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RICCI PSEUDO η-PARALLEL REAL HYPERSURFACES OF A COMPLEX SPACE FORM

MAYUKO KON

ABSTRACT. We prove that the Ricci tensor of a real hypersurface of a complex space form $M^n(c)$, $c \neq 0$, $n \geq 3$, satisfies Ricci pseudo η-parallel condition if and only if M is pseudo-Einstein.

1. Introduction

Let $M^n(c)$ be an n-dimensional complex space form with constant holomorphic sectional curvature $4c$, and let M be a real hypersurface of $M^n(c)$. We denote by J the complex structure of $M^n(c)$. Then M has an induced almost contact metric structure (ϕ, ξ, η, g).

As a generalization of Einstein manifolds, Riemannian manifolds with parallel Ricci tensor have been intensively studied. Ki [3] proved that there are no real hypersurfaces in a complex space form $M^n(c)$, $c \neq 0$, with parallel Ricci tensor S. Moreover, Kimura and Maeda [6] showed that no real hypersurface in $M^n(c)$, $c \neq 0$, $n \geq 3$, satisfies semi-parallel condition, that is, $R(X, Y)S = 0$ for any X and Y tangent to the real hypersurface. Ki, Nakagawa and Suh [4] proved that the Ricci tensor S of a real hypersurface M of a complex space form $M^n(c)$, $c \neq 0$, $n \geq 3$, is cyclic semi-parallel, that is,

$$(R(X, Y)S)Z + (R(Y, Z)S)X + (R(Z, X)S)Y = 0$$

for any X, Y and Z tangent to M if and only if M is a pseudo-Einstein real hypersurface. On the other hand, Niebergall and Ryan [10] considered the condition $g((R(X, Y)S)Z, W) = 0$ for any X, Y, Z, W orthogonal to ξ, which is called pseudo-Ryan, under the assumption that M is Hopf hypersurface. In [8], the author showed that M satisfies pseudo-Ryan condition if and only if it is Pseudo-Einstein when $n \geq 3$.

One of the generalizations of Ricci semi-parallelity is the *Ricci pseudo-parallelity*:

$$R(X, Y)S = F((X \wedge Y)S),$$

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where F is a function. The Ricci pseudo-parallelity is an interest property for hypersurfaces. In fact, every Cartan's isoparametric hypersurface in spheres has pseudo-parallel Ricci tensor (see [2]).

In this paper, we study Ricci pseudo-parallel condition on the holomorphic distribution for real hypersurfaces of a complex space form. If the curvature tensor R and the Ricci tensor S of M satisfy

$$g((R(X,Y)S)Z,W) = Fg(((X \wedge Y)S)Z,W)$$

for any tangent vector fields X, Y, Z and W orthogonal to ξ, F being a function, we call S the *pseudo η-parallel* Ricci tensor. We prove the following

Theorem 3.1. *Let M be a real hypersurface of a complex space form $M^n(c)$, $c \neq 0$, $n \geq 3$. Then S is pseudo η-parallel if and only if M is pseudo-Einstein.*

Using Theorem 3.1, we obtain the following results.

Theorem 3.2. *Let M be a real hypersurface of a complex projective space $\mathbb{C}P^n$, $n \geq 3$. If S is pseudo η-parallel, then M is locally congruent to one of the following*:

(i) *a geodesic hypersphere of radius r $(0 < r < \pi/2)$,*

(ii) *a minimal tube of radius $\pi/4$ over a complex projective space $\mathbb{C}P^{\frac{n-1}{2}}$ with principal curvatures 1, -1 and 0 whose multiplicities are $n-1$, $n-1$ and 1, respectively.*

Theorem 3.3. *Let M be a real hypersurface of a complex hyperbolic space $\mathbb{C}H^n$, $n \geq 3$. If S is pseudo η-parallel, then M is locally congruent to one of the following*:

(i) *a geodesic hypersphere,*

(ii) *a tube over a complex hyperbolic hyperplane,*

(iii) *a horosphere.*

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2. Preliminaries

Let $M^n(c)$ denote the complex space form of complex dimension n with constant holomorphic sectional curvature $4c$. For the sake of simplicity, if $c > 0$, we only use $c = +1$ and call it the complex projective space $\mathbb{C}P^n$, and if $c < 0$, we just consider $c = -1$, so that we call it the complex hyperbolic space $\mathbb{C}H^n$. We denote by J the almost complex structure of $M^n(c)$. The Kähler metric of $M^n(c)$ will be denoted by G.

Let M be a real $(2n-1)$-dimensional hypersurface immersed in $M^n(c)$. We denote by g the Riemannian metric induced on M from G. We can take the unit normal vector field N of M in $M^n(c)$, locally. For any vector field X tangent to M, we define ϕ, η and ξ by

$$JX = \phi X + \eta(X)N, \qquad JN = -\xi,$$

where ϕX is the tangential part of JX, ϕ is a tensor field of type (1,1), η is a 1-form, and ξ is the unit vector field on M. Then they satisfy

$$\begin{aligned} &\phi^2 X = -X + \eta(X)\xi, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \\ &g(\phi X, Y) + g(X, \phi Y) = 0, \quad \eta(X) = g(X, \xi), \\ &g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y). \end{aligned}$$

Thus (ϕ, ξ, η, g) defines an almost contact metric structure on M. Let H_0 denote the holomorphic distribution on M defined by $H_0(x) = \{X \in T_x(M) | \eta(X) = 0\}$.

We denote by $\tilde{\nabla}$ the operator of covariant differentiation in $M^n(c)$, and by ∇ the one in M determined by the induced metric. Then the *Gauss and Weingarten formulas* are given respectively by

$$\tilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)N, \qquad \tilde{\nabla}_X N = -AX$$

for any vector fields X and Y tangent to M. We call A the *shape operator* of M derived from N. If the shape operator A of M is of the form $AX = \lambda X + \mu\eta(X)\xi$ for some functions λ and μ, then M is said to be *η-umbilical* (see Tashiro-Tachibana [12]).

For the contact metric structure on M, we have

$$\nabla_X \xi = \phi AX, \qquad (\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi.$$

We denote by R the Riemannian curvature tensor field of M. Then the *equation of Gauss* is given by

$$\begin{aligned} R(X, Y)Z = c(&g(Y, Z)X - g(X, Z)Y + g(\phi Y, Z)\phi X \\ &- g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z) \\ &+ g(AY, Z)AX - g(AX, Z)AY, \end{aligned}$$

and the *equation of Codazzi* by

$$(\nabla_X A)Y - (\nabla_Y A)X = c(\eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi).$$

From the equation of Gauss, the Ricci tensor S of type $(1,1)$ of M is given by

$$\begin{aligned} g(SX, Y) = &(2n+1)cg(X, Y) - 3c\eta(X)\eta(Y) \\ &+ \mathrm{tr}A g(AX, Y) - g(AX, AY), \end{aligned} \tag{1}$$

where tr$A$ is the trace of A. When the Ricci tensor S satisfies $g(SX, Y) = ag(X, Y) + b\eta(X)\eta(Y)$ for constants a and b, M is said to be pseudo-Einstein.

We use the following theorems.

Theorem A ([1], [11]). *Let M be a η-umbilical real hypersurface of a complex projective space $\mathbb{C}P^n$, $n \geq 2$, then M is locally congruent to a geodesic hypersphere.*

The following theorem is the direct consequence of theorems in Montiel [9].

Theorem B. *Let M be a η-umbilical real hypersurface of a complex hyperbolic space $\mathbb{C}H^n$, $n \geq 3$. Then M is locally congruent to one of the following*:

(a) *a geodesic hypersphere,*
(b) *a tube over a complex hyperbolic hyperplane,*
(c) *a horosphere.*

Theorem C ([1], [7]). *Let M be real hypersurface of a complex projective space $\mathbb{C}P^n$. We suppose that the Ricci tensor S satisfies $g(SX, Y) = ag(X, Y) + b\eta(X)\eta(Y)$ for funcions a and b. Then a and b must be constant and M is locally congruent to one of the following*:

(a) *a geodesic hypersphere,*
(b) *a tube of radius r over a complex projective subspace $\mathbb{C}P^p$, $1 \leq p \leq n-2$, $0 < r < \pi/2$ and $\cot^2 r = p/(n-p-1)$.*
(c) *a tube over a complex quadric Q^{n-1}.*

Theorem D ([9]). *A real hypersurface M of a complex hyperbolic space $\mathbb{C}H^n$, $n \geq 3$, is pseudo-Einstein if and only if it is η-umbilical.*

3. Characterization of pseudo-Einstein real hypersurfaces

First, we prepare the following lemmas.

Lemma 3.1. *Let M be a real hypersurface of a complex space form $M^n(c)$, $c \neq 0$, $n \geq 3$. Suppose that the curvature tensor R and the Ricci tensor S of M satisfy*

$$g((R(X, Y)S)Z + (R(Y, Z)S)X + (R(Z, X)S)Y, W) = 0$$

for any tangent vectors X, Y, Z and W orthogonal to ξ. Then we have

$$g(SX, Y) = \frac{1}{2n-2}(r - g(S\xi, \xi))g(X, Y),$$

for any tangent vectors X and Y orthogonal to ξ, where r denotes the scalar curvature of M.

Proof. We suppose that R and the Ricci tensor S of M satisfy

$$g((R(X,Y)S)Z + (R(Y,Z)S)X + (R(Z,X)S)Y, W) = 0.$$

for any tangent vectors X, Y, Z and W orthogonal to ξ. Since

$$(R(X,Y)S)Z = R(X,Y)SZ - SR(X,Y)Z,$$

the first Bianchi identity gives

$$g(R(X,Y)SZ + R(Y,Z)SX + R(Z,X)SY, W) = 0.$$

We take an orthonormal basis $\{e_1, \cdots, e_{2n-2}, e_{2n-1} = \xi\}$ of the tangent space $T_x(M)$. Then we have

$$g(\sum_{i=1}^{2n-2} R(e_i, \phi e_i)SX + \sum_{i=1}^{2n-2} R(\phi e_i, X)Se_i + \sum_{i=1}^{2n-2} R(X, e_i)S\phi e_i, Y) = 0.$$

By $\phi\xi = 0$,

$$g(\sum_{i=1}^{2n-1} R(e_i, \phi e_i)SX + \sum_{i=1}^{2n-1} R(\phi e_i, X)Se_i + \sum_{i=1}^{2n-1} R(X, e_i)S\phi e_i, Y) = 0.$$

Since we have

$$g(\sum_{i=1}^{2n-1} R(\phi e_i, X)Se_i, Y) = -g(\sum_{i=1}^{2n-1} R(e_i, X)S\phi e_i, Y),$$

it follows that

$$\sum_{i=1}^{2n-1} g(R(e_i, \phi e_i)SX, Y) = 2\sum_{i=1}^{2n-1} g(R(e_i, X)S\phi e_i, Y).$$

On the other hand, by the equation of Gauss,

$$\begin{aligned}
&\sum_i g(R(e_i, \phi e_i)SX, Y) \\
&\quad = -4ncg(\phi SX, Y) + 2g(SX, A\phi AY), \\
&2\sum_i g(R(e_i, X)S\phi e_i, Y) \\
&\quad = c\{-6g(\phi SX, Y) + 2g(S\phi X, Y) - 2\sum_i g(S\phi e_i, \phi e_i)g(\phi X, Y)\} \\
&\qquad + 2g(AX, S\phi AY) - 2\sum_i g(AX, Y)g(Ae_i, S\phi e_i).
\end{aligned}$$

Thus we have

$$\begin{aligned} & c\{(-4n+6)g(\phi SX,Y)-2g(S\phi X,Y)\} \\ & \quad = -2c\sum_i g(S\phi e_i,\phi e_i)g(\phi X,Y)+2g(AX,S\phi AY) \\ & \qquad -2\sum_i g(AX,Y)g(Ae_i,S\phi e_i)-2g(A\phi AY,SX). \end{aligned}$$

Using (1), for $X, Y \in H_x$, we obtain

$$\begin{aligned} & g(AX,S\phi AY)-\sum_i g(AX,Y)g(Ae_i,S\phi e_i)-g(A\phi AY,SX) \\ & = -\sum_i (2n+1)cg(AX,Y)g(Ae_i,\phi e_i) \\ & \quad -\sum_i \mathrm{tr}Ag(AX,Y)g(Ae_i,A\phi e_i)+\sum_i g(AX,Y)g(Ae_i,A^2\phi e_i) \\ & = 0. \end{aligned}$$

From these equations and the assumption $c \neq 0$, we have

$$(2n-3)g(\phi SX,Y)+g(S\phi X,Y)=\sum_i g(S\phi e_i,\phi e_i)g(\phi X,Y),$$

for any $X, Y \in H_x$. Since $\phi X, \phi Y \in H_x$, we also have

$$(2n-3)g(\phi S\phi X,\phi Y)+g(S\phi^2 X,\phi Y)=\sum_i g(S\phi e_i,\phi e_i)g(\phi X,Y),$$

and hence

$$(2n-3)g(S\phi X,Y)+g(\phi SX,Y)=\sum_i g(S\phi e_i,\phi e_i)g(\phi X,Y).$$

From these equations, we obtain

$$(2n-4)g(S\phi X,\phi Y)=(2n-4)g(\phi X,Y).$$

Since $n \geq 3$, we have $g(S\phi X,\phi Y)=g(SX,Y)$. Thus, by the definition of the scalar curvature r of M, we get

$$\begin{aligned} (2n-2)g(SX,Y) &= \sum_i g(S\phi e_i,\phi e_i)g(X,Y) \\ &= (r-g(S\xi,\xi))g(X,Y), \end{aligned}$$

which proves our assertion. □

Lemma 3.2. *Let M be a real hypersurface of a complex space form $M^n(c)$, $c \neq 0$, $n \geq 3$. If S is pseudo η-parallel, then we have*

$$g(SX,Y)=\frac{1}{2n-2}(r-g(S\xi,\xi))g(X,Y),$$

for any tangent vectors X and Y orthogonal to ξ.

Proof. We suppose $g((R(X,Y)S)Z,W) = Fg(((X\wedge Y)S)Z,W)$ for any tangent vectors X, Y, Z and W orthogonal to ξ, F being a function. Since we have

$$(X\wedge Y)Z = g(Y,Z)X - g(X,Z)Y,$$

we obtain

$$((X\wedge Y)S)Z = g(Y,SZ)X - g(SZ,X)Y - g(Y,Z)SX + g(Z,X)SY.$$

So we have

$$((X\wedge Y)S)Z + ((Y\wedge Z)S)X + ((Y\wedge Z)S)X = 0.$$

Since $g((R(X,Y)S)Z,W) = Fg(((X\wedge Y)S)Z,W)$, we have

$$g((R(X,Y)S)Z + (R(Y,Z)S)X + (R(Z,X)S)Y, W) = 0.$$

for any tangent vectors X, Y, Z and W orthogonal to ξ. From Lemma 3.1, we have our result. □

Using Lemma 3.2, we prove our main theorem.

Theorem 3.1. *Let M be a real hypersurface of a complex space form $M^n(c)$, $c\neq 0$, $n\geq 3$. Then S is pseudo η-parallel if and only if M is pseudo-Einstein.*

Proof. We suppose that M satisfies $g((R(X,Y)S)Z,W) = Fg(((X\wedge Y)S)Z,W)$ for any tangent vector fields X, Y, Z and W orthogonal to ξ. We can choose an orthonormal basis $\{e_1,\cdots,e_{2n-2},\xi\}$ at a point p of M such that the shape operator A is represented by a matrix form

$$A = \begin{pmatrix} \lambda_1 & \cdots & 0 & h_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \lambda_{2n-2} & h_{2n-2} \\ h_1 & \cdots & h_{2n-2} & \alpha \end{pmatrix}.$$

Then, we have

$$\begin{aligned} Se_i &= (2n+1)ce_i - 3c\eta(e_i)\xi + hAe_i - A^2e_i \\ &= ((2n+1)c + h\lambda_i - \lambda_i^2)e_i + h_i(h-\lambda_i-\alpha)\xi - \sum_{k=1}^{2n-2} h_ih_ke_k, \\ S\xi &= (2n+1)c\xi - 3c\eta(\xi)\xi + hA\xi - A^2\xi \\ &= (2n-2)c\xi + h(\sum_{k=1}^{2n-2} h_ke_k + \alpha\xi) - A(\sum_{k=1}^{2n-2} h_ke_k + \alpha\xi) \\ &= \sum_{k=1}^{2n-2} h_k(h-\lambda_k-\alpha)e_k + ((2n-2)c + \alpha h - \sum_{k=1}^{2n-2} h_k^2 - \alpha^2)\xi, \end{aligned}$$

where we have put $h = \mathrm{tr}A$. By Lemma 2.2, we have

$$\begin{aligned} &g(Se_i, e_j) = -h_i h_j = 0 \quad (i \neq j), \\ &g(Se_i, e_i) = \frac{1}{2n-2}(r - g(S\xi, \xi)) \quad (i = 1, \cdots, 2n-2). \end{aligned} \tag{2}$$

Equation (3.1) shows that at most one h_i does not vanish at p. Thus we can assume that $h_i = 0$ for $i = 2, \cdots, 2n-2$. We set $a = g(Se_i, e_i)$. Then we have

$$\begin{aligned} &Se_1 = ae_1 + h_1(h - \lambda_1 - \alpha)\xi, \\ &Se_i = ae_i \quad (i = 2, \cdots, 2n-2), \\ &S\xi = h_1(h - \lambda_1 - \alpha)e_1 + ((2n-2)c + \alpha h - h_1^2 - \alpha^2)\xi. \end{aligned} \tag{3}$$

Since $g((R(X,Y)S)Z, W) = Fg(((X \wedge Y)S)Z, W)$ for any tangent vector fields X, Y, Z and W orthogonal to ξ, we have

$$g(R(X,Y)SZ - SR(X,Y)Z, W) = Fg(((X \wedge Y)SZ - S(X \wedge Y)Z, W).$$

By the equation of Gauss, for any $j \geq 2$,

$$\begin{aligned} &g(R(e_1, e_j)Se_1, e_j) - g(SR(e_1, e_j)e_1, e_j) \\ &= ag(R(e_1, e_j)e_1, e_j) + h_1(h - \lambda_1 - \alpha)g(R(e_1, e_j)\xi, e_j) \\ &\qquad - ag(R(e_1, e_j)e_1, e_j) \\ &= h_1(h - \lambda_1 - \alpha)g(R(e_1, e_j)\xi, e_j) \\ &= -h_1^2 \lambda_j (h - \lambda_1 - \alpha). \end{aligned}$$

On the other hand, for any $j \geq 2$,

$$\begin{aligned} &F(g((e_1 \wedge e_j)Se_1, e_j) - g(S(e_1 \wedge e_j)e_1, e_j)) \\ &= F(-g(Se_1, e_1)g(e_j, e_j) + g(e_1, e_1)g(Se_j, e_j)) \\ &= F(a - a) = 0. \end{aligned}$$

From these equations, we have

$$-h_1^2 \lambda_j (h - \lambda_1 - \alpha) = 0$$

for any $j \geq 2$. If $h_1(h - \lambda_1 - \alpha) \neq 0$, then we obtain $\lambda_j = 0$ for $j \geq 2$. Since $h = \mathrm{tr}A$, we have $h = \lambda_1 + \alpha$. This is a contradiction. So we have $h_1(h - \lambda_1 - \alpha) = 0$. By (3.2), $SX = aX$ and $g(S\xi, X) = 0$ for any X orthogonal to ξ at $p \in M$, and hence at any point of M. Thus we M is pseudo-Einstein and $h_1 = 0$ (see [7]). We remark that a and $g(S\xi, \xi)$ are costant.

Conversely, if M is pseudo-Einstein, we have $SZ = aZ + b\eta(Z)\xi = aZ$ and $SW = aW$ for any tangent vectors Z and W orthogonal to ξ, where a and b are constant. Then we have

$$g((R(X,Y)S)Z,W) = g(R(X,Y)SZ,W) - g(SR(X,Y)Z,W) = 0,$$
$$Fg(((X \wedge Y)S)Z,W) = Fg((X \wedge Y)SZ,W) - Fg(S(X \wedge Y)Z,W) = 0.$$

□

Next, we prove the following theorem (see [5]).

Theorem 3.2. *Let M be a real hypersurface of a complex projective space $\mathbb{C}P^n$, $n \geq 3$. If S is pseudo η-parallel, then M is locally congruent to one of the following*:

(i) *a geodesic hypersphere of radius r $(0 < r < \pi/2)$,*

(ii) *a minimal tube of radius $\pi/4$ over a complex projective space $\mathbb{C}P^{\frac{n-1}{2}}$ with principal curvatures 1, -1 and 0 whose multiplicities are $n-1$, $n-1$ and 1, respectively.*

Theorem 3.3. *Let M be a real hypersurface of a complex hyperbolic space $\mathbb{C}H^n$, $n \geq 3$. If S is pseudo η-parallel, then M is locally congruent to one of the following*:

(i) *a geodesic hypersphere,*

(ii) *a tube over a complex hyperbolic hyperplane,*

(iii) *a horosphere.*

Proof of Theorem 3.2 and Theorem 3.3. We suppose $g((R(X,Y)S)Z,W) = Fg(((X \wedge Y)S)Z,W)$ for any tangent vector fields X, Y, Z and W. From Theorem 3.1, M is pseudo-Einstein, so we can put $SX = aX + b\eta(X)\xi$, where a and b are constant. We notice that M is a Hopf hypersuface. Then we have

$$SX = aX, \quad S\xi = (a+b)\xi$$

for any tangent vector X orthogonal to ξ. Thus we obtain

$$\begin{aligned} &g((R(X,\xi)S)X,\xi) \\ &= g(R(X,\xi)SX,\xi) - g(SR(X,\xi)X,\xi) \\ &= -bg(R(X,\xi)X,\xi) \\ &= b(c + g(AX,X)g(A\xi,\xi)). \end{aligned}$$

On the other hand, we have

$$\begin{aligned} &Fg(((X \wedge \xi)S)X,\xi) \\ &= Fg((X \wedge \xi)SX,\xi) - Fg(S(X \wedge \xi)X,\xi) \\ &= bFg(X,X). \end{aligned}$$

Since $b \neq 0$, we have $c + g(AX,X)g(A\xi,\xi) = F$ for any unit tangent vector X orthogonal to ξ. If $g(A\xi,\xi) \neq 0$, then M is η-umbilical. If $A\xi = 0$, then we have $c = F$. When $c > 0$, Theorem C implies that M is a tube over a complex projective space $\mathbb{C}P^{\frac{n-1}{2}}$ with constant principal curvatures 1, -1 and 0 whose multiplicities are $n-1$, $n-1$ and 1, respectively (see [7]). Otherwise, when $c<0$, from Theorem D, pseudo-Einstein real hypersurface M does not satisfy $A\xi=0$ (see [9]).

Conversely, we suppose M is η-umbilical. Then the shape operator A can be represented by $AX=\lambda X+\mu\eta(X)\xi$, λ and μ being constant. Moreover, M is pseudo-Einstein and $SX=aX+b\eta(X)\xi$ for some constants a and b. By the straightforward computation, we have

$$\begin{aligned}
&g((R(X,Y)S)Z,W)-(c+\lambda(\lambda+\mu))g(((X\wedge Y)S)Z,W)\\
&=-\lambda(\lambda+\mu)\big(b\eta(Z)\eta(Y)g(X,W)-b\eta(Z)\eta(X)g(Y,W)\\
&\qquad -b\eta(X)\eta(W)g(Y,Z)+b\eta(Y)\eta(W)g(Z,X)\big)\\
&\quad +b\eta(Z)\big(g(AY,\xi)g(AX,W)-g(AX,\xi)g(AY,W)\big)\\
&\quad -b\eta(W)\big(g(AX,\xi)g(AY,Z)-g(AY,\xi)g(AX,Z)\big)\\
&=0.
\end{aligned}$$

Next we suppose that M is a pseudo-Einstein real hypersurface of $M^n(c)$, $c>0$, $n\geq 3$, which satisfies $A\xi=0$. We put $SX=aX+b\eta(X)\xi$ for some constant a and b. Thus we have

$$\begin{aligned}
&g((R(X,Y)S)Z,W)-cg(((X\wedge Y)S)Z,W)\\
&=b\eta(Z)\big(g(AY,\xi)g(AX,W)-g(AX,\xi)g(AY,W)\big)\\
&\quad -b\eta(W)\big(g(AX,\xi)g(AY,Z)-g(AY,\xi)g(AX,Z)\big)\\
&=0.
\end{aligned}$$

So we have our theorem.

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(Mayuko Kon) Faculty of Education, Shinshu University, 6-Ro, Nishinagano, Nagano City 380-8544, Japan
E-mail address: mayuko_k@shinshu-u.ac.jp (M. Kon),

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