

THE THREE-SEPARATED-ARC PROPERTY OF THE MODULAR FUNCTION

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Let D be the open unit disk and I be the unit circle in the complex plane, and denote the Riemann sphere by Ω . If $f(z)$ is a function defined on D with values belonging to Ω , if $\zeta \in I$, and if A is an arc at ζ , then $C_A(f, \zeta)$ denotes the cluster set of f at ζ along A . If there exist three mutually exclusive arcs A_1, A_2, A_3 at ζ such that

$$C_{A_1}(f, \zeta) \cap C_{A_2}(f, \zeta) \cap C_{A_3}(f, \zeta) = \emptyset,$$

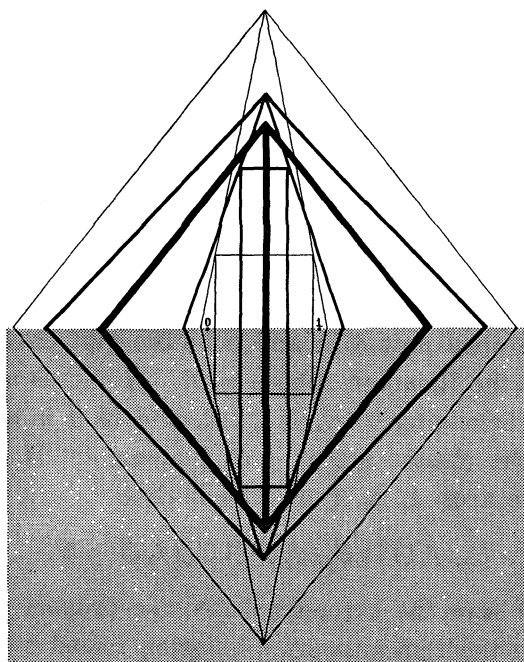
then f is said to have the three-separated-arc property at ζ .

The following theorem answers a question raised by Belna [1, p. 220] concerning the modular function $\mu(z)$ that maps D onto the universal covering surface W of the extended w -plane punctured at the points $w = 0, 1, \infty$.

THEOREM. *The modular function $\mu(z)$ has the three-separated-arc property at every point of I .*

Proof. For convenience and clarity, we refer the reader to the Figure, which represents the w -plane. The shaded lower half is the lower half-plane, the unshaded upper half is the upper half-plane. We consider three graphs, g_1, g_2, g_3 ; g_1 is represented by the lightest lines, g_2 by the heavier lines, and g_3 by the heaviest lines.

For $j = 1, 2, 3$, let G_j denote the set of points on W that overlie the set g_j , and let γ_j be the preimage of G_j under the mapping $\mu(z)$. One readily infers from the Figure that if $\zeta \in I$, then there are in D three mutually exclusive arcs A_1, A_2, A_3 at ζ such that $A_j \subset \gamma_j$ ($j = 1, 2, 3$). The cluster set $C_{A_j}(\mu, \zeta)$ is clearly a subset of g_j ($j = 1, 2, 3$). Since it is evident that $g_1 \cap g_2 \cap g_3 = \emptyset$, the theorem is proved.



Figure

REFERENCE

- [1] C. L. Belna, Intersections of arc-cluster sets for meromorphic functions, Nagoya Math. J. 40 (1970), 213-220.

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