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Supplements and Corrections to my paper; "On Algebras of Bounded Representation Type"

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In the present paper we give supplements and corrections to the paper mentioned in the title. We abbreviate this paper by [A].

Supplements

In [A] we showed the proof of the "if" part of Theorem 2 in outline because it was quite long but we are afraid that it is too rough to be understood. Therefore in this supplements we shall show it in some detail and moreover we are going to clear the proof of my paper [1].

1) Let A be an associative algebra over an algebraically closed field k, N its radical and $\sum_{\kappa}\sum_{\lambda} Ae_{\kappa,\lambda}$ the direct decomposition of A into directly indecomposable left ideals where $Ae_{\kappa\lambda}\simeq Ae_{\kappa 1}=Ae_{\kappa}$. Moreover we assume that $N^2=0$ and A is the basic algebra.

If Ne, where e is a primitive idempotent, is the direct sum at most two simple components an A-left module $\mathfrak{m}=\sum_i Aem_i$ is the direct sum of direct components of the type Aen_i. Next if $Ne=\sum_{i=1}^{3} Au_i$ an A-left module $\mathfrak{m}=\sum_i Aem_i$ is the direct sum of direct components of the following types;

(1) Aen_i

(2) Aen_j+Aen_{j+1} where $u_1n_j\neq 0,\ u_2n_j=0,\ u_3n_j=u_3n_{j+1},$
$u_2n_{j+1}\neq 0,\ u_1n_{j+1}=0.$

These proof was shown in detail in [A]. Hence we shall use these results without proof.

Now let $\mathfrak{m}=\sum_i\sum_{\lambda_i} Ae_im_{i,\lambda_i}$ be an arbitrary A-left module and $\{Ne_1, \cdots, Ne_r\}$ be a chain of A. From the results of [A], we have to prove it in the following four cases:

(1) $\{Ne_1, \cdots, Ne_r\}$ is such a chain that each Ne_i is the direct sum of at most two simple components.

(2) $\{Ne_1, \cdots, Ne_r\}$ is such a chain that either Ne_1 or Ne_r is the direct sum of three simple components and all other Ne_i are the direct sums of at most two simple components.

(3) $\{Ne_1, Ne_2, Ne_3, Ne_4\}$ is such a chain that Ne_3 is the direct sum of three simple components, Ne_2 is the direct sum of two simple components and Ne_1 and Ne_4 are simple.

(4) $\{Ne_1, Ne_2, Ne_3\}$ is such a chain that Ne_2 is the direct sum of three simple components and Ne_1 and Ne_3 are the direct sums of at most two simple components.

[The case I] Suppose that $\{Ne_1, \cdots, Ne_r\}$ is such a chain that $Ne_i = Au_i^{(\xi_i)} + Au_i^{(\xi_{i+1})}$ where $Au_i^{(\xi_i)} \cong \bar{A}\bar{e}_{\xi_i}$ and $\xi_i \neq \xi_{i+1}$. Then it is clear from the proof of [A] that an arbitrary A-left module $\mathfrak{m} = \sum_i \sum_{\lambda_i} Ae_i m_{i,\lambda_i}$ is decomposed into directly indecomposable components M_j of the following type;

$$M_j = Ae_1 n_{1,j} * Ae_2 n_{2,j} * \cdots * Ae_r n_{r,j}$$

where $Ae_i n_{i,j} * Ae_{i+1} n_{i+1,j}$ means that $Ae_i n_{ij} + Ae_{i+1} n_{i+1,j}$ and $Ae_i n_{i,j} \cap Ae_{i+1} n_{i+1,j} = Au_i^{(\xi_{i+1})} n_{i,j} = Au_{i+1}^{(\xi_{i+1})} n_{i+1,j}$.

If we express it by the matrix form we have the following form;

3) $R(a) = \begin{bmatrix} X & 0 \\ Z & Y \end{bmatrix}$ for an arbitrary element a of A where X and Y are the direct sums of $I_{s_i} \times x_i$ and $I_{t_j} \times y_j$ [1)] and

$$Z = \begin{pmatrix} x_{\xi_1,1} & x_{\xi_1,2} & & & \\ & x_{\xi_2,2} & x_{\xi_2,3} & & \\ & & x_{\xi_3,3} & & \\ & & & \ddots & \\ & & & & x_{\xi_r,r} \end{pmatrix}^{2)}.$$

From now on we have only to consider about the form of Z.

[The case II] $\{Ne_1, \cdots, Ne_r\}$ is supposed to be such a chain that

1) $I_{s_i} \times x_i = \underbrace{\begin{pmatrix} x_i & & 0 \\ & \ddots & \\ & & x_i \end{pmatrix}}_{s_i}$.

2) See [A], [1], [2] or [3] for $x_{\xi_i,j}$.

$Ne_1 = Au_1^{(\xi_1)} \oplus Au_1^{(\xi_0)} \oplus Au_1^{(\xi_2)}$ and $Ne_i = Au_i^{(\xi_i)} \oplus Au_i^{(\xi_{i+1})}$ where $i \neq 1$ and $Au_i^{(\xi_i)} \cong \bar{A}\bar{e}_{\xi_i}$.[3)]

Now we put $\mathfrak{m} = \sum_{\kappa_i} \mathfrak{m}_i$ where $\mathfrak{m}_i = \sum Ae_i m_{i,\kappa_i}$. From the result of [$A$], $\mathfrak{m}_1$ is the direct sum of $Ae_1 n_{1,i_1}$ or $Ae_1 n_{1,j} + Ae_1 n_{1,j+1}$ which have the type 1) or 2). Then we may arrange $\mathfrak{m}_1$ in the following way;

$$\begin{aligned}\mathfrak{m}_1 = &\overset{\oplus}{\sum_i} Ae_1 n_{1,i}^{(1)} \oplus \overset{\oplus}{\sum_i} Ae_1 n_{1,i}^{(2)} \oplus \overset{\oplus}{\sum_i} Ae_1 n_{1,i}^{(3)} \oplus \overset{\oplus}{\sum_i} Ae_1 n_{1,i}^{(4)} \oplus \overset{\oplus}{\sum_i} Ae_1 n_{1,i}^{(5)} \\ &\oplus \overset{\oplus}{\sum_i} Ae_1 n_{1,i}^{(6)} \oplus \overset{\oplus}{\sum_i} Ae_1 n_{1,i} \oplus \overset{\oplus}{\sum_j} (Ae_1 \bar{n}_{1,j}^{(4)} + Ae_1 \bar{n}_{1,j}^{(5)})\end{aligned}$$

where $Ne_1 n_{1,i}^{(1)} = Au_1^{(\xi_2)} n_{1,i}^{(1)}$, $Ne_1 n_{1,i}^{(2)} = Au_1^{(\xi_0)} n_{1,i}^{(2)}$, $Ne_1 n_{1,i}^{(3)} = Au_1^{(\xi_1)} n_{1,i}^{(3)}$, $Ne_1 n_{1,i}^{(4)} = Au_1^{(\xi_0)} n_{1,i}^{(4)} \oplus Au_1^{(\xi_2)} n_{1,i}^{(4)}$, $Ne_1 n_{1,i}^{(5)} = Au_1^{(\xi_1)} n_{1,i}^{(5)} \oplus Au_1^{(\xi_2)} n_{1,i}^{(5)}$, $Ne_1 n_{1,i}^{(6)} = Au_1^{(\xi_1)} n_{1,i}^{(6)} \oplus Au_1^{(\xi_0)} n_{1,i}^{(6)}$, $Ne_1 n_{1,i} \cong Ne_1$ and $(Ae_1 \bar{n}_{1,j}^{(4)} + Ae\bar{n}^{(5)})$ have the type 2).

It is clear that $\overset{\oplus}{\sum} Ae_1 n_{1,i}^{(2)} \oplus \overset{\oplus}{\sum} Ae_1 n_{1,i}^{(3)} \oplus \overset{\oplus}{\sum} Ae_1 n_{1,i}^{(6)}$ is the direct summand of $\mathfrak{m}$. Such a components is called the trivial component. Now by the same way as the case I we have $\mathfrak{m}_2 + \mathfrak{m}_3 + \cdots + \mathfrak{m}_r = \overset{\oplus}{\sum_i} (Ae_2 n_{2,i} * \cdots * Ae_r n_{r,i})$. Moreover we put $n_{2,i} = \hat{N}_{2,i}^{(p)}$ if $Ae_2 n_{2,i} * \cdots * Ae_p \hat{n}_{p,i}$ where $Ne_p \hat{n}_{p,i} = Au_p^{(\xi_p)} \hat{n}_{p,i}$ and $n_{2,i} = N_{2,i}^{(q)}$ if $Ae_2 n_{2,i} * \cdots * Ae_q n_{q,i}$ where $Ne_q n_{q,i} \cong Ne_q$. Other components are the direct summands of $\mathfrak{m}$ and need not be deliberated. Now suppose that $\sum_i \beta_i u_1^{(\xi_2)} n_{1i}^{(1)} + \sum_j \gamma_j u_1^{(\xi_2)} n_{1j}^{(4)} + \sum \delta_i u_1^{(\xi_2)} n_{1i}^{(5)} + \sum \rho_i u_1^{(\xi_2)} n_{1i} + \sum \varphi_i u_1^{(\xi_2)} \bar{n}_{1i}^{(5)} = \sum \beta_i^{(1)} u_2^{(\xi_2)} \hat{N}_{2i}^{(2)} + \sum \beta_i^{(2)} u_2^{(\xi_2)} \hat{N}_{2i}^{(2)} + \sum \beta_i^{(3)} u_2^{(\xi_2)} \hat{N}_{2i}^{(3)} + \cdots + \sum \beta_i^{(s)} u_2^{(\xi_2)} N_{2i}^{(r)}$. Then in the left hand side one of n_{1i} is replaced by $N_{1i} = \sum \beta_i N_{1i}^{(1)} + \sum \gamma_j n_{1j}^{(4)}$ if $\rho_i \neq 0$, and one of $n_{1j}^{(4)}$ is replaced by $N_{1j}^{(4)} = \sum \beta_i n_{1i}^{(1)} + \sum \gamma_j n_{1j}^{(4)}$ and one of $n_{1i}^{(5)}$ is replaced by $N_{1i}^{(5)} = \sum \delta_1 n_{1i}^{(5)} + \sum \varphi_1 \bar{n}_{1i}^{(5)}$ if $\rho_i = 0$. Next in the right hand side one of $N_{2i}^{(t)}$ is replaced by $M_{2i}^{(t)} = \sum \beta_i^{(1)} \hat{N}_{2i}^{(2)} + \cdots + \sum \beta_j^{(s)} N_{2j}^{(t)}$ where t is the minimum of all ρ of $N_{2i}^{(\rho)}$ or, if $\beta_i^{(\rho)} = 0$ for all $N_{\kappa i}^{(\rho)}$ one of $\hat{N}_{2i}^{(s)}$ is replaced by $\hat{M}_{2i}^{(s)} = \sum \beta_i^{(1)} \hat{N}_{2i}^{(2)} + \cdots + \sum \beta_i^{(s)} \hat{N}_{2i}^{(s)}$ where s is the maximum of all η of $\hat{N}_{2i}^{(\eta)}$.

Moreover suppose that $(Ae_1 N_{1i} * Ae_2 M_{2i}^{(s)} * \cdots * Ae_s n_{si}) + Ae_1 N_{1j}^{(4)} + (Ae_2 M_{2j}^{(r)} * \cdots * Ae_r n_{rj})$ where $r < s$ and $u_2^{(\xi_2)} M_{2j}^{(r)} = \eta_1 u_1^{(\xi_2)} N_{1i} + \eta_2 u_1^{(\xi_2)} N_{1j}^{(4)}$. Then if N_{1i} is replaced by $N'_{1i} = \eta_1 N_{1i} + \eta_2 N_{1j}^{(4)}$ and $M_{2i}^{(s)}$ is replaced by $M'^{(s)}_{2i} = M_{2i}^{(s)} - \frac{1}{\eta_1} M_{2j}^{(r)}$, $\cdots$, n_{si} by $n_{si} - \frac{1}{\eta_1} n_{rj}$ we have $(Ae_1 N'_{1i} * Ae_2 M_{2j}^{(r)} * \cdots * Ae_r n_{rj}) \oplus (Ae_1 N_{1j}^{(4)} * Ae_2 M'^{(s)}_{2i} * \cdots * Ae_s n'_{si})$.

In this way $\mathfrak{m}$ is the direct sum of directly indecomposable components of the following types;

(2, 1) $\quad Ae_1 N_{1i}^{(1)} * Ae_2 M_{2i}^{(s)} * \cdots * Ae_s n_{si}$

3) $\oplus$ denotes the direct sum.

(2, 2) $\quad Ae_1N_{1i}^{(4)} * Ae_2M_{2i}^{(s)} * \cdots * Ae_sn_{si}$

(2, 3) $\quad Ae_1N_{1i}^{(5)} * Ae_2M_{2i}^{(s)} * \cdots * Ae_sn_{si}$

(2, 4) $\quad Ae_1N_{1i} * Ae_2M_{2i}^{(s)} * \cdots * Ae_sn_{si}$

(2, 5) $\quad (Ae_1\overline{N}_{1j}^{(4)} + Ae_1\overline{N}_{1j}^{(5)}) * Ae_2M_{2i}^{(s)} * \cdots * Ae_sn_{si}$

(2, 6) $\quad (Ae_1N_{1j}^{(4)} * Ae_2M_{2j}^{(s)} * \cdots * Ae_sn_{sj}) + Ae_1N_{1,j+1}^{(5)} + (Ae_2M_{2i}^{(s)} * \cdots * Ae_rn_{ri})$

where $u_2^{(\xi_2)}M_{2i}^{(s)} = \eta_1u_1^{(\xi_2)}N_{1j}^{(4)} + \eta_2u_1^{(\xi_2)}N_{1,j+1}^{(5)}$ and $r \gneqq s$.

(2, 7) $\quad (Ae_1N_{1,j}^{(4)} * Ae_2M_{2,j}^{(s)} * \cdots * Ae_sn_{s,j}) + Ae_1N_{1,j+1}^{(1)} + (Ae_2\hat{M}_{2,i}^{(r)} * \cdots * Ae_r\hat{n}_{r,i})$

where $u_2^{(\xi_2)}M_{2,i}^{(r)} = \eta_1u_1^{(\xi_2)}N_{1,j}^{(4)} + \eta_2u_1^{(\xi_2)}N_{1,j+1}^{(5)}$ and $r \leq s$.

If we use the matrix form (3) these types are as follows;

$$(2, 1') \qquad Z = \begin{pmatrix} x_{\xi_1,1} & 0 & & \\ x_{\xi_0,1} & 0 & 0 & \\ x_{\xi_2,1} & x_{\xi_2,2} & & \\ 0 & x_{\xi_3,2} & & \\ & 0 & \ddots & \\ & & \ddots & \\ & & & x_{\xi_s,s} \end{pmatrix}.$$

This is the type 2,4) and contains 2,1), 2,2) and 2,3).

$$(2, 2') \qquad Z = \begin{pmatrix} x_{\xi_1,1} & 0 & 0 & & \\ x_{\xi_0,1} & x_{\xi_0,2} & 0 & 0 & \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} & & \\ 0 & 0 & x_{\xi_3,3} & & \\ & & & \ddots & \\ & 0 & & \ddots & \\ & & & & x_{\xi_s,s} \end{pmatrix}.$$

This the type 2,5).

$$(2, 3') \qquad Z = \begin{pmatrix} x_{\xi_1,1} & 0 & 0 & 0 & 0 & & \\ 0 & \hat{x}_{\xi_0,2} & 0 & 0 & 0 & & \\ x_{\xi_2,1} & 0 & x_{\xi_2,3} & \hat{x}_{\xi_2,4} & 0 & & \\ 0 & \hat{x}_{\xi_2,2} & 0 & \hat{x}'_{\xi_2,4} & 0 & & \\ 0 & 0 & x_{\xi_3,3} & 0 & x_{\xi_3,5} & & \\ 0 & 0 & 0 & \hat{x}_{\xi_3,4} & 0 & & \\ & & & & & \ddots & \\ & & & & & & \ddots \\ & & & & & & \hat{x}_{\xi_s,t} \end{pmatrix}.$$

This is the type (2,5) and contains the type (2,7).

[The case III] Suppose that $\{Ne_1, Ne_2, Ne_3, Ne_4\}$ is such a chain that $Ne_1 = Au_1^{(\xi_1)}$, $Ne_2 = Au_2^{(\xi_1)} \oplus Au_2^{(\xi_2)}$, $Ne_3 = Au_3^{(\xi_2)} \oplus Au_3^{(\xi_0)} Au_3^{(\xi_3)}$ and $Ne_4 = Au_4^{(\xi_3)}$. Moreover in this case and the next case we shall consider the proof by the matrix form. Hence we have only to considera about Z of 3).

Generally Z has the following form;

$$Z = \begin{pmatrix} Z_{\xi_1,1} & Z_{\xi_1,2} & 0 & 0 \\ 0 & Z_{\xi_2,2} & Z_{\xi_2,3} & 0 \\ 0 & 0 & Z_{\xi_0,3} & 0 \\ 0 & 0 & Z_{\xi_3,3} & Z_{\xi_3,4} \end{pmatrix}.$$

Now $\begin{pmatrix} Z_{\xi_1,1} & Z_{\xi_1,2} & 0 \\ 0 & Z_{\xi_2,2} & Z_{\xi_2,3} \\ 0 & 0 & Z_{\xi_0,3} \\ 0 & 0 & Z_{\xi_3,3} \end{pmatrix}$ is the direct sum of the following components from the result of the case II.

$$(3,1)\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x_{\xi_2,3} \\ 0 & 0 & 0 \\ 0 & 0 & x_{\xi_3,3} \end{pmatrix}, \quad (3,2)\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} \\ 0 & 0 & 0 \\ 0 & 0 & x_{\xi_3,3} \end{pmatrix}, \quad (3,3)\ \begin{pmatrix} 0 & x_{\xi_1,2} & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} \\ 0 & 0 & 0 \\ 0 & 0 & x_{\xi_3,3} \end{pmatrix},$$

$$(3,4)\ \begin{pmatrix} x_{\xi_1,1} & x_{\xi_1,2} & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} \\ 0 & 0 & 0 \\ 0 & 0 & x_{\xi_3,3} \end{pmatrix}, \qquad (3,5)\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} & 0 \\ 0 & x'_{\xi_2,2} & 0 & x'_{\xi_2,3} \\ 0 & 0 & 0 & x'_{\xi_0,3} \\ 0 & 0 & x_{\xi_3,3} & 0 \end{pmatrix},$$

$$(3,6)\ \begin{pmatrix} 0 & x_{\xi_1,2} & 0 & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} & 0 \\ 0 & x'_{\xi_2,2} & 0 & x'_{\xi_2,3} \\ 0 & 0 & 0 & x'_{\xi_0,3} \\ 0 & 0 & x_{\xi_3,3} & 0 \end{pmatrix}, \qquad (3,7)\ \begin{pmatrix} x_{\xi_1,1} & x_{\xi_1,2} & 0 & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} & 0 \\ 0 & x'_{\xi_2,2} & 0 & x'_{\xi_2,3} \\ 0 & 0 & 0 & x'_{\xi_0,3} \\ 0 & 0 & x_{\xi_3,3} & 0 \end{pmatrix},$$

$$(3,8)\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & x_{\xi_2,3} & 0 \\ 0 & 0 & x_{\xi_0,3} & x'_{\xi_0,3} \\ 0 & 0 & 0 & x_{\xi_3,3} \end{pmatrix}, \qquad (3,9)\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} & 0 \\ 0 & 0 & x_{\xi_0,3} & x'_{\xi_0,3} \\ 0 & 0 & 0 & x_{\xi_3,3} \end{pmatrix},$$

$$(3,10)\quad \begin{pmatrix} 0 & x_{\xi_1,2} & 0 & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} & 0 \\ 0 & 0 & x_{\xi_0,3} & x'_{\xi_0,3} \\ 0 & 0 & 0 & x_{\xi_3,3} \end{pmatrix},\qquad (3,11)\quad \begin{pmatrix} x_{\xi_1,1} & x_{\xi_1,2} & 0 & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} & 0 \\ 0 & 0 & x_{\xi_0,3} & x_{\xi'_0,3} \\ 0 & 0 & 0 & x_{\xi_3,3} \end{pmatrix},$$

$$(3,12)\quad \begin{pmatrix} 0 & x_{\xi_1,2} & x'_{\xi_1,2} & 0 & 0 \\ 0 & 0 & x'_{\xi_2,2} & x'_{\xi_2,3} & 0 \\ 0 & x_{\xi_2,2} & 0 & 0 & x'_{\xi_2,3} \\ 0 & 0 & 0 & 0 & x'_{\xi_0,3} \\ 0 & 0 & 0 & x'_{\xi_3,3} & 0 \end{pmatrix},$$

$$(3,13)\quad \begin{pmatrix} x_{\xi_1,1} & x_{\xi_1,2} & x'_{\xi_1,2} & 0 & 0 \\ 0 & 0 & x'_{\xi_2,2} & x'_{\xi_2,3} & 0 \\ 0 & x_{\xi_2,2} & 0 & 0 & x_{\xi_2,3} \\ 0 & 0 & 0 & 0 & x_{\xi_0,3} \\ 0 & 0 & 0 & x'_{\xi_3,3} & 0 \end{pmatrix},$$

$$(3,14)\quad \begin{pmatrix} x_{\xi_1,1} & x_{\xi_1,2} & 0 & 0 & 0 \\ x'_{\xi_1,1} & 0 & x'_{\xi_1,2} & 0 & 0 \\ 0 & x_{\xi_2,2} & 0 & x_{\xi_2,3} & 0 \\ 0 & 0 & x'_{\xi_2,2} & 0 & x'_{\xi_2,3} \\ 0 & 0 & 0 & 0 & x'_{\xi_0,3} \\ 0 & 0 & 0 & x_{\xi_3,3} & 0 \end{pmatrix},$$

$$(3,15)\quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x_{\xi_2,3} \\ 0 & 0 & x_{\xi_0,3} \\ 0 & 0 & x_{\xi_3,3} \end{pmatrix},\qquad (3,16)\quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} \\ 0 & 0 & x_{\xi_0,3} \\ 0 & 0 & x_{\xi_3,3} \end{pmatrix},$$

$$(3,17)\quad \begin{pmatrix} 0 & x_{\xi_1,2} & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} \\ 0 & 0 & x_{\xi_0,3} \\ 0 & 0 & x_{\xi_3,3} \end{pmatrix},\qquad (3,18)\quad \begin{pmatrix} x_{\xi_1,1} & x_{\xi_1,2} & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} \\ 0 & 0 & x_{\xi_0,3} \\ 0 & 0 & x_{\xi_3,3} \end{pmatrix},$$

$$(3,19)\quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x_{\xi_0,3} \\ 0 & 0 & x_{\xi_3,3} \end{pmatrix}.$$

Now let $Z^{(i)}$ and $Z^{(j)}$ be the components of the type $(3,i)$ and $(3,j)$. Moreover we put

$$Z = \begin{pmatrix} Z^{((i)} & 0 & x_{\xi_{3},4} \\ 0 & Z^{(j)} & x'_{\xi_{3},4} \end{pmatrix}$$

where $Z^{(i)}$ is on the different rows and columns from those of $Z^{(j)}$ and $x_{\xi_{3},4}$ is on the same row as $x_{\xi_{3},3}$ of $Z^{(i)}$ and $x'_{\xi_{3},4}$ is on the same row as $x_{\xi_{3},3}$ of $Z^{(j)}$. Then if $R(a)$ is not decomposed into at least two direct components,[4] $Z^{(i)}$ and $Z^{(j)}$ are said to be unseparated. Then if there exists a group which contains at least four unseparated components, we can construct an arbitrary large directly indecomposable representation by the same way as Lemma 6 or Lemma 7 of [A].

But if not, it is proved by the same way as Theorem 1 or [A] that an arbitrary representation is decomposed into directly indecomposable components of finite degrees. Hence we have only to show that there is no group which contains at least four unseparted components.

Now suppose that {1)→2), 3)} denotes that the components of the type (3,1) is unseparated from the component of the type (3,2) or (3,3). Then

{ 1) ⟶ 16), 17), 18), 10)}
{ 2) ⟶ 8), 10), 11), 19)}
{ 3) ⟶ 5), 7), 8), 16), 18), 19)}
{ 4) ⟶ 5), 8), 10), 12), 16), 19)}
{ 5) ⟶ 14), 19)}
{ 6) ⟶ 16), 18), 19)}
{ 7) ⟶ 16), 17), 19)}
{ 8) ⟶ 12), 13), 14)}
{10) ⟶ 13)}
{12) ⟶ 18), 19)}
{13) ⟶ 19)}
{14) ⟶ 16), 19)} .

Hence the groups of unseparated components are as follows:

(1,16), (1,17), (1,18), (1,19), (2,8), (2,10), (2,11), (2,19), (3,5), (3,7), (3,8), (3,16), (3,18), (3,19), (4,5), (4,8), (4,10), (4,12), (4,16), (4,19), (5,15), (5,19), (6,16), (6,18), (6,19), (7,16), (7,17), (7,19), (8,12), (8,13),

4) We denote the representation which has Z in the left lower corner by Z.

(8,14), (10,13), (12,18), (12,19), (13,19), (14,16), (14,19), (3,5,19), (3,7,16), (3,7,19), (4,5,19), (4,8,12), (4,12,19), (5,14,19).

From these groups we have indecomposable components of different types from above and if we repeat the same process as above we have the following types of indecomposable components and an arbitrary representation is the direct sum of these components. (3,1′), ⋯, (3,19′) are obtained from (3,1), ⋯, (3,19) such that $Z_{\xi_3,4}=x_{\xi_3,4}$ is on the same row as $x_{\xi_3,3}$ and to the right of it.

$$(3,\ 20')\qquad \begin{pmatrix} x_{\xi_2,3} & 0 & 0 & 0 & 0 \\ x_{\xi_3,3} & 0 & 0 & 0 & x_{\xi_3,4} \\ 0 & x'_{\xi_1,1} & x'_{\xi_1,2} & 0 & 0 \\ 0 & 0 & x'_{\xi_2,2} & x'_{\xi_2,3} & 0 \\ 0 & 0 & 0 & x'_{\xi_0,3} & 0 \\ 0 & 0 & 0 & x'_{\xi_3,3} & x'_{\xi_3,4} \end{pmatrix}$$

where if $x'_{\xi_1,1}$, $x'_{\xi_1,2}$ and $x'_{\xi_2,2}=0$, $x'_{\xi_2,3}=0$.

$$(3,\ 21')\qquad \begin{pmatrix} x_{\xi_2,2} & x_{\xi_2,3} & 0 & 0 & 0 & 0 & 0 \\ 0 & x_{\xi_3,3} & 0 & 0 & 0 & 0 & x_{\xi_3,4} \\ 0 & 0 & x'_{\xi_1,1} & x'_{\xi_1,2} & 0 & 0 & 0 \\ 0 & 0 & 0 & x'_{\xi_2,2} & x'_{\xi_2,3} & 0 & 0 \\ 0 & 0 & 0 & 0 & x_{\xi_0,3} & x'_{\xi_0,3} & 0 \\ 0 & 0 & 0 & 0 & 0 & x'_{\xi_3,3} & x'_{\xi_3,4} \end{pmatrix}$$

where if $x'_{\xi_1,1}$, $x'_{\xi_1,2}$ and $x'_{\xi_2,2}=0$, $x'_{\xi_2,3}=0$ and $x'_{\xi_0,3}=0$.

$$(3,\ 22')\qquad \begin{pmatrix} x''_{\xi_2,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_{\xi_0,3} & x''_{\xi_0,3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x''_{\xi_3,3} & 0 & 0 & 0 & 0 & 0 & x_{\xi_3,4} \\ 0 & 0 & x_{\xi_1,1} & x_{\xi_1,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & x'_{\xi_1,1} & 0 & x'_{\xi_1,2} & 0 & 0 & 0 \\ 0 & 0 & 0 & x_{\xi_2,2} & 0 & x_{\xi_2,3} & 0 & 0 \\ 0 & 0 & 0 & 0 & x'_{\xi_2,2} & 0 & x'_{\xi_2,3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x'_{\xi_0,3} & 0 \\ 0 & 0 & 0 & 0 & 0 & x_{\xi_3,3} & 0 & x'_{\xi_3,4} \end{pmatrix}$$

where $x'_{\xi_i,j}$ may be zero.

$$(3,\ 23')\quad \begin{pmatrix} x_{\xi_1,2} & 0 & 0 & 0 & 0 & 0 \\ x_{\xi_2,2} & x_{\xi_2,3} & 0 & 0 & 0 & 0 \\ 0 & x_{\xi_3,3} & 0 & 0 & 0 & x_{\xi_3,4} \\ 0 & 0 & x'_{\xi_1,1} & x'_{\xi_1,2} & 0 & 0 \\ 0 & 0 & 0 & x'_{\xi_2,2} & x'_{\xi_2,3} & 0 \\ 0 & 0 & 0 & 0 & x'_{\xi_0,3} & 0 \\ 0 & 0 & 0 & 0 & x'_{\xi_3,3} & x_{\xi'_{3,4}} \end{pmatrix}$$

where $x'_{\xi_2,2} \neq 0$ and if $x'_{\xi_1,1}=0$, $x'_{\xi_1,2}=0$.

$$(3,\ 24')\quad \begin{pmatrix} x_{\xi_1,1} & x_{\xi_1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_{\xi_2,2} & x_{\xi_2,3} & 0 & 0 & 0 & 0 & 0 \\ 0 & x'_{\xi_2,2} & 0 & x'_{\xi_2,3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x'_{\xi_0,3} & 0 & 0 & 0 & 0 \\ 0 & 0 & x_{\xi_3,3} & 0 & 0 & 0 & 0 & x_{\xi_3,4} \\ 0 & 0 & 0 & 0 & x''_{\xi_1,1} & x''_{\xi_1,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & x''_{\xi_2,2} & x''_{\xi_2,3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x''_{\xi_0,3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x''_{\xi_3,3} & x''_{\xi_3,4} \end{pmatrix}$$

where if $x_{\xi_1,1}=0$, $x''_{\xi_1,1} \neq 0$ and if $x''_{\xi_1,1}=0$, $x_{\xi_1,1} \neq 0$.

$$(3,\ 25')\quad \begin{pmatrix} x_{\xi_1,1} & x_{\xi_1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ x'_{\xi_1,1} & 0 & x'_{\xi_1,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & x_{\xi_2,2} & 0 & x_{\xi_2,3} & 0 & 0 & 0 & 0 \\ 0 & 0 & x'_{\xi_2,2} & 0 & x'_{\xi_2,3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x'_{\xi_0,3} & 0 & 0 & 0 \\ 0 & 0 & 0 & x_{\xi_3,3} & 0 & 0 & 0 & x_{\xi_3,4} \\ 0 & 0 & 0 & 0 & 0 & x''_{\xi_2,2} & x''_{\xi_2,3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x''_{\xi_0,3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x''_{\xi_3,3} & x''_{\xi_3,4} \end{pmatrix}$$

where $x'_{\xi_1,1}$, $x'_{\xi_1,2}$, $x'_{\xi_2,2}$, $x'_{\xi_2,3}$ and $x'_{\xi_0,3}$ may be zero.

$$
(3,\ 26') \quad \begin{pmatrix}
x_{\xi_1,1} & x_{\xi_1,2} & x'_{\xi_1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & x'_{\xi_2,2} & 0 & x_{2,3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & x'_{\xi_2,2} & 0 & x'_{\xi_2,3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & x'_{\xi_0,3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & x_{\xi_3,3} & 0 & 0 & 0 & 0 & x_{\xi_3,4} \\
0 & 0 & 0 & 0 & 0 & x''_{\xi_1,2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & x''_{\xi_2,2} & x''_{\xi_2,3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & x''_{\xi_0,3} & x'''_{\xi_0,3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & x''_{\xi_3,3} & x''_{\xi_3,4}
\end{pmatrix}
$$

where if $x''_{\xi_1,2}=0$, $x''_{\xi_2,2}=0$.

$$
(3,\ 27') \quad \begin{pmatrix}
x_{\xi_1,2} & x'_{\xi_1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\
x_{\xi_2,2} & 0 & x'_{\xi_2,3} & 0 & 0 & 0 & 0 & 0 \\
0 & x'_{\xi_2,2} & 0 & x'_{\xi_2,3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & x'_{\xi_0,3} & 0 & 0 & 0 & 0 \\
0 & 0 & x_{\xi_3,3} & 0 & 0 & 0 & 0 & x_{\xi_3,4} \\
0 & 0 & 0 & 0 & x''_{\xi_1,1} & x''_{\xi_1,2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & x''_{\xi_2,2} & x''_{\xi_2,3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & x''_{\xi_0,3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & x''_{\xi_3,3} & x''_{\xi_3,4}
\end{pmatrix}
$$

$$
(3,\ 28')\qquad
\left(\begin{array}{cccccccccccccc}
x_{\xi_1,1} & x_{\xi_1,2} & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & x''_{\xi_1,2} & 0 & 0 & 0 & 0 & 0 & & & & & & \\
0 & x_{\xi_2,2} & 0 & 0 & 0 & x_{\xi_2,3} & 0 & 0 & & & & 0 & & \\
0 & x'_{\xi_2,2} & 0 & 0 & 0 & 0 & x'_{\xi_2,3} & 0 & & & & & & \\
0 & 0 & x''_{\xi_2,2} & 0 & 0 & 0 & 0 & x''_{\xi_2,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & x'''_{\xi_2,2} & 0 & 0 & 0 & 0 & x'''_{\xi_2,3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & x^{(4)}_{\xi_2,2} & 0 & 0 & 0 & 0 & 0 & x^{(4)}_{\xi_2,3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & x^{(5)}_{\xi_2,2} & 0 & 0 & 0 & 0 & 0 & x^{(5)}_{\xi_2,3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & x'_{\xi_0,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x^{(4)}_{\xi_0,3} & 0 & 0 & 0 & 0 \\
 & & & & & 0 & 0 & 0 & 0 & 0 & 0 & x^{(6)}_{\xi_0,3} & 0 & 0 \\
 & & & & & 0 & 0 & 0 & 0 & 0 & x^{(5)}_{\xi_0,3} & 0 & 0 & 0 \\
 & & & & & x_{\xi_3,3} & 0 & 0 & 0 & 0 & 0 & 0 & & \\
 & & 0 & & & 0 & 0 & 0 & x'''_{\xi_3,3} & 0 & 0 & 0 & & \\
 & & & & & 0 & 0 & 0 & 0 & 0 & x^{(5)}_{\xi_3,3} & 0 & Z_{\xi_3,4} & \\
 & & & & & 0 & 0 & 0 & 0 & 0 & 0 & x^{(6)}_{\xi_3,3} & & \\
 & & & & & 0 & 0 & x''_{\xi_3,3} & 0 & 0 & 0 & 0 & &
\end{array}\right)
$$

where

$$
Z_{\xi_3,4}=\begin{pmatrix} x_{\xi_3,4} & 0 \\ 0 & 0 \\ 0 & 0 \\ x^{(6)}_{\xi_3,4} & 0 \\ x''_{\xi_3,4} & 0 \end{pmatrix},\quad
\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ x^{(5)}_{\xi_3,4} & 0 \\ x^{(6)}_{\xi_3,4} & 0 \\ x''_{\xi_3,4} & 0 \end{pmatrix},\quad
\begin{pmatrix} x_{\xi_3,4} & 0 \\ 0 & 0 \\ 0 & 0 \\ x^{(6)}_{\xi_3,4} & x^{(6)}_{\xi_3,4} \\ 0 & x''_{\xi_3,4} \end{pmatrix},\quad
\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ x^{(5)}_{\xi_3,4} & 0 \\ x^{(6)}_{\xi_3,4} & \bar{x}^{(6)}_{\xi_3,4} \\ 0 & x''_{\xi_3,4} \end{pmatrix},
$$

$$
\begin{pmatrix} x_{\xi_3,4} & \bar{x}_{\xi_3,4} \\ 0 & 0 \\ 0 & x^{(5)}_{\xi_3,4} \\ x^{(6)}_{\xi_3,4} & 0 \\ x''_{\xi_3,4} & 0 \end{pmatrix}
\quad \text{or} \quad
\begin{pmatrix} x_{\xi_3,4} & 0 \\ 0 & x'''_{\xi_3,4} \\ 0 & 0 \\ x^{(6)}_{\xi_3,4} & \bar{x}^{(6)}_{\xi_3,4} \\ x''_{\xi_3,4} & 0 \end{pmatrix}.
$$

$$
(3,\,29') \qquad
\left(\begin{array}{cccccccccccccc}
x_{\xi_1,1} & 0 & x_{\xi_1,2} & 0 & 0 & 0 & 0 & 0 & & & & & & \\
0 & x'_{\xi_1,1} & 0 & x'_{\xi_1,2} & 0 & 0 & 0 & 0 & & & & & & \\
0 & x''_{\xi_1,1} & 0 & 0 & x''_{\xi_1,2} & 0 & 0 & 0 & & & & 0 & & \\
0 & 0 & x_{\xi_2,2} & 0 & 0 & 0 & x_{\xi_2,3} & 0 & & & & & & \\
0 & 0 & 0 & x'_{\xi_2,2} & 0 & 0 & 0 & x'_{\xi_2,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & x''_{\xi_2,2} & 0 & 0 & 0 & x''_{\xi_2,3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & x^{(3)}_{\xi_2,2} & 0 & 0 & 0 & x^{(3)}_{\xi_2,3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & x^{(4)}_{\xi_2,2} & 0 & 0 & 0 & 0 & x^{(4)}_{\xi_2,3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x''_{\xi_0,3} & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & 0 & 0 & 0 & 0 & x^{(4)}_{\xi_0,3} & 0 & 0 & 0 \\
 & & & & & & 0 & 0 & 0 & 0 & 0 & x^{(5)}_{\xi_0,3} & 0 & 0 \\
 & & & & & & x_{\xi_3,3} & 0 & 0 & 0 & 0 & 0 & & \\
 & & 0 & & & & 0 & x'_{\xi_3,3} & 0 & 0 & 0 & 0 & \multicolumn{2}{c}{Z_{\xi_{3,4}}} \\
 & & & & & & 0 & 0 & 0 & x^{(3)}_{\xi_3,3} & 0 & 0 & & \\
 & & & & & & 0 & 0 & 0 & 0 & 0 & x^{(5)}_{\xi_3,3} & &
\end{array}\right)
$$

where

$$
Z_{\xi_{3,4}} = \begin{pmatrix} x_{\xi_3,4} & 0 \\ 0 & 0 \\ x^{(3)}_{\xi_3,4} & 0 \\ x^{(5)}_{\xi_3,4} & 0 \end{pmatrix},\quad
\begin{pmatrix} x_{\xi_3,4} & 0 \\ 0 & 0 \\ x^{(3)}_{\xi_3,4} & \bar{x}^{(3)}_{\xi_3,4} \\ 0 & x^{(5)}_{\xi_3,4} \end{pmatrix},\quad
\begin{pmatrix} 0 & 0 \\ x'_{\xi_3,4} & 0 \\ x^{(3)}_{\xi_3,4} & 0 \\ x^{(5)}_{\xi_3,4} & 0 \end{pmatrix},\quad
\begin{pmatrix} 0 & 0 \\ x'_{\xi_3,4} & 0 \\ x^{(3)}_{\xi_3,4} & \bar{x}^{(3)}_{\xi_3,4} \\ 0 & x^{(5)}_{\xi_3,4} \end{pmatrix},
$$

$$
\begin{pmatrix} x_{\xi_3,4} & 0 \\ 0 & x'_{\xi_3,4} \\ x^{(3)}_{\xi_3,4} & \bar{x}^{(3)}_{\xi_3,4} \\ x^{(5)}_{\xi_3,4} & 0 \end{pmatrix}
\quad \text{or} \quad
\begin{pmatrix} 0 & x_{\xi_3,4} \\ x'_{\xi_3,4} & 0 \\ x^{(3)}_{\xi_3,4} & 0 \\ x^{(5)}_{\xi_3,4} & \bar{x}^{(5)}_{\xi_3,4} \end{pmatrix}.
$$

$$
(3,\,30')\qquad
\left(\begin{array}{ccccccccccc}
x_{\xi_1,1} & x_{\xi_1,2} & 0 & 0 & 0 & 0 & 0 & & & & \\
0 & 0 & x'_{\xi_1,2} & x''_{\xi_1,2} & 0 & 0 & 0 & & & & \\
0 & x_{\xi_2,2} & 0 & 0 & x_{\xi_2,3} & 0 & 0 & & & 0 & \\
0 & 0 & x'_{\xi_2,2} & 0 & 0 & x'_{\xi_2,3} & 0 & & & & \\
0 & 0 & 0 & x''_{\xi_2,2} & 0 & 0 & x''_{\xi_2,3} & & & & \\
 & & & & 0 & 0 & 0 & x^{(3)}_{\xi_2,3} & x^{(4)}_{\xi_2,3} & 0 & 0 \\
 & & & & 0 & 0 & x''_{\xi_0,3} & 0 & 0 & 0 & 0 \\
 & & & & 0 & 0 & 0 & 0 & x^{(4)}_{\xi_0,3} & 0 & 0 \\
 & 0 & & & x_{\xi_3,3} & 0 & 0 & 0 & 0 & & \\
 & & & & 0 & x'_{\xi_3,3} & 0 & 0 & 0 & & Z_{\xi_3,4} \\
 & & & & 0 & 0 & 0 & x^{(3)}_{\xi_3,3} & 0 & &
\end{array}\right)
$$

where

$$
Z_{\xi_3,4} = \begin{pmatrix} x_{\xi_3,4} & 0 \\ x'_{\xi_3,4} & 0 \\ x^{(3)}_{\xi_3,4} & 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} x_{\xi_3,4} & 0 \\ x'_{\xi_3,4} & \bar{x}'_{\xi_3,4} \\ 0 & x^{(3)}_{\xi_3,4} \end{pmatrix}.
$$

[The case IV] Suppose that $\{Ne_1, Ne_2, Ne_3\}$ is such a chain that $Ne_1 = Au_1^{(\xi_1)} \oplus Au_1^{(\xi_2)}$, $Ne_2 = Au_2^{(\xi_2)} \oplus Au_2^{(\xi_0)} \oplus Au_2^{(\xi_3)}$ and $Ne_3 = Au_3^{(\xi_3)} \oplus Au_3^{(\xi_4)}$. Now by the same way as above we use the matrix form. Then Z has the following forms;

$$
Z = \begin{pmatrix} Z_{\xi_1,1} & 0 & 0 \\ Z_{\xi_2,1} & Z_{\xi_2,2} & 0 \\ 0 & Z_{\xi_0,2} & 0 \\ 0 & Z_{\xi_3,2} & Z_{\xi_3,3} \\ 0 & 0 & Z_{\xi_4,3} \end{pmatrix}
$$

and $\begin{pmatrix} Z_{\xi_3,3} \\ Z_{\xi_4,3} \end{pmatrix}$ is assumed to have the following form;

$$
\begin{pmatrix} Z_{\xi_3,3} \\ \\ Z_{\xi_4,3} \end{pmatrix} = \begin{pmatrix} Z'_{\xi_3,3} & Z''_{\xi_3,3} \\ \\ 0 & Z_{\xi_4,3} \end{pmatrix} \quad \text{where} \quad Z_{\xi_4,3} = \begin{pmatrix} x_{\xi_4,3} & & 0 \\ & x'_{\xi_4,3} & \\ 0 & & \ddots \\ & & & x^{(t)}_{\xi_4,3} \end{pmatrix}.
$$

Now $\begin{pmatrix} Z_{\xi_1,1} & 0 & 0 \\ Z_{\xi_2,1} & Z_{\xi_2,2} & 0 \\ 0 & Z_{\xi_0,2} & 0 \\ 0 & Z_{\xi_3,2} & Z'_{\xi_3,3} \end{pmatrix}$, is the direct sum of the components of the the following types;

$$(4,1)\quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x_{\xi_0,2} & 0 \\ 0 & x_{\xi_3,2} & 0 \end{pmatrix},\qquad (4,2)\quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & x_{\xi_2,2} & 0 \\ 0 & 0 & 0 \\ 0 & x_{\xi_3,2} & 0 \end{pmatrix},\qquad (4,3)\quad \begin{pmatrix} 0 & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & 0 \\ 0 & 0 & 0 \\ 0 & x_{\xi_3,2} & 0 \end{pmatrix},$$

$$(4,4)\quad \begin{pmatrix} x_{\xi_1,1} & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & 0 \\ 0 & 0 & 0 \\ 0 & x_{\xi_3,2} & 0 \end{pmatrix},\qquad (4,5)\quad \begin{pmatrix} x_{\xi_2,1} & x_{\xi_2,2} & 0 & 0 \\ x'_{\xi_2,1} & 0 & x'_{\xi_2,2} & 0 \\ 0 & 0 & x'_{\xi_0,2} & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 \end{pmatrix},$$

$$(4,6)\quad \begin{pmatrix} x_{\xi_1,1} & 0 & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & 0 & 0 \\ x'_{\xi_2,1} & 0 & x'_{\xi_2,2} & 0 \\ 0 & 0 & x'_{\xi_0,2} & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 \end{pmatrix},\qquad (4,7)\quad \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & x_{\xi_2,2} & x'_{\xi_2,2} & 0 \\ 0 & 0 & x'_{\xi_0,2} & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 \end{pmatrix},$$

$$(4,8)\quad \begin{pmatrix} 0 & 0 & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & x'_{\xi_2,2} & 0 \\ 0 & 0 & x'_{\xi_0,2} & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 \end{pmatrix},\qquad (4,9)\quad \begin{pmatrix} x_{\xi_1,1} & 0 & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & x'_{\xi_2,2} & 0 \\ 0 & 0 & x'_{\xi_0,2} & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 \end{pmatrix},$$

$$(4,10)\quad \begin{pmatrix} x_{\xi_1,1} & x'_{\xi_1,1} & 0 & 0 & 0 \\ x_{\xi_2,1} & 0 & x_{\xi_2,2} & 0 & 0 \\ 0 & x'_{\xi_2,1} & 0 & x'_{\xi_2,2} & 0 \\ 0 & 0 & 0 & x'_{\xi_0,2} & 0 \\ 0 & 0 & x_{\xi_3,2} & 0 & 0 \end{pmatrix},\qquad (4,11)\quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & x_{\xi_2,2} & 0 \\ 0 & x_{\xi_0,2} & 0 \\ 0 & x_{\xi_3,2} & 0 \end{pmatrix},$$

$$(4,12)\quad \begin{pmatrix} 0 & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & 0 \\ 0 & x_{\xi_0,2} & 0 \\ 0 & x_{\xi_3,2} & 0 \end{pmatrix},\qquad (4,13)\quad \begin{pmatrix} x_{\xi_1,1} & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & 0 \\ 0 & x_{\xi_0,2} & 0 \\ 0 & x_{\xi_3,2} & 0 \end{pmatrix},$$

$$(4,14)\quad \begin{pmatrix} x_{\xi_0,2} & 0 & 0 \\ x_{\xi_3,2} & 0 & x_{\xi_3,3} \\ 0 & x'_{\xi_2,2} & 0 \\ 0 & x'_{\xi_3,2} & x'_{\xi_3,3} \end{pmatrix},\qquad (4,15)\quad \begin{pmatrix} x_{\xi_0,2} & 0 & 0 & 0 \\ x_{\xi_3,2} & 0 & 0 & x_{\xi_3,3} \\ 0 & x'_{\xi_2,1} & x_{\xi_2,2} & 0 \\ 0 & 0 & x'_{\xi_3,2} & x'_{\xi_3,4} \end{pmatrix},$$

$$(4,16)\quad \begin{pmatrix} x_{\xi_0,2} & 0 & 0 & 0 \\ x_{\xi_3,2} & 0 & 0 & x_{\xi_3,3} \\ 0 & x'_{\xi_1,1} & 0 & 0 \\ 0 & x'_{\xi_2,1} & x'_{\xi_2,2} & 0 \\ 0 & 0 & x'_{\xi_3,2} & x'_{\xi_3,3} \end{pmatrix},\qquad (4,17)\quad \begin{pmatrix} x_{\xi_0,2} & 0 & 0 & 0 & 0 \\ x_{\xi_3,2} & 0 & 0 & 0 & x_{\xi_3,3} \\ 0 & x'_{\xi_2,1} & x'_{\xi_2,2} & 0 & 0 \\ 0 & x''_{\xi_2,1} & 0 & x''_{\xi_2,2} & 0 \\ 0 & 0 & 0 & x''_{\xi_0,2} & 0 \\ 0 & 0 & x'_{\xi_3,2} & 0 & x'_{\xi_3,3} \end{pmatrix},$$

$$(4,18)\quad \begin{pmatrix} x_{\xi_0,2} & 0 & 0 & 0 & 0 \\ x_{\xi_3,2} & 0 & 0 & 0 & x_{\xi_3,3} \\ 0 & x'_{\xi_1,1} & 0 & 0 & 0 \\ 0 & x'_{\xi_2,1} & x'_{\xi_2,2} & 0 & 0 \\ 0 & x''_{\xi_2,1} & 0 & x''_{\xi_2,2} & 0 \\ 0 & 0 & 0 & x''_{\xi_0,2} & 0 \\ 0 & 0 & x'_{\xi_3,2} & 0 & x'_{\xi_3,3} \end{pmatrix},\qquad (4,19)\quad \begin{pmatrix} x_{\xi_0,2} & 0 & 0 & 0 & 0 & 0 \\ x_{\xi_3,2} & 0 & 0 & 0 & 0 & x_{\xi_3,3} \\ 0 & x'_{\xi_1,1} & x''_{\xi_1,1} & 0 & 0 & 0 \\ 0 & x'_{\xi_2,1} & 0 & x'_{\xi_2,2} & 0 & 0 \\ 0 & 0 & x''_{\xi_2,1} & 0 & x''_{\xi_2,2} & 0 \\ 0 & 0 & 0 & 0 & x''_{\xi_0,2} & 0 \\ 0 & 0 & 0 & x'_{\xi_3,2} & 0 & x'_{\xi_3,3} \end{pmatrix},$$

$$(4,20)\quad \begin{pmatrix} x_{\xi_2,1} & x_{\xi_2,2} & 0 & 0 & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 & x_{\xi_3,3} \\ 0 & 0 & x'_{\xi_2,2} & x''_{\xi_2,2} & 0 \\ 0 & 0 & 0 & x''_{\xi_0,2} & 0 \\ 0 & 0 & x'_{\xi_3,2} & 0 & x'_{\xi_3,3} \end{pmatrix},\qquad (4,21)\quad \begin{pmatrix} x_{\xi_2,1} & x_{\xi_2,2} & 0 & 0 & 0 & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 & 0 & x_{\xi_3,3} \\ 0 & 0 & x'_{\xi_1,1} & 0 & 0 & 0 \\ 0 & 0 & x'_{\xi_2,1} & x'_{\xi_2,2} & x''_{\xi_2,2} & 0 \\ 0 & 0 & 0 & 0 & x''_{\xi_0,2} & 0 \\ 0 & 0 & 0 & x'_{\xi_3,2} & 0 & x_{\xi_3,3} \end{pmatrix},$$

$$(4,22)\quad \begin{pmatrix} x_{\xi_1,1} & 0 & 0 & 0 & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & 0 & 0 & 0 & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 & 0 & x_{\xi_3,3} \\ 0 & 0 & x'_{\xi_2,1} & x'_{\xi_2,2} & 0 & 0 \\ 0 & 0 & x''_{\xi_2,1} & 0 & x''_{\xi_2,2} & 0 \\ 0 & 0 & 0 & 0 & x''_{\xi_0,2} & 0 \\ 0 & 0 & 0 & x'_{\xi_3,2} & 0 & x'_{\xi_3,3} \end{pmatrix},\qquad (4,23)\quad \begin{pmatrix} x_{\xi_1,1} & 0 & 0 & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & 0 & 0 & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 & x_{\xi_3,3} \\ 0 & 0 & x'_{\xi_2,2} & x''_{\xi_2,2} & 0 \\ 0 & 0 & 0 & x''_{\xi_0,2} & 0 \\ 0 & 0 & x'_{\xi_2,2} & 0 & x'_{\xi_3,3} \end{pmatrix},$$

$$(4,24)\quad \begin{pmatrix} x_{\xi_1,1} & 0 & 0 & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & 0 & 0 & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 & x_{\xi_3,3} \\ 0 & 0 & x'_{\xi_2,1} & x'_{\xi_2,2} & 0 \\ 0 & 0 & 0 & x''_{\xi_0,2} & 0 \\ 0 & 0 & 0 & x'_{\xi_3,2} & x'_{\xi_3,3} \end{pmatrix},\qquad (4,25)\quad \begin{pmatrix} x_{\xi_1,1} & 0 & 0 & 0 & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & 0 & 0 & 0 & 0 \\ x'_{\xi_2,1} & 0 & x'_{\xi_2,2} & 0 & 0 & 0 \\ 0 & 0 & x'_{\xi_0,2} & 0 & 0 & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 & 0 & x_{\xi_3,3} \\ 0 & 0 & 0 & x''_{\xi_2,1} & x''_{\xi_2,2} & 0 \\ 0 & 0 & 0 & 0 & x''_{\xi_0,2} & 0 \\ 0 & 0 & 0 & 0 & x''_{\xi_2,2} & x''_{\xi_3,3} \end{pmatrix},$$

$$(4, 26) \quad \begin{pmatrix} x_{\xi_2,2} & x'_{\xi_2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & x'_{\xi_0,2} & 0 & 0 & 0 & 0 & 0 \\ x_{\xi_3,2} & 0 & 0 & 0 & 0 & 0 & x_{\xi_3,3} \\ 0 & 0 & x''_{\xi_1,1} & x'''_{\xi_1,1} & 0 & 0 & 0 \\ 0 & 0 & x''_{\xi_2,1} & 0 & x''_{\xi_2,2} & 0 & 0 \\ 0 & 0 & 0 & x'''_{\xi_2,1} & 0 & x'''_{\xi_2,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & x'''_{\xi_0,2} & 0 \\ 0 & 0 & 0 & 0 & x''_{\xi_0,2} & 0 & x''_{\xi_3,3} \end{pmatrix},$$

$$(3, 27) \quad \begin{pmatrix} x_{\xi_1,1} & 0 & 0 & 0 & 0 & 0 & 0 \\ x_{\xi_2,1} & x_{\xi_2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & x_{\xi_3,2} & 0 & 0 & 0 & 0 & x_{\xi_3,3} \\ 0 & 0 & x'_{\xi_2,1} & x'_{\xi_2,2} & 0 & 0 & 0 \\ 0 & 0 & x''_{\xi_2,1} & 0 & x''_{\xi_2,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & x''_{\xi_0,2} & 0 & 0 \\ 0 & 0 & 0 & x'_{\xi_3,2} & 0 & 0 & x'_{\xi_3,3} \\ 0 & 0 & 0 & 0 & 0 & x'''_{\xi_0,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & x'''_{\xi_3,2} & x'''_{\xi_3,3} \end{pmatrix},$$

$$(3, 28) \quad \begin{pmatrix} x_{\xi_2,2} & 0 & 0 & 0 \\ x_{\xi_3,2} & 0 & 0 & x_{\xi_3,3} \\ 0 & x'_{\xi_2,1} & x'_{\xi_2,2} & 0 \\ 0 & 0 & x'_{\xi_0,2} & 0 \\ 0 & 0 & x'_{\xi_3,2} & x'_{\xi_3,3} \end{pmatrix}, \qquad (3, 29) \quad \begin{pmatrix} x_{\xi_2,2} & 0 & 0 & 0 \\ x_{\xi_3,2} & 0 & 0 & x_{\xi_3,3} \\ 0 & x'_{\xi_1,1} & 0 & 0 \\ 0 & x'_{\xi_2,1} & x'_{\xi_2,2} & 0 \\ 0 & 0 & x'_{\xi_0,2} & 0 \\ 0 & 0 & x'_{\xi_3,2} & x'_{\xi_3,3} \end{pmatrix}.$$

Then

1) $\longrightarrow$ 2), 3), 4), 5), 6), 10), 22), 24), 25), 27), 28), 29),
2) $\longrightarrow$ 12), 13),
3) $\longrightarrow$ 7), 9), 14), 16), 17), 18), 19), 23), 26),
4) $\longrightarrow$ 5), 7), 12), 14), 17), 18), 25), 27), 28),
5) $\longrightarrow$ 14), 16), 18), 24), 27),
6) $\longrightarrow$ 12), 14), 28),

7) ⟶ 10), 15), 16), 19), 22), 24), 27),
9) ⟶ 15), 20),
10) ⟶ 14), 16), 17), 18), 23),
12) ⟶ 29),
14) ⟶ 22), 24), 25),
15) ⟶ 22), 23), 26),
16) ⟶ 17), 22),
17) ⟶ 22), 23), 24),
18) ⟶ 22), 24),
19) ⟶ 22), 23).

Hence the groups of unseparated components are as follows;

(1,2), (1,3), (1,4), (1,5), (1,6), (1,10), (1,22), (1,24), (1,25), (1,27), (1,28), (1,27), (2,12), (2,13), (3,7), (3,9), (3,14), (3,16), (3,17), (3,18), (3,19), (3,23), (3,26), (4,5), (4,7), (4,12), (4,14), (4,17), (4,18), (4,25), (4,28), (5,14), (5,16), (5,18), (5,24), (5,27), (6,12), (6,14), (6,28), (7,10), (7,15), (7,16), (7,19), (7,22), (7,24), (9,15), (9,20), (10,14), (10,16), (10,17), (10,18), (10,23), (12,29), (14,24), (14,25), (15,22), (15,23), (15,26), (16,17), (16,22), (17,22), (17,23), (17,24), (18,22), (18,24), (19,22), (19,23), (14,25), (1,5,24), (3,7,16), (3,7,19), (3,7,19), (3,17,23), (3,19,23), (4,5,14), (4,5,18), (5,18,24), (4,14,25), (5,14,24), (7,10,16), (7,15,22), (7,16,22), (7,19,22), (10,16,17), (10,17,23), (16,17,22).

From these groups we have different types of indecomposable components from above types and if we repeat the same process we have a finite number of types of indecomposable components and an arbitrary representation is the direct sum of these components. Now we shall omit to arrange all the types, because the number of them is large and they are also obtained by the same way as the case III.

2) In [1] we showed that if k is algebraically closed and $N^2=0$ the class of algebras of bounded representation type is that of algebras of finite representation type but the proof was rough and was hard to be understood. But from the above results it is clear that we have only to show the following lemma. Namely

[Lemma] *Let $R(a)$ have the following type*;

$$
Z=\left(\begin{array}{ccccccccccccc}
x_{\xi_2,2} & \hat{x}_{\xi_2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & x_{\xi_0,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x_{\xi_3,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & y_{\xi_3,3} \\
0 & 0 & x'_{\xi_1,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & x'_{\xi_2,1} & x'_{\xi_2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & x'_{\xi_3,2} & 0 & x'_{\xi_3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & x''_{\xi_0,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & x''_{\xi_3,2} & x''_{\xi_3,3} & 0 & 0 & 0 & 0 & 0 & 0 & y''_{\xi_3,3} \\
 & & & & 0 & 0 & x'''_{\xi_1,1} & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & 0 & 0 & x'''_{\xi_2,1} & x'''_{\xi_2,2} & 0 & 0 & 0 & 0 & 0 \\
 & & & & 0 & 0 & 0 & x'''_{\xi_3,2} & 0 & 0 & 0 & x'''_{\xi_3,3} & y'''_{\xi_3,3} \\
 & & & & 0 & 0 & 0 & 0 & x^{(4)}_{\xi_2,1} & x^{(4)}_{\xi_2,2} & 0 & 0 & 0 \\
 & 0 & & & 0 & 0 & 0 & 0 & \hat{x}^{(4)}_{\xi_2,1} & 0 & \hat{x}^{(4)}_{\xi_2,2} & 0 & 0 \\
 & & & & 0 & 0 & 0 & 0 & 0 & 0 & x^{(4)}_{\xi_0,2} & 0 & 0 \\
 & & & & 0 & 0 & 0 & 0 & 0 & x^{(4)}_{\xi_3,2} & 0 & x^{(4)}_{\xi_3,3} & 0 \\
 & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_{\xi_4,3}
\end{array}\right)
$$

Then there is a non-singular matrix P such that $PR(a)=R'(a)P$ where $R(a)$ and $R'(a)$ have Z and Z' of the above type.

Because the indecomposable components of other types have the same constructions as this and the number of different types are finite. The proof of this lemma is clear from [1] or [A].

Corrections

The following corrections should be made in the paper [A].

1) In Lemma 2 of [A], if $e=e_i$, the form of $R(a)$ is not used. But it is shown by the simple computation that this lemma is true.

This correction should be made to other lemmas.

2) In Theorem 2 we showed the types of indecomposable components of the case 3 but they do not include all the types. Now we shall omit to show all the types but they are obtained from above results.

3) In Theorem 2 the form of Q'_{ij} or D'_{ij} are not complete.

Generaly I_t must be $\begin{pmatrix} x_1 & & \\ & \ddots & \\ & & x_t \end{pmatrix}$.

4) Errata; p. 104, line 21. For 8 read 14.

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References

[1] T. Yoshii: Note on Algebras of Bounded Representation Type, Proc. Japan Acad. **32**, 441–445 (1956).

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[3] James P. Jans: On the Indecomposable Representation of Algebras, Dissertation, University of Michigan (1954).