k>1$, (ii) $q=1$ and $k=0$,
(iii) $q=2$ and $k=-1$, (iv) $q=3$ and $k<-2$,
$SO(6)/U(3)$ (i) $q=0$ and $k>1$, (ii) $q=1$ and $k=0$,
(iii) $q=3$ and $k<-2$,
$SO(8)/U(4)$ (i) $q=0$ and $k>1$, (ii) $q=1$ and $k=0$,
(iii) $q=5$ and $k=-4$, (iv) $q=6$ and $k<-5$.

3. Hypersurfaces of Hermitians symmetric spaces of compact type

We retain the notations and assumptions introduced in the previous sections. Let V be a hypersurface, that is closed codimension 1 complex submanifold, in an irreducible Hermitian symmetric space M. Taking a sufficiently fine finite covering $\{U_j\}$ of V, V is defined in each U_j by a holomorphic equation $s_j=0$. We associate with V the complex line bundle $\{V\}$ over M determined by the system $\{s_{jk}\}$ of non-vanishing holomorphic functions $s_{jk}=s_j/s_k$ on $U_j\cap U_k$. There is an integer d such that $\{V\}=E_{-d\omega_j}$. Since $\{V\}$ has a holomorphic section, $d>0$. We call d the degree of V. If $M=P_n(\boldsymbol{C})$, this definition coincides with the usual definition of the degree of the hypesurface of $P_n(\boldsymbol{C})$. We denote by Θ (resp. Ω) the sheaf of germs of holomorphic vector fields (resp. holomorphic functions) on V. We shall compute the dimensions of $H^q(V, \Theta)$ and $H^q(V, \Omega)$.

By Serre's duality theorem, we have

$$\dim H^0(V, \Theta) = \dim H^n(V, \Omega^1(K_V)) .$$

Denote by $E|_V$ the restriction to V of a holomorphic vector bundle E over M. Since $K_V=(K_M\otimes\{V\})|_V$, we have

$$(3.1) \qquad \dim H^0(V, \Theta) = \dim H^n(V, \Omega^1(E_{-(d-\lambda)\omega_j}|_V)) .$$

Let us recall the following vanishing theorem of Akizuki-Nakano [1]. Let L be a holomorphic line bundle over a compact complex manifold N. Then we have

$$(3.2) \qquad H^q(N, \Omega^p(L)) = 0 \quad \text{for} \quad p+q\geqq n+1\,, \quad \text{if } L \text{ is positive.}$$

Therefore we get

$$H^0(V, \Theta) = 0 \quad \text{if} \quad d>\lambda\,,$$

by (3.1).

Theorem 8. *Let M be an irreducible Hermitian symmetric space of compact type BDI, EIII or EVII, and let V be a hypersurface of M whose degree is d. Then we have*

$$H^0(V, \Theta) = 0 \qquad \text{if} \quad d \geqq 2 .$$

The following lemma follows from Theorems 3, 4 and 5.

Lemma 3. *Let M be an n-dimensional irreducible Hermitian symmetric space of compact type BDI, EIII or EVII. Then we have*

$$H^q(M, \Omega^p(E_{-k\omega_j})) = 0 , \quad H^{q+1}(M, \Omega^p(E_{-(k-d)\omega_j})) = 0$$

for $p+q=n+2$, $k=pd-\lambda$ if $2 \leqq p \leqq n$ and $d \geqq 2$.

Proof of Theorem 8. Recall the pair of exact sequences (Kodaira and Spencer [6]).

$$\cdots \to H^{q-1}(V, \Omega^p(E_{-k\omega_j}|_V)) \to H^q(M, \Omega''^p(E_{-k\omega_j})) \to H^q(M, \Omega^p(E_{-k\omega_j})) \cdots \to ,$$
$$\cdots \to H^q(M, \Omega''^p(E_{-k\omega_j})) \to H^q(V, \Omega^{p-1}(E_{-(k-d)\omega_j}|_V)) \to H^{q+1}(M, \Omega^p(E_{-(k-d)\omega_j})) \to \cdots ,$$

where $\Omega''^p(L)$ is the kernel of the canonical map of $\Omega^p(L)$ onto $\Omega^p(L|_V)$ for a holomorphic line bundle L over M. We see from the above pair of exact sequences and Lemma 3 that

$$H^{n-p+1}(V, \Omega^p(E_{-(pd-\lambda)\omega_j}|_V)) \to H^{n-p+2}(M, \Omega''^p(E_{-(pd-\lambda)\omega_j})) \to 0 ,$$
$$H^{n-p+2}(M, \Omega''^p(E_{-(pd-\lambda)\omega_j})) \to H^{n-p+2}(V, \Omega^{p-1}(E_{-((p-1)d-\lambda)\omega_j}|_V)) \to 0 .$$

Thus $H^{n-p+1}(V, \Omega^p(E_{-(pd-\lambda)\omega_j}|_V))=0$ implies $H^{n-p+2}(V, \Omega^{p-1}(E_{-((p-1)d-\lambda)\omega_j}|_V))=0$, while we have $H^1(V, \Omega^n(E_{-(nd-\lambda)\omega_j}|_V))=0$ by (3.2). Hence we obtain $H^n(V, \Omega^1(E_{-(d-\lambda)\omega_j}|_V))=0$. q.e.d.

REMARK. The above proof is motivated by Kodaira and Spencer [5].

Let N be a complex manifold and let $W \to N$ be a holomorphic vector bundle over N. Assume that V is a hypersurface of N. We denote by $\hat{\Omega}(W|_V)$ the trivial extension of $\Omega(W|_V)$ to N. Then we have the following exact sequence (Kodaira and Spencer [6])

$$0 \to \Omega(W \otimes \{V\})^{-1} \to \Omega(W) \to \hat{\Omega}(W|_V) \to 0 . \tag{3.3}$$

Assume that V is a hypersurface of M with degree d. It is easy to see that the normal bundle of V is equivalent to $\{V\}|_V$. Hence, by Kimura [4], the nullity of V as a minimal submanifold of M is given as follows:

$$n(V) = \dim_R H^0(V, \Omega(\{V\}|_V)) . \tag{3.4}$$

Denote by C the trivial line bundle over M. Then, by (3.3), we have the exact sequence:

$$0 \to \Omega(C) \to \Omega(\{V\}) \to \hat{\Omega}\{(V\}\,|_V) \to 0\,.$$

Since $H^1(M, \Omega(C))=0$,

$$\dim H^0(V, \Omega(\{V\}\,|_V)) = \dim H^0(M, \Omega(\{V\}))-1\,.$$

Since $\{V\}=E_{-d\omega_j}$, we get

$$\dim H^0(M, \Omega(\{V\})) = \dim V_{-d\omega_j}$$

by the theorem of Bott. Therefore,

$$\dim H^0(V, \Omega(\{V\}\,|_V)) = \dim V_{-d\omega_j} \quad 1\,, \tag{3.5}$$

and by (3.4)

$$n(V) = 2(\dim V_{-d\omega_j}-1)\,.$$

We prove the following lemma.

Lemma 4. *Let M be an irreducible Hermitian symmetric space of compact type of dimension>3. Assume that M is not $P_n(\boldsymbol{C})$, $Sp(2)/U(2)$, $SO(6)/U(3)$ or $SO(8)/U(4)$. Then for a hypersurface V of M, we have*

$$\dim H^0(V, (T\Omega(M)\,|_V)) = \dim H^0(M, \Omega T(M))\,,$$
$$H^1(V, \Omega(T(M)\,|_V)) = 0\,.$$

Proof. We have the exact sequence:

$$\cdots \to H^q(M, \Omega(T(M)\otimes E_{d\omega_j})) \to H^q(M, \Omega T(M)) \to H^q(V, (T\Omega(M)\,|_V)) \to \cdots$$

by (3.3). On the other hand, by Serre's duality theorem

$$\dim H^j(M, \Omega(T(M)\otimes E_{d\omega_j})) = \dim H^{n-j}(M, \Omega^1(E_{-(d-\lambda)\omega_j}))\,.$$

Hence, since $H^j(M, \Omega T(M))=0, j=1, 2$, the lemma follows from Theorem 7.

From this lemma we get the following.

Theorem 9. *Let M be an irreducible Hermitian symmetric space of compact type of dimension>3. Assume that M is not $P_n(\boldsymbol{C})$, $Sp(2)/U(2)$, $SO(6)/U(3)$ or $SO(8)/U(4)$. Then for a hypersurface V of M, we have*

$$\dim H^1(V, \Theta) = \dim H^0(V, \{V\}\,|_V) + \dim H^0(V, \Theta) - \dim H^0(M, \Omega T(M))\,.$$

By Theorems 8, 9 and (3.5), we obtain the following theorem.

Theorem 10 *Let M be an irreducible Hermitian symmetric space of compact type: BDI, EIII or EVII, and let V be a hypersurface of M. Assume that* dim $M>3$ *and the degree of* $V\geq 2$. *Then we have*

$$\dim H^1(V, \Theta) = \dim V_{-d\omega_j} - \dim H^0(M, \Omega T(M)) - 1 .$$

Finally the following theorem follows from Theorem 6.

Theorem 11. *Let M be an n-dimensional irreducible Hermitian symmetric space of compact type, and let V be a hypersurface of M with degree d. Then the group* $H^q(V, \Omega)$ *vanishes except for the following cases*:

$$q=0 \quad or \quad n-1 \qquad if \quad d\geqq\lambda ,$$
$$q=0 \qquad\qquad if \quad d<\lambda .$$

Proof. By Serre's duality theorem we have

$$\dim H^q(V, \Omega) = \dim H^{n-1-q}(V, \Omega(E_{-(d-\lambda)\omega_j}|_V)) \tag{3.7}$$

for $q=0, \cdots, n-1$. On the other hand, by applying (3.3), we obtain the exact sequence:

$$\begin{aligned} \cdots &\to H^j(M, \Omega(E_{\lambda\omega_j})) \to H^j(M, \Omega(E_{-(d-\lambda)\omega_j})) \\ &\to H^j(V, \Omega(E_{-(d-\lambda)\omega_j}|_V)) \to \cdots . \end{aligned} \tag{3.8}$$

It follows from Theorem 6 that:

$$\begin{aligned} &H^q(M, \Omega(E_{\lambda\omega_j})) = 0 , && \text{for} \quad q = 0, 1, \cdots, n-1 , \\ &H^n(M, \Omega(E_{\lambda\omega_j})) \neq 0 , && \\ &H^q(M, \Omega(E_{-(d-\lambda)\omega_j})) = 0 , && \text{for any } q , \quad \text{if } d<\lambda , \\ &H^q(M, \Omega(E_{-(d-\lambda)\omega_j})) = 0 , && \text{for } q>0 , \quad \text{if } d\geqq\lambda , \\ &H^0(M, \Omega(E_{-(d-\lambda)\omega_j})) \neq 0 , && \text{if} \quad d\geqq\lambda . \end{aligned}$$

Hence the theorem is obtained by (3.7) and (3.8).

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