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INTERPOLATION PROBLEM FOR ℓ^1 AND AN F-SPACE

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ABSTRACT. Let B be an F-space and B_1^* the unit ball of the dual space. A sequence (ϕ_n) in B_1^* is called ℓ^1-interpolating if for every sequence (w_n) in ℓ^1 there exists an element f in B such that $\phi_n(f) = w_n$ for all n. In order to study an interpolation problem for ℓ^1, we introduce two quantities ρ_n and $\prod_{k\neq n}\sigma(\phi_n, \phi_k)$. For arbitrary Banach space, we show that (ϕ_n) is an ℓ^1-interpolating sequence if and only if $\inf_n \rho_n > 0$. Moreover, when a Banach space has a predual, we show that if $\inf_n \prod_{k\neq n}\sigma(\phi_n, \phi_k) > 0$ then (ϕ_n) is an ℓ^1-interpolating sequence. When (ϕ_n) is embeded in the open unit disc in the complex plane, we show that (ϕ_n) is an ℓ^1-interpolating sequence if and only if $\inf_n \prod_{k\neq n}\sigma(\phi_n, \phi_k) > 0$, for a Hardy space $H^p(D)(1 \leq p \leq \infty)$ and the Smirnov class $N_+(D)$.

1. Introduction

Let B be an F-space with an invariant metric d and B^* its dual space. B_1^* denotes the unit ball of B^*. Throughout this paper we assume that (ϕ_n) is an infinite sequence of distinct points in B^*. Let ℓ be a sequence space of (w_n) where $w_n \in \mathbb{C}$. A sequence (ϕ_n) is called ℓ-interpolating if for every sequence (w_n) in ℓ there exists an element f in B sucn that $\phi_n(f) = w_n$ for all n. For (ϕ_n) in B^* put

$$\begin{aligned} J &= \{f \in B \ ; \ f = 0 \ \text{ on } \ (\phi_k)\}, \\ J_n &= \{f \in B \ ; \ f = 0 \ \text{ on } \ (\phi_k)_{k\neq n}\}, \\ \rho_n &= \sup\{|\ \phi_n(f)\ | \ ; \ f \in J_n, \ d(f, 0) \leq 1\} \end{aligned}$$

and

$$\sigma(\phi_n, \phi_k) = \sup\{|\ \phi_n(f)\ | \ ; \ \phi_k(f) = 0, \ d(f, 0) \leq 1\}.$$

In general, $\rho_n > 0$ if and only if $J_n \supset J$ and $J_n \neq J$. Hence $\rho_n > 0$ if and only if there exists an element f_n in B such that $\phi_k(f_n) = \delta_{kn}$. In this paper, we assume that $\rho_n > 0$ for all n and so $J_n = \langle f_n \rangle + J$.

In this paper, we study an ℓ^1-interpolation problem for an F-space. The following two natural problems will be considered.

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Problem 1 *For a given F-space, prove that (ϕ_n) is an ℓ^1-interpolation sequence if and only if* $\inf_n \rho_n > 0$.

Problem 2 *For a given F-space, suppose that (ϕ_n) is in B_1^*. Then, prove that (ϕ_n) is an ℓ^1-interpolation sequence if and only if* $\inf_n \prod_{k \neq n} \sigma(\phi_n, \phi_k) > 0$.

In this paper, we solve Problem 1 for arbitrary Banach space and the Smirnov class $N_+(D)$ on the open unit disc D. Problem 2 is studied for arbitrary Banach space with the predual space and the Smirnov class $N_+(D)$.

The first contribution for an ℓ^1-interpolation problem was by Shapiro and Shields [7]. In fact they solved Problem 2 for a Hardy space $H^1(D)$. Snyder [8] has solved Problem 2 for a Hardy space $H^\infty(D)$. Hatori [3] has solved Problem 2 for a Hardy space $H^p(D)$ when $1 < p < \infty$. In fact he proved it for a Hardy spaceH^p on a finite connected domain. In the previous paper [6], we have solved Problem 1 for arbitrary uniform algebra A when (ϕ_n) is in the maximal ideal space. Moreover we have solved Problem 2 for several special uniform algebras.

In Section 2, we show that (ϕ_n) is an ℓ^1-interpolating sequence if and only if $\inf_n \rho_n > 0$ for arbitrary Banach space. We also study an ℓ^p-interpolating sequence when $0 < p < 1$. Moreover, when a Banach space has a predual, we show that if $\inf_n \prod_{k \neq n} \sigma(\phi_n, \phi_k) > 0$ and (ϕ_n) is in the predual then (ϕ_n) is an ℓ^1-interpolating sequence. In Section 3, we solve Problem 1 for the Smirnov class $N_+(D)$ when (ϕ_n) is embeded in D. In Section 4, we solve Problem 2 for $N_+(D)$ when (ϕ_n) is embeded in D. This is a little bit surprising because we have not a complete interpolation theorem for $N_+(D)$.

2. General theorem for F-space

Corollary 2.1 generalizes Theorem 2.1 in the previous paper [6].

Lemma 2.1 *Let B an F-space with an invariant metric d. If (ϕ_n) is an ℓ^p-interpolating sequence and $0 < p \leq 1$ then* $\sup_n d(f_n + J, 0) < \infty$.

Proof. Put $S = (\phi_n)$. Then there exists a sequence (f_n) in B such that $\phi_k(f_n) = \delta_{nk}$. For $(w_n) \in \ell^p$, put

$$T(w_n) = \sum_{n=1}^{\infty} w_n (f_n \mid S)$$

then by the hypothesis there exists f in B such that $T(w_n) = f \mid S$. Since $B \mid S$ is isomorphic to the quotient space B/J, we put the quotient norm of B/J on $B \mid S$. By the closed graph theorem, T is bounded from ℓ^p to $B \mid S$ and so

$$d(f_k + J, 0) \leq \| T \|$$

because $T((\delta_{nk})_n) = f_k \mid S$. This implies that $\sup_n d(f_n + J, 0) < \infty$. □

Lemma 2.2 *Let B be an F-space with an invariant metric d and $d(\alpha f, 0) = \mid \alpha \mid^p d(f, 0)$ $(f \in B, \alpha \in \mathbb{C})$ for some p with $0 < p \leq 1$. If $\sup_n d(f_n + J, 0) < \infty$ then (ϕ_n) is an ℓ^p-interpolating sequence.*

Proof. Suppose that $M = \sup_n d(f_n + J, 0) < \infty$. Let ε be arbitrary positive constant. For each n there exists g_n in J such that $d(f_n + g_n, 0) \leq M + \varepsilon$. If $(w_n) \in \ell^p$, put

$$f = \sum_{n=1}^{\infty} w_n (f_n + g_n)$$

then f belongs to B and $\phi_k(f) = w_k$ for $k = 1, 2, \ldots$. □

Lemma 2.3 *Let B be an F-space with an invariant metric d. If (ϕ_n) is a sequence in B^* such that $\phi_k(f_n) = \delta_{nk}$, then $d(f_n + J, 0) = 1/\rho_n$ for $n = 1, 2, \ldots$.*

Proof. Note that $J_n = \langle f_n \rangle + J$ for any n. By the definition of ρ_n, $1 = \mid \phi_n(f_n) \mid \leq \rho_n d(f_n + J, 0)$. On the other hand, for any $\varepsilon > 0$ there exists $F_\varepsilon \in J_n$ such that $\mid \phi_n(F_\varepsilon) \mid + \varepsilon \geq \rho_n d(F_\varepsilon + J, 0)$. Since $f_n + J = F_\varepsilon + J$, this implies that $1 + \varepsilon \geq \rho_n d(f_n + J, 0)$ and so $1 \geq \rho_n d(f_n + J, 0)$ as $\varepsilon \to 0$. □

Theorem 2.1 *Let B be an F-space and (ϕ_n) in B^* and $0 < p \leq 1$.*

(i) *If (ϕ_n) is an ℓ^p-interpolating sequence then $\inf_n \rho_n > 0$.*

(ii) *If $d(\alpha f, 0) = \mid \alpha \mid^p d(f, 0)$ $(f \in B, \alpha \in \mathbb{C})$ and $\inf_n \rho_n > 0$ then (ϕ_n) is an ℓ^p-interpolating sequence.*

Proof. Lemmas 2.1 and 2.3 imply (i). Lemmas 2.2 and 2.3 imply (ii). □

Corollary 2.1 *Let B be a Banach space and (ϕ_n) in B^*. Then (ϕ_n) is an ℓ^1-interpolating sequence if and only if $\inf_n \rho_n > 0$.*

For (ϕ_n) in B^* where B is an F-space, $\ell(B, (\phi_n))$ is a sequence space as the following:

$$\ell(B, (\phi_n)) = \left\{ (w_n) \; ; \; \sum_{n=1}^{\infty} d(w_n f_n + J, 0) < \infty \right\}.$$

If B is a Banach space then

$$\begin{aligned} \ell(B, (\phi_n)) &= \left\{ (w_n) \; ; \; \sum_{n=1}^{\infty} \mid w_n \mid d(f_n + J, 0) < \infty \right\} \\ &= \left\{ (w_n) \; ; \; \sum_{n=1}^{\infty} \mid w_n \mid / \rho_n < \infty \right\} \end{aligned}$$

by Lemma 2.3. If $d(\alpha f, 0) = \mid \alpha \mid^p d(f, 0)$ then

$$\ell(B, (\phi_n)) = \left\{ (w_n) \; ; \; \sum_{n=1}^{\infty} \mid w_n \mid^p / \rho_n < \infty \right\}.$$

Proposition 2.1 *Let B be an F-space with an invariant metric d. Then for any (ϕ_n) in B^*, (ϕ_n) is an $\ell(B, (\phi_n))$-interpolating sequence, that is, for any (w_n) in $\ell(B, (\phi_n))$, there exists f in B such that $\phi_n(f) = w_n$ $(n = 1, 2, \ldots)$.*

Proof. For each n, there exists g_n in J such that

$$d(w_n f_n + g_n, 0) \leq d(w_n f_n + J, 0) + \frac{1}{n^2}.$$

Put

$$f = \sum_{n=1}^{\infty} (w_n f_n + g_n),$$

then

$$d(f, 0) \leq \sum_{n=1}^{\infty} d(w_n f_n + g_n, 0) \leq \sum_{n=1}^{\infty} d(w_n f_n + J, 0) + \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

Hence f belongs to B and $\phi_k(f) = w_k$ for $k = 1, 2, \ldots$. □

Lemma 2.4 *Let B be an F-space with an invariant metric d and (ϕ_n) in B^*. Then $\rho_n \leq \sigma(\phi_n, \phi_k)$ if $n \neq k$.*

Proof. For any $n \geq 1$,

$$\begin{aligned} \rho_n &= \sup\{\mid \phi_n(f) \mid \; ; \; f \in J_n, \; d(f, 0) \leq 1\} \\ &\leq \sup\{\mid \phi_n(f) \mid \; ; \; \phi_k(f) = 0, \; d(f, 0) \leq 1\} \\ &= \sigma(\phi_n, \phi_k) \end{aligned}$$

if $n \neq k$. □

Proposition 2.2 *et B be a Banach space and (ϕ_n) be in B^*. Then if (ϕ_n) is an ℓ^1-interpolating sequence, then $\inf_n \inf_{k \neq n} \sigma(\phi_n, \phi_k) > 0$.*

Proof. It is a result of Lemma 2.4 and Corollary 2.1. □

Theorem 2.2 *Let B be a Banach space whose predual is E, that is, $E^* = B$. If (ϕ_n) is in E and $\inf_n \prod_{k \neq n} \sigma(\phi_n, \phi_k) > 0$ then (ϕ_n) is an ℓ^1-interpolating sequence.*

Proof. By Corollary 2.1 it is enough to prove that if $\inf_n \prod_{k\neq n} \sigma(\phi_n, \phi_k) > 0$ then $\inf_n \rho_n > 0$. For $1 \leq n \leq \ell < \infty$, put

$$J_n^\ell = \{f \in B \ ; \ \phi_k(f) = 0 \ \text{ if } \ 1 \leq k \leq \ell, \ k \neq n\}$$

and

$$\rho_{n,\ell} = \sup\{|\phi_n(f)| \ ; \ f \in J_n^\ell, \| f \| \leq 1\}.$$

Claim 1. *For any* $\ell \geq 1$, $\rho_{n,\ell} \geq \prod_{k\neq n}^{\ell} \sigma(\phi_n, \phi_k)$. For if ε is any positive constant then for each k with $1 \leq k \leq \ell$, there exists f_k^ε in B such that

$$\sigma(\phi_n, \phi_k) \geq | \phi_n(f_k^\varepsilon) | \geq \sigma(\phi_n, \phi_k) - \varepsilon,$$

$\prod_{k\neq n}^{\ell} f_k^\varepsilon \in$ the unit ball of B and $\phi_j(\prod_{k\neq n}^{\ell} f_k^\varepsilon) = 0$ if $j \neq n$. Put $f^\varepsilon = \prod_{k\neq n}^{\ell} f_k^\varepsilon$ then $f^\varepsilon \in J_n^\ell$ and $\| f^\varepsilon \| \leq 1$. Then

$$\rho_{n,\ell} \geq | \phi_n(f^\varepsilon) | \geq \prod_{k\neq n}^{\ell} \{\sigma(\phi_n, \phi_k) - \varepsilon\}.$$

As $\varepsilon \to 0$ $\rho_{n,\ell} \geq \prod_{k\neq n}^{\ell} \sigma(\phi_n, \phi_k)$ for any $\ell \geq 1$.

Claim 2. $\lim_{\ell\to\infty}\rho_{n,\ell} = \rho_n$ *for any* $n \geq 1$. For $\rho_{n,\ell} \geq \rho_{n,\ell+1}$ and $\lim_{\ell\to\infty}\rho_{n,\ell} \geq \rho_n$ for any $n \geq 1$. If $\lim_{\ell\to\infty}\rho_{n,\ell} > \varepsilon > 0$, then for each ℓ there exists $g_\ell \in J_n^\ell$ such that $\| g_\ell \| \leq 1$ and $| \phi_n(g_\ell) | \geq \varepsilon > 0$. Then there exists $g \in J_n$ such that $\| g \| \leq 1$ and $g_\ell \to g$ weak star in B. Then $| \phi_n(g) | \geq \varepsilon > 0$ because ϕ_n is continuous in the weak star topology. Thus $\lim_{\ell\to\infty}\rho_{n,\ell} \leq \rho_n$ and so $\lim_{\ell\to\infty}\rho_{n,\ell} = \rho_n$.

Claims 1 and 2 imply that $\rho_n \geq \prod_{k\neq n}^{\infty}\sigma(\phi_n, \phi_k)$. □

3. Answer for Problem 1

In this section we study Problem 1 for concrete examples which are F-spaces defined by analytic functions. For $0 < p \leq \infty$ $H^p(G)$ denotes a Hardy space on G and $L_a^p(G)$ denotes a Bergman space on G, where G is a domain in $\mathbb{C}^n$. When $1 \leq p \leq \infty$, $H^p(G)$ and $L_a^p(G)$ are Banach spaces and so we can apply Corollary 2.1 for them. When $0 < p < 1$, we could not solve it but Corollary 3.1 solves it partially.

Corollary 3.1 *Let* $0 < p \leq 1$ *and let* $B = H^p(G)$ *or* $L_a^p(G)$. *If* (ϕ_n) *is in* B^*, *then* (ϕ_n) *is an* ℓ^p*-interpolating sequence if and only if* $\inf_n\rho_n > 0$.

Proof. Since $d(f, g) = \| f - g \|_p^p$, $d(\alpha f, 0) = | \alpha |^p \ d(f, 0)$ for $f \in B$ and $\alpha \in \mathbb{C}$. The corollary is a result of Theorem 2.1. □

Let D be the open unit disc in $\mathbb{C}$ and $N_+(D)$ denotes the Smirnov class on D. Then $d(f, g) = \int_0^{2\pi} \log(1+ | f(e^{i\theta}) - g(e^{i\theta}) |)d\theta/2\pi$ is an invariant metric on $N_+(D)$. For a in D, $\gamma(a)$ denotes the norm of the evaluation functional on $N_+(D)$.

Lemma 3.1 *Let $B = N_+(D)$ and (a_n) in D. If $\phi_n(f) = f(a_n)/\gamma(a_n)$ for $n = 1, 2, \ldots$ and $\sum_n (1 - |a_n|) < \infty$ then*

$$\rho_n = \prod_{j \neq n} \left| \frac{a_n - a_j}{1 - \bar{a}_j a_n} \right|.$$

Proof. Since $J = QN_+(D)$ and $J_n = Q_n N_+(D)$ where

$$Q(z) = \prod_{j=1}^{\infty} - \frac{a_j}{| a_j |} \frac{z - a_j}{1 - \bar{a}_j z}$$

and

$$Q_n(z) = Q(z) / \frac{z - a_n}{1 - \bar{a}_n z},$$

$$\begin{aligned} \rho_n &= \sup\{| f(a_n)/\gamma(a_n) | \ ; \ f \in Q_n N_+(D), \ d(f, 0) \leq 1\} \\ &= \sup\{| Q_n(a_n) h(a_n)/\gamma(a_n) | \ ; \ h \in N_+(D), \ d(h, 0) \leq 1\} \\ &= \prod_{j \neq n} \left| \frac{a_n - a_j}{1 - \bar{a}_j a_n} \right|. \end{aligned}$$

□

Theorem 3.1 *Let $B = N_+(D)$ and (a_n) in D. Suppose $\phi_n(f) = f(a_n)/\gamma(a_n)$ $(n = 1, 2, \ldots)$, then the following* (i)–(iv) *are equivalent.*

(i) (ϕ_n) *is an ℓ^1-interpolating sequence.*

(ii) (ϕ_n) *is an ℓ^p-interpolating sequence for any $0 < p \leq 1$.*

(iii) (ϕ_n) *is an ℓ^p-interpolating sequence for some $0 < p \leq 1$.*

(iv) $\inf_n \ \rho_n > 0$.

Proof. (i)⇒(ii)⇒(iii) is clear. (iii)⇒(iv) is a result of (i) in Theorem 2.1. We show (iv)⇒(i). By Lemma 3.1

$$\rho_n = \prod_{j \neq n} \left| \frac{a_n - a_j}{1 - \bar{a}_j z_n} \right|$$

and so $\inf_n \ \rho_n > 0$ implies that (a_n) is an ℓ^1-interpolating sequence for $H^\infty(D)$ by Corollary 2.1. Since $N_+(D) \supset H^\infty(D)$, (iv) implies (i). □

4. Answer for Problem 2

In this section we study Problem 2 for concrete examples which are F-spaces defined by analytic functions, $H^p(G)$, $N_+(G)$ and $L^p_a(G)$ where G is a domain in $\mathbb{C}^n$, $1 \le p \le \infty$ and (ϕ_n) is embeded in G. When $H^p(G)$ or $L^p_a(G)$ has a predual, we can apply Theorem 2.2. Theorem 4.1 is a result of Shapiro and Shields [7] for $H^1(D)$, one of Snyder [8] for $H^\infty(D)$ and one of Hatori [3] for $H^p(D)$ $(1 < p < \infty)$, essentially. Proposition 4.1 is a result of Kabaila [4]. Theorem 4.1 for $N_+(D)$ is a main theorem in this paper. We could not solve Problem 2 for $H^p(D)$ $(0 < p < 1)$.

When $B = H^p(D)$ or $N_+(D)$, if (a_n) is in D and $\phi_n(f) = f(a_n)$ $(f \in B)$ for $n = 1, 2, \ldots$ then

$$\sigma(\phi_n, \phi_k) = \gamma(a_n) \left| \frac{a_n - a_k}{1 - \bar{a}_k a_n} \right|$$

and so

$$\prod_{k \ne n} \sigma(\phi_n, \phi_k) = \prod_{k \ne n} \gamma(a_n) \left| \frac{a_n - a_k}{1 - \bar{a}_k a_n} \right| = \rho_n$$

where $\gamma(a)$ denotes the norm of the evaluation functional at a on B. In order to study Problem 2, we assume that $\phi_n(f) = f(a_n)/\gamma(a_n)$.

Theorem 4.1 *Let $B = H^p(D)$ $(1 \le p \le \infty)$ or $N_+(D)$ and let (a_n) be in D. If $\phi_n(f) = f(a_n)/\gamma(a_n)$ $(f \in B)$ for $n = 1, 2, \ldots$, then (ϕ_n) is an ℓ^1-interpolation sequence if and only if* $\inf_n \prod_{k \ne n} \sigma(\phi_n, \phi_k) > 0$.

Proof. By Corollary 2.1 and Theorem 3.1, if (ϕ_n) is an ℓ^1-interpolation sequence then $\inf_n \rho_n > 0$. When $B = H^p(D)$ or $N_+(D)$, if

$$Q_k = Q / \frac{z - a_k}{1 - \bar{a}_k z}$$

and

$$Q = \prod_{j=1}^{\infty} - \frac{a_j}{| a_j |} \, \frac{z - a_j}{1 - \bar{a}_j z}$$

then for any $k \ge 1$

$$\{ f \in B \ ; \ f = 0 \ \text{ on } \ (a_n)_{n \ne k} \} = Q_k B.$$

Hence $J_k = Q_k B$ and so

$$\begin{aligned} \rho_k &= \sup\{ | \phi_k(f) | \ ; \ f \in Q_k B, \ d(f, 0) \le 1 \} \\ &= | \phi_k(Q_k) | \sup\{ | \phi_k(h) | \ ; \ h \in B, \ d(h, 0) \le 1 \} \\ &= \prod_{n \ne k} \left| \frac{a_n - a_k}{1 - \bar{a}_k a_n} \right| \end{aligned}$$

because $\| \phi_k \|= 1$ where $d(f,0) =\| f \|_p$ or

$$d(f,0) = \int_0^{2\pi} \log(1+ \mid f(e^{i\theta}) \mid)d\theta/2\pi.$$

Since

$$\{f \in B \ ; \ \phi_k(f) = 0\} = \frac{z-a_k}{1-\bar{a}_k z}B, \ \sigma(\phi_n,\phi_k) = \left|\frac{a_n-a_k}{1-\bar{a}_k a_n}\right|$$

and so $\rho_n = \prod_{k\neq n}\sigma(\phi_n,\phi_k)$. Hence $\inf_n \prod_{k\neq n}\sigma(\phi_n,\phi_k) > 0$.

Conversely if $\inf_n \prod_{k\neq n}\sigma(\phi_n,\phi_k) > 0$ then $\inf_n \rho_n > 0$ by the equality above. Hence (ϕ_n) is an ℓ^1-interpolation sequence by Corollary 2.1 and Theorem 3.1. □

Proposition 4.1 *Let* $B = H^p(D)$ $(0 < p < 1)$ *and let* (a_n) *be in* D*. If* $\phi_n(f) = f(a_n)/\gamma(a_n)(f \in B)$ *for* $n = 1, 2, \ldots,$ *then* (ϕ_n) *is an* ℓ^p*-interpolation sequence if and only if* $\inf_n \prod_{k\neq n}\sigma(\phi_n,\phi_k) > 0$.

Proof. By Corollary 3.1, (ϕ_n) is an ℓ^p-interpolation sequence if and only if $\inf_n \rho_n > 0$. By the proof of Lemma 3.1,

$$\rho_n = \prod_{k\neq n}\left|\frac{a_n-a_k}{1-\bar{a}_k a_n}\right| = \prod_{k\neq n}\sigma(\phi_n,\phi_k).$$

This implies the proposition. □

5. Remarks

Let B be an F-space of analytic functions on D and let (a_n) in D. Suppose $\phi_n(f) = f(a_n)/\gamma(a_n)$ $(f \in B)$ for $n = 1, 2, \ldots$ where $\gamma(a_n)$ denotes the norm of the evaluation functional on B. It will be nice to give a big sequence space ℓ_B such that (ϕ_n) is an ℓ_B-interpolation sequence if and only if $\inf \prod_{k\neq n}\sigma(\phi_n,\phi_k) > 0$. When $B = H^\infty(D)$, Carleson [1] showed that $\ell_B = \ell^\infty$. When $B = H^p(D)$ $(1 \leq p < \infty)$, Shapiro and Shields [7] proved that $\ell_B = \ell^p$. When $B = H^p(D)$ $(0 < p < 1)$, Kabaila [4] proved that $\ell_B = \ell^p$. When $B = N_+(D)$, it is not known what is ℓ_B. Yanagihara [9] studied an interpolation problem for $N_+(D)$ and he gave a sufficient condition when $\phi_n(f) = f(a_n)$ but not $\phi_n(f) = f(a_n)/\gamma(a_n)$ (see [2]). That is, if $\inf_n \rho_n > 0$ and (w_n) satisfies $\sum_{n=1}^\infty (1 - |a_n|) \log^+ |w_n| < \infty$ then there exists a function f in $N_+(D)$ such that $f(a_n) = w_n (n = 1, 2, \ldots)$. It is clear that

$$\ell^\infty \subset \{(w_n) : \sum_{n=1}^{\infty}(1 - |a_n|) \log^+ |w_n| < \infty\}.$$

If $1 \leq p \leq \infty$ then $\ell^1 \subset \ell^p$ and if $0 < p < 1$ then $\ell^p \subset \ell^1$. We could not prove that if $\inf_k \prod_{n\neq k}\sigma(\phi_n,\phi_k) > 0$ then (ϕ_n) is an ℓ^1-interpolation sequence for

H^p $(0 < p < 1)$. It seems to be a little bit surprising that (ϕ_n) is an ℓ^1-interpolation sequence for $N_+(D)$ if and only if $\inf_k \prod_{n \neq k} \sigma(\phi_n, \phi_k) > 0$.

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References

[1] L. Carleson, *An interpolation problem for bounded analytic functions*, Amer. J. Math. **80** (1958), 921–930.

[2] A. Hartmann, X. Massaneda, A. Nicolau and P. Thomas, *Interpolation in the Nevanlinna and Smirnov classes and harmonic majorants*, J. Funct. Anal. **217** (2004), 1–37.

[3] O. Hatori, *The Shapiro-Shields theorem on finite connected domains*, Surikaisekikenkyūsho Kōkyūroku **1049** (1998), 21–29 (in Japanese).

[4] V. Kabaila, *Interpolation sequnces for the H_p classes in the case* $p < 1$, Litovsk. Mat. Sb. **3 (1)** (1963), 141–147 (in Russian).

[5] H. Hedenmalm, B. Korenblum and K. Zhu, *Theory of Bergman Spaces*, 199, Springer-Verlag, New York, 2000.

[6] T. Nakazi, *Interpolation problem for ℓ^1 and a uniform algebra*, J. Austral. Math. Soc. **72** (2002), 1–11.

[7] H. S. Shapiro and A. L. Shields, *On some interpolation problems for analytic functions*, Amer. J. Math. **83** (1961), 513–532.

[8] A. K. Snyder, *Sequence spaces and interpolation problems for analytic functions*, Studia Math. **39** (1971), 137–153.

[9] N. Yanagihara, *Interpolation theorems for the class N^+*, Illinois J. Math. **18** (1974), 427–435.

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