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CLASSIFICATION OF NON-GORENSTEIN Q-FANO d-FOLDS OF FANO INDEX GREATER THAN $d-2$

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Introduction

First of all we recall some definitions.

DEFINITION 0.1. A d-dimensional normal projective variety X is called a **Q**-*Fano d-fold* if it has only terminal singularities and if the anti-canonical Weil divisor $-K_X$ is ample. The *singularity index* $I = I(X)$ of X is defined to be the smallest positive integer such that $-IK_X$ is Cartier. Then there is a positive integer r and a Cartier divisor H such that $-IK_X \sim rH$. Taking the largest number of such r, we call r/I the *Fano index* of X.

Since $\chi(xH)$ is a polynomial of degree d, the vanishing theorem 1.1 implies that $r/I \leq d+1$. In the Gorenstein case (i.e. $-K_X$ is Cartier), it is well known that if its Fano index is d, then $(X, H) \cong$ (quadric, $\mathcal{O}(1)$), and if its Fano index is $d+1$, then $(X, H) \cong (\mathbf{P}^d, \mathcal{O}(1))$.

DEFINITION 0.2. A Gorenstein **Q**-Fano d-fold is called *Del Pezzo variety* if its Fano index is $d-1$.

There are remarkable works for Del Pezzo varieties by T. Fujita [Fu1, 2].

In this paper we shall prove the following

THEOREM. *Let X be a* **Q**-*Fano d-fold* $(d \geq 3)$, *I the singularity index of X and r an integer such that $-IK_X \sim rH$ for a Cartier divisor H. Assume that $1 < I$ and $d-2 < r/I$. Then (X, H) has one of the following expressions as weighted hypersurfaces or weighted projective spaces.*

[1] $((6) \subset \mathbf{P}(1,1,2,3,I,\ldots,I), \mathcal{O}(I))$ $\qquad I = 2,3,4,5,6, \quad d \geq 3$

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[2]	$((4) \subset \mathbf{P}(1,1,1,2,I,\dots,I), \mathcal{O}(I))$	$I = 2,3, \quad d \geq 3$
[3]	$((3) \subset \mathbf{P}(1,1,1,1,2,\dots,2), \mathcal{O}(2))$	$I = 2, \quad d \geq 3$
[4]	$((2) \subset \mathbf{P}(1,1,1,1,1,2,\dots,2), \mathcal{O}(2))$	*the defining equation does not contain the coordinate of weight* 2, $I = 2$, $d \geq 4$
[5]	$(\mathbf{P}(1,1,1,2,\dots,2), \mathcal{O}(2))$	$I = 2, \quad d \geq 3$
[6]	$(\mathbf{P}(1,1,1,2,4,\dots,4), \mathcal{O}(4))$	$I = 4, \quad d \geq 4$
[7]	$(\mathbf{P}(1,1,1,1,3,\dots,3), \mathcal{O}(3))$	$I = 3, \quad d \geq 4$
[8]	$(\mathbf{P}(1,1,1,1,1,2,\dots,2), \mathcal{O}(2))$	$I = 2, \quad d \geq 5$

In particular, $\operatorname{Pic} X \cong \mathbf{Z}$. *Conversely, general varieties having above expressions are* $\mathbf{Q}$*-Fano varieties, and their Fano indices are*

$$\begin{cases} d - 2 + 1/I & \textit{except type } [5] \\ d - 1 + 1/I & \textit{type } [5]. \end{cases}$$

Smooth Fano 3-folds are classified by Fano, Iskovskih, Shokurov, Mori, Mukai, et. al. (cf. [Is1,2], [Sh1,2], [MM], [Mu]). According to the minimal model program (cf. [KMM]), we have to extend this to the case of the varieties with $\mathbf{Q}$-factorial terminal singularities. In this case, K_X is not Cartier. This is just the point of difficulties and of interests. Y. Kawamata proved in [Ka2] that singularity indices I and the degree $(-K_X)^3$ are bounded for all the $\mathbf{Q}$-Fano 3-folds whose Picard number $\rho(X) = 1$. Thanks to this we have a hope to classify $\mathbf{Q}$-Fano 3-folds.

In the case of $\mathbf{Q}$-Fano d-folds with $1 < I$ and $d - 2 < r/I$, the main methods of classification are (1) to bound the numerical invariants by Riemann-Roch formula, (2) ladder argument, and (3) a criterion for terminal singularity. For (2), we have the next theorem.

THEOREM 0.1 (V. Alexeev [Al]). *Let* (X, Δ) *be a* d*-dimensional* $(d \geq 2)$ *log Fano variety* (*i.e. normal projective variety with only log terminal singularities with* $\lfloor \Delta \rfloor = 0$ *and* $-(K_X + \Delta)$ *is ample*) *with the property that there exists an* $r \in \mathbf{Q}_{>0}$ $(r > d - 2)$ *and an ample Cartier divisor* H *such that*

$$-(K_X + \Delta) \sim_{\mathbf{Q}} rH.$$

Then a general member of $|H|$ *is a normal variety with only log terminal singularities.*

In Section 1, we obtain a $\mathbf{Q}$-Fano version of above Theorem 0.2 as its corollary, and construct a ladder of subvarieties. We can reduce our problem to the 3-dimensional case by using this ladder. In Section 2, we classify the invariants in 3-dimensional case by the Riemann-Roch formula for singular varieties and the ladder argument. Then we can find a very good member in $|H|$ which is a non-singular Del Pezzo surface and a weighted hypersurface. In Section 3, we show that X can also be written as weighted hypersurface. Using a criterion for terminal singularities, the proof is completed in Section 4.

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Notation. In this paper we always assume that the ground field k is algebraically closed of characteristic 0, and we will follow the notation and the terminology of [KMM]. The following symbols will be used.

$\sim$: linear equivalence
$\sim_{\mathbf{Q}}$: $\mathbf{Q}$-linear equivalence
$\equiv$: numerical equivalence
K_X : the canonical divisor of X
$\rho(X)$: the Picard number of X
$\mathrm{h}^i(D) := \dim_k \mathrm{H}^i(D)$
$\chi(D) := \sum_i (-1)^i \mathrm{h}^i(X, D)$
$c_i(X)$: the i-th Chern class of X

1. Ladder

Recall here some definitions about ladder (cf. [Ful]). Let V be a variety and L an ample line bundle on V. A sequence $(V, L) = (V_d, L_d) > (V_{d-1}, L_{d-1}) > \cdots > (V_1, L_1)$ is called *a ladder* if each V_{j-1} ($j = 2,3,\ldots, d$) is an irreducible and reduced member of $|L_j|$, where L_j is the restriction of L to V_j. A ladder is called *regular* if each restriction map $r : \mathrm{H}^0(V_j, L_j) \to \mathrm{H}^0(V_{j-1}, L_{j-1})$ is surjective.

The next theorem is fundamental.

THEOREM 1.1 (Vanishing Theorem [KMM]). *Let X be a normal projective variety with only log terminal singularities, and D a $\mathbf{Q}$-Cartier Weil divisor on X. If $D -$*

K_X is ample, then

$$\mathrm{H}^i(X, \mathcal{O}_X(D)) = 0 \quad \forall i > 0.$$

As a corollary of Theorem 0.1, the next proposition holds.

PROPOSITION 1.2. *With the same hypotheses of Theorem 0.1, $|H|$ has at most isolated base points, which are regular points of X and their multiplicities are one. In particular if X is a $\mathbf{Q}$-Fano d-fold $(d \geq 3)$, then the general member of $|H|$ is also $\mathbf{Q}$-Fano.*

Proof. By Theorem 0.1, we have a ladder,

$$(X, H) = (X_d, H_d) > (X_{d-1}, H_{d-1}) > \cdots > (X_2, H_2),$$

where X_i $(2 \leq i \leq d)$ is an i-dimensional log Fano variety. Note that this ladder is regular since $H^1(X_i, \mathcal{O}_{X_i}) = 0$ by the Vanishing Theorem. So it is sufficient to prove the assertion only in the case $\dim X = 2$.

In the proof of Theorem 0.1, Alexeev showed the following claim.

CLAIM. *Let Y be a nonsingular projective variety, $f: Y \to X$ a proper birational morphism, $|L|$ a free linear system on Y and $\sum F_j$ a normal crossing divisor on Y such that*

(1) $K_Y \sim_{\mathbf{Q}} f^*K_X + \sum a_jF_j$, *with* $a_j \in \mathbf{Q}$, $a_j > -1$ *whenever* F_j *is exceptional for* f.

(2) $|f^*H| = |L| + \sum r_jF_j$ *with* $r_j \in \mathbf{Z}$, $r_j \geq 0$ *and* $r_j \neq 0$ *iff* $f(F_j) \in \mathrm{Bs}\,|H|$, *then*

$$a_j - r_j > -1.$$

Hence if $r_j \neq 0$, then $a_j > 0$. Since $\dim X = 2$, this means that $f(F_j)$ is a smooth point of X and $r_j = 1$ if $a_j = 1$. □

LEMMA 1.3. *Let X be a $\mathbf{Q}$-Fano d-fold $(d \geq 3)$. Assume that $1 < I$ and $d - 2 < \frac{r}{I}$, then I and r are coprime.*

Proof. Assume the opposite, then I and r have a common divisor $c > 1$; put $I = cI'$ and $r = cr'$. Then we have a non-trivial torsion Weil divisor $D := I'K_X + r'H$. Now take a ladder,

$$(X, H) = (X_d, H_d) > (X_{d-1}, H_{d-1}) > \cdots > (X_2, H_2)$$

where X_i $(1 \leq i \leq d)$ is a $\mathbf{Q}$-Fano i-fold. We have the next exact sequence.

$$0 \to \mathcal{O}_X(D - H) \to \mathcal{O}_X(D) \to \mathcal{O}_{X_{d-1}}(D \cap X_{d-1}) \to 0$$

Since $-K_X + D - H \equiv -K_X - H$ is ample, $\mathrm{H}^1(X, D - H) = 0$ by the Vanishing Theorem. Therefore the restriction map is surjective;

$$\mathrm{H}^0(X, D) \twoheadrightarrow H^0(X_{d-1}, D \cap X_{d-1}).$$

Note that $\mathrm{H}^0(X, D) = 0$. So if $D \cap X_{d-1}$ is trivial, then $\mathrm{h}^0(X_{d-1}, D \cap X_{d-1}) = 1$ and this is absurd. Hence $D \cap X_{d-1}$ is a non-trivial torsion Weil divisor. By repeating this procedure and so on, we see that $D \cap X_2$ is a non-trivial torsion Weil divisor. But X_2 is a nonsingular Del Pezzo surface. This is a contradiction. □

2. Riemann-Roch

In this section we restrict the possible values of $(-K_{X_2})^2$ by using a ladder and the Riemann-Roch formula for singular varieties.

THEOREM 2.1 (Y. Kawamata [Ka1], [Re]). *Let X be a 3-fold with only terminal singularities. Then,*

$$\chi(\mathcal{O}_X) = \frac{1}{24}(-K_X) \cdot c_2(X) + \frac{1}{24}\sum_p \left(i_p - \frac{1}{i_p}\right)$$

where

$$(-K_X) \cdot c_2(X) := f^*(-K_X) \cdot c_2(Y)$$

for a resolution $f : Y \to X$, i_p is the singularity index of $p \in X$ and the summation is taken for all singular points on X counted with multiplicities.

LEMMA 2.2. *Let I' be the singularity index of X_{d-1} and put $I = mI'$. Then there exists a Cartier divisor L of X_{d-1} such that $mL \sim H_{d-1}$ and*

$$-I'K_{X_{d-1}} \sim (r - I)L, \quad (d-3)m < \frac{r - I}{I'}.$$

Proof. By the adjunction formula,

$$-mI'K_{X_{d-1}} \sim (r - I)H_{d-1}.$$

Since m is coprime to $r - I$ and the Picard group of a $\mathbf{Q}$-Fano variety has no torsion part (Indeed, if $\pi ; Y \to X$ is a n-sheeted etale map of $\mathbf{Q}$-Fano varieties, then $\chi(\mathcal{O}_Y) = n\chi(O_X)$), there is a Cartier divisor L of X_{d-1} with $mL \sim H_{d-1}$. The next inequality follows from $d - 2 < r/I$. $\sqcup$

LEMMA 2.3. *Let X be a $\mathbf{Q}$-Fano 3-fold, and assume that $1 < I$ and $1 < r/I$. Take a general member $S \in |H|$ which is a nonsingular Del Pezzo surface. If the Fano index of S is 1, then $(-K_S)^2 \leq 3$.*

Proof. By the preceding lemma,

$$-K_S \sim (r - I)L$$

where L is a Cartier divisor with $IL \sim H_S$. Since the Fano index of S is 1, $r - I = 1$. Hence $(-K_S)^2 = H^3/I^2$.

Next, by Theorem 2.1 and the ordinary Riemann-Roch formula, we have

$$\begin{aligned}\chi(-H) &= 1 - \frac{H^3}{12}\left(-1 + \frac{r}{I}\right)\left(-2 + \frac{r}{I}\right) - \frac{1}{12}\frac{1}{r}(-K_X)c_2(X) \\ &= 1 - \frac{H^3}{12}\left(-1 + \frac{r}{I}\right)\left(-2 + \frac{r}{I}\right) - \frac{I}{12r}\left(24 - \frac{N}{I}\right),\end{aligned}$$

where we put

$$N := I\sum_p \left(i_p - \frac{1}{i_p}\right).$$

Since $-K_X - H$ is ample, the Vanishing Theorem implies that $0 = \mathrm{h}^0(-H) = \chi(-H)$. Therefore

$$N = (2I - r)(12 - (-K_S)^2 r(r - I)) = (I - 1)(12 - (-K_S)^2(I + 1)).$$

Note that $N > 0$, so $(-K_S)^2 \leq 3$. $\square$

3. Weighted complete intersection

Recall some definitions about weighted complete intersections (cf. [Do], [Mo]). Let $a_0, \ldots, a_t$ be positive integers and $T = k[X_0, \ldots, X_t]$ a graded polynomial ring with $\deg X_i = a_i$. Let $\{f_i\}_{i=1,2,\ldots,s}$ be a regular sequence of homogeneous elements with $\deg f_i = b_i$ and J the homogeneous ideal generated by the $\{f_i\}_{i=12,\ldots,s}$. In this situation, $\mathbf{P}(a_0, \ldots, a_t) := \mathrm{Proj}\, T$ is called a *weighted projective space of type* $(a_0, \ldots, a_t)$, and $((b_1, \ldots, b_s) \subset \mathbf{P}(a_0, \ldots, a_t)) := (\mathrm{Proj}\, T/J \subset \mathrm{Proj}\, T)$ a *weighted complete intersection of type* $(b_1, \ldots, b_s)$. Especially in the case $s = 1$, we call it a *weighted hypersurface.*

We saw in Lemma 2.3 that the general member S of $|H|$ is a nonsingular Del Pezzo surface of $(-K_S)^2 \leq 3$, quadric or $\mathbf{P}^2$. It is well known that these S can be written as weighted hypersurfaces (cf. [HW]).

THEOREM 3.1. *Let S be nonsingular Del Pezzo surface of $(-K_S)^2 = 1,2$ or 3. Then $(S, \mathcal{O}_S(-K_S))$ is expressed as follows.*

$(-K_S)^2$	
1	$((6) \subset \mathbf{P}(1,1,2,3), \mathcal{O}_S(1))$
2	$((4) \subset \mathbf{P}(1,1,1,2), \mathcal{O}_S(1))$
3	$((3) \subset \mathbf{P}(1,1,1,1), \mathcal{O}_S(1))$

We shall prove that X can also be written as weighted hypersurface by using this fact and the next lemma.

LEMMA 3.2. *Let X be a **Q**-Fano variety of* $\dim X \geq 3$, *I the singularity index of X and H a Cartier divisor of X such that $-IK_X \sim rH$ for a positive r. Assume that (X, H) satisfies the following conditions.*

(1) *I and r are coprime.*

(2) *There exists a member Y in $|H|$ which can be expressed as*

$$(Y, H_Y) \cong ((b_1, \ldots, b_s) \subset \mathbf{P}(a_0, \ldots, a_t), \mathcal{O}_Y(I)).$$

Then (X, H) can be expressed as

$$((b_1, \ldots, b_s) \subset \mathbf{P}(a_0, \ldots, a_t, I), \mathcal{O}_X(I)).$$

Proof. Since I and r are coprime, there exist integers p and q such that $pr + qI = 1$. We define the Weil divisor D as

$$D := -pK_X + qH.$$

Then

$$ID \sim H, \quad \mathcal{O}_Y(D \cap Y) \cong \mathcal{O}_Y(1).$$

And obviously, the next exact sequences hold.

$$0 \to \mathcal{O}_X((n-I)D) \to \mathcal{O}_X(nD) \to \mathcal{O}_Y(n) \to 0 \quad (\forall n \in \mathbf{Z})$$

$\mathrm{H}^1(X, (n-I)D) = 0$ by the Theorem 1.1 and Serre's duality. Then we have next exact sequences;

$$0 \to \mathrm{H}^0(X, (n-I)D) \xrightarrow{\times\varphi} \mathrm{H}^0(X, nD) \to \mathrm{H}^0(Y, \mathcal{O}_Y(n)) \to 0 \quad (\forall n \in \mathbf{Z})$$

where $\varphi \in \mathrm{H}^0(X, ID)$ is a section corresponding to Y. The rest of proof is shown by standard argument, so we omit it (cf. [Mo] Theorem 3.6). □

4. Classification

In this section we complete the proof of the theorem stated in the introduction. The next criterion of terminal singularities for weighted hypersurfaces is a direct consequence of [Re] Theorem 4.6.

LEMMA 4.1. *Let $X = (b) \subset \mathbf{P}(a_0, \ldots, a_t)$ be a weighted hypersurface with the assumption that its defining polynomial does not contain the t-th coordinate. If X has only terminal singularities, then*

$$b < a_0 + \cdots + a_{t-1} - a_t.$$

We also use the next theorem frequently.

THEOREM 4.2. ([Re] Theorem 4.11). *A quotient singularity $X = \mathbf{A}^n/\mu_r$ of type $\frac{1}{r}(a_1, \ldots, a_n)$ is terminal if and only if*

$$0 < \sum_{i=1}^{n} ka_i \bmod r - r \quad \textit{for } k = 1, \ldots, r-1.$$

We note here the next fact.

If the defining equation f of a weighted hypersurface $X = (b) \subset \mathbf{P}(a_0, \ldots, a_t)$ can be written as $f = X_i + g$, then X is isomorphic to $\mathbf{P}(a_0, \ldots, \hat{a}_i, \ldots, a_t)$.

Proof of the theorem. First we consider the case in which X_{d-1} is a Gorenstein $\mathbf{Q}$-Fano variety, i.e., $I' = 1$ and $m = I$ with the notation in Lemma 2.2. Since the Fano index of X_{d-1} is greater than $(d-1)-2$, (X_{d-1}, H_{d-1}) is (Del Pezzo, IL), (Quadric, $\mathcal{O}(I)$) or $(\mathbf{P}^{d-1}, \mathcal{O}(I))$.

1. *Case* $(X_{d-1}, H_{d-1}) \cong$ (Del Pezzo, IL).

In this case $r - I = d - 2$, hence $d = 3$ by Lemma 2.2. Then by Lemmas 2.3, 3.2 and Theorem 3.1, (X, H) has the one of the following expressions.

[1] $((6) \subset \mathbf{P}(1,1,2,3,I), \mathcal{O}(I))$
[2] $((4) \subset \mathbf{P}(1,1,1,2,I), \mathcal{O}(I))$
[3] $((3) \subset \mathbf{P}(1,1,1,1,I), \mathcal{O}(I))$

If (X, H) is type [1], I is not more than 6. Indeed, if I is more than 6, then its defining equation does not contain the weight I's coordinate. Hence we can use Lemma 4.1 and lead a contradiction. By the same reason, if (X, H) is type [2] (or type [3]), then I is not more than 4 (resp. 3). We claim that the case type [2] and $I = 4$, and type [3] and $I = 3$ does not occur. In this case, if its defining equation contains the homogeneous coordinate X_4, then its singular index is not I. So we can apply Lemma 4.1 and lead a contradiction.

2. *Case* $(X_{d-1}, H_{d-1}) \cong$ (Quadric, $\mathcal{O}(I)$).

In this case $r - I = d - 1$, hence $(d, I) = (3, *)$ or $(4,2)$ by Lemma 2.2. If $d = 3$, by Lemma 3.1, X can be written as

$$X = (2) \subset \mathbf{P}(1,1,1,1,I).$$

The case $d = 3$ cannot occur. Indeed, if the defining equation f is written as $f = g + X_4$, then X is isomorphic to $\mathbf{P}^3$, and if f does not contain X_4, we get a contradiction by Lemma 4.1. In the case $d = 4$, we get type [4].

3. *Case* $(X_{d-1}, H_{d-1}) \cong (\mathbf{P}^{d-1}, \mathcal{O}_{X_{d-1}}(I))$.

In this case $r - I = d$, hence $(d, I) = (3, *)$, $(4,2)$, $(4,3)$ or $(5,2)$ by Lemma 2.2. Since I and r are coprime, the case $(d, I) = (4,2)$ cannot occur. In the case $d = 3$, by Lemma 3.2, (X, H) can be written as

$$(\mathbf{P}(1,1,1,I), \mathcal{O}_X(I)).$$

By Theorem 4.2, I must be 2 and we get type [5]. In the case (4,3) (or (5,2)), we get type [7] (resp. [8]) by Lemma 3.2.

Next we consider the case in which the general member $X_{d-1} \in |H|$ is not Gorenstein. It is enough to show that if (X_{d-1}, H_{d-1}) has an expression of type [1] $\sim$ [8], then (X, H) can also be expressed as [1]$\sim$[8]. If $I = I'$, then by Lemma 3.2, (X, H) has an expression of type [1]$\sim$[8]. So we may assume that $1 < I' < I$. Note that the Fano index of X_{d-1} is smaller than $d - 1$. Therefore by Lemma 2.2,

$$2(d-3) \leq m(d-3) < d-1.$$

Hence

$$d = 4,\ m = 2 \quad \text{and} \quad 2 < \frac{r - I}{I'} = \text{Fano index of } X_{d-1}.$$

Thus we conclude that $X_{d-1} \cong \mathbf{P}(1,1,1,2,4)$ and $I = mI' = 4$ since this is the only type for which the dimension is 3 and the Fano index is greater than 2. Then

$$(X, H) \cong (\mathbf{P}(1,1,1,2,4), \mathcal{O}(4)),$$

this is of type [6].

Let X be a $\mathbf{Q}$-Fano of type [1]~[8]. The Weil divisor class group $\mathrm{Div}\,X$ is isomorphic to $\mathbf{Z}$, and $\mathcal{O}_X(1)$ generates $\mathrm{Pic}X$. This follows from the same argument of [Mo] Theorem 3.7. Next we take X generally from [1]~[8], then X is quasismooth and the adjunction formula of quasismooth weighted complete intersections (cf. [Do] 3.3.4) and Theorem 4.2 implies that X is a $\mathbf{Q}$-Fano whose Fano-index is as written in the last part of the theorem. □

Remark 4.1. We can see by the next well known lemma (cf. [H] IV. 3.2) that $|H|$ is free for all type [1]~[8] and very ample except the type [1] and $I = 2$:

Let C be a nonsingular curve of genus $g(C)$ and D a divisor, then

$$\begin{aligned} \deg D \geq 2g(C) \quad &\Rightarrow |D| \text{ free} \\ \deg D \geq 2g(C) + 1 \quad &\Rightarrow |D| \text{ very ample.} \end{aligned}$$

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