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JACOBI OPERATORS ON A SEMI-INVARIANT SUBMANIFOLD OF CODIMENSION 3 IN A COMPLEX PROJECTIVE SPACE

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ABSTRACT. In this paper, we characterize some semi-invariant submanifolds of codimension 3 in a complex projective space $\mathbb{C}P^{n+1}$ in terms of the shape operator A, the structure tensor field ϕ and the Jacobi operator R_ξ with respect to the structure vector field ξ.

0. Introduction

A submanifold M is called a CR submanifold of a Kaehlerian manifold $\tilde{M}$ with complex structure J if it is endowed with a pair of mutually orthogonal and complementary differentiable distribution $(T, T^\perp)$ such that T is J-invariant, and $T^\perp$ is totally real ([1], [19]). In particular, M is said to be a *semi-invariant submanifold* if $dim T^\perp = 1$, and the unit normal in $JT^\perp$ is called a *distinguished normal* to M ([2], [17]). In this case, M admits an induced almost contact metric structure (ϕ, ξ, g).

A typical example of a semi-invariant submanifold is real hypersurfaces. Takagi([15]) classified homogeneous real hypersurfaces of a complex projective space by means of six model spaces of type A_1, A_2, B, C, D and E, further he explicitly write down their principal curvatures and multiplicities in the table in [16].

Cecil and Ryan [3] extensively investigated a real hypersurface which is realized a tube of constant radius r over a complex submanifold of $\mathbb{C}P^n$ on which ξ is principal curvature vector with principal curvature $\alpha = 2\cot 2r(A\xi = \alpha\xi)$ and the corresponding focal map φ_r has constant rank, where we denote by A the shape operator of a real hypersurface in $\mathbb{C}P^n$.

On the other hand, Okumura [10] characterized real hypersurfaces of type A_1 and A_2 by the property that the shape operator A and structure tensor field ϕ commute. Namely he proved

Theorem O [10]. *Let M be a connected real hypersurface of $\mathbb{C}P^n$. If M satisfies $\phi A = A\phi$, then M is locally congruent to one of the following spaces:*

(A_1) *a geodesic hypersphere (that is, a tube of radius r over a hyperplane $\mathbb{C}P^{n-1}$, where $0 < r < \frac{\pi}{2}$),*
(A_2) *a tube of radius r over a totally geodesic $\mathbb{C}P^k (1 \le k \le n-2)$, where $0 < r < \frac{\pi}{2}$.*

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We denote by ∇ the Levi-Civita connection with respect to g. The curvature tensor field R on the submanifold M is defined by $R(X,Y) = [\nabla_X, \nabla_Y] - \nabla_{[X,Y]}$, where X and Y are vector fields on M. We define the Jacobi operator $R_\xi = R(\cdot,\xi)\xi$ with respect to the structure vector field ξ. Then R_ξ is a self-adjoint endomorphism on the tangent space of a CR submanifold. In the preceding work [4], Cho and the present author give another characterization of real hypersurfaces of type A_1 and A_2 in a complex projective space $\mathbb{C}P^n$ in terms of the shape operator A, the structure tensor field ϕ and the Jacobi operator R_ξ. More specifically, they proved the following:

Theorem CK [4]. *Let M be a connected real hypersurface of $\mathbb{C}P^n$. If M satisfies $R_\xi\phi A = A\phi R_\xi$, then M is locally congruent to one of the following spaces:*

(A_1) *a geodesic hypersphere (that is, a tube of radius r over a hyperplane $\mathbb{C}P^{n-1}$, where $0 < r < \frac{\pi}{2}$),*

(A_2) *a tube of radius r over a totally geodesic $\mathbb{C}P^k (1 \leq k \leq n-2)$, where $0 < r < \frac{\pi}{2}$.*

For the real hypersurface of a complex space form many results are known. And new examples of nontrivial semi-invariant submanifolds in a complex projective space are constructed in [8], [14]. Therefore we may except to generalize some results which are valid in a real hypersurface to a semi-invariant submanifold. From this point of view, a semi-invariant submanifold of codimension 3 in a complex projective space are investigated in [6], [7], [8], [18] and so on by using properties of the third fundamental form of the submanifold and those of induced almost contact metric structure. One of them, Takagi and the present authors [8] assert the following:

Theorem KST [8]. *Let M be a real $(2n-1)$-dimensional $(n > 2)$ semi-invariant submanifold of codimension 3 in a complex projective space $\mathbb{C}P^{n+1}$ such that the third fundamental tensor n satisfies $dn = 2\theta\omega$ for a certain scalar $\theta(< \frac{c}{2})$, where $\omega(X,Y) = g(X,\phi Y)$ for any vectors X and Y on M. Then M has constant eigenvalues corresponding the shape operator A in the direction of distinguished normal and the structure vector ξ is an eigenvector of A if and only if M is locally congruent to a homogeneous real hypersurface of $\mathbb{C}P^n$.*

The main purpose of the present paper is to extend Theorem CK under certain conditions on a semi-invariant submanifold of codimension 3 in a complex projective space. Namely, we prove

Theorem. *Let M be a real $(2n-1)$-dimensional semi-invariant $(n > 2)$ submanifold of codimension 3 in a complex projective space $\mathbb{C}P^{n+1}$ such that the third fundamental form n satisfies $dn = 2\theta\omega$ for a certain scalar $\theta(< \frac{c}{2})$, where $\omega(X,Y) = g(X,\phi Y)$ for any vectors X and Y on M. If M satisfies $R_\xi\phi A = A\phi R_\xi$, then M is locally congruent to one of the following spaces in $\mathbb{C}P^n$:*

(A_1) *a geodesic hypersphere (that is, a tube of radius r over a hyperplane $\mathbb{C}P^{n-1}$, where $0 < r < \frac{\pi}{2}$),*

(A_2) *a tube of radius r over a totally geodesic $\mathbb{C}P^k (1 \leq k \leq n-2)$, where $0 < r < \frac{\pi}{2}$.*

Remark. The above Theorem can be considered by fact that the submanifolds of type A_1 and A_2 satisfy the condition $R_\xi \phi A = A\phi R_\xi$ respectively.

All manifolds in this paper are assumed to be connected and of class C^∞ and the semi-invariant submanifolds are supposed to be orientable.

1. Preliminaries

Let $\tilde{M}$ be a real $2(n+1)$-dimensional Kaehlerian manifold equipped with parallel almost complex structure J and a Riemannian metric tensor G and covered by a system of coordinate neighborhoods $\{\tilde{V}; y^A\}$.

Let M be a real $(2n-1)$-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{V; x^h\}$ and immersed isometrically in $\tilde{M}$ by the immersion $i : M \to \tilde{M}$.

Throughout this paper the following convention on the range of indices are used, unless otherwise stated:

$$A, B, C, \cdots = 1, 2, 3, \cdots, 2(n+1); \; i, j, k, \cdots = 1, 2, 3, \cdots, 2n-1.$$

The summation convention will be used with respect to those system of indices. In the sequel we identify $i(M)$ with M itself and represent the immersion by $y^A = y^A(x^h)$.

We put

$$B_i{}^A = \partial_i y^A, \quad \partial_i = \partial/\partial x^i$$

and denote by C, D and E three mutually orthogonal unit normals to M. Then denoting by g the fundamental metric tensor with components g_{ji} on M, we have $g_{ji} = G(B_j, B_i)$ since the immersion is isometric, where we have put $B_j = (B_j{}^A)$.

As is well-known, a submanifold M of a Kaehlerian manifold $\tilde{M}$ is said to be a *CR submanifold* ([1], [19]) if it is endowed with a pair of mutually orthogonal and complementary differentiable distribution $(T, T^\perp)$ such that for any $p \in M$ we have $JT_p = T_p$, $JT_p{}^\perp \subset T_p{}^\perp M$, where $T_p{}^\perp M$ denotes the normal space of M at p. In particular, M is said to be a ***semi-invariant submanifold*** ([2], [17]) provided that $dimT^\perp = 1$ or to be CR submanifold with CR dimension $n-1$ ([13]). In this case the unit vector field in $JT^\perp$ is called a ***distinguished normal*** to the semi-invariant submanifold and denoted this by C. Then we have

$$(1.1) \qquad JB_i = \phi_i{}^h B_h + \xi_i C, \quad JC = -\xi^h B_h, \quad JD = -E, JE = D,$$

where we have put $\phi_{ji} = G(JB_j, B_i), \xi_i = G(JB_i, C), \xi^h$ being associated components of ξ_h (see [8]). A tensor field of type (1,1) with components $\phi_i{}^h$ will be denoted by ϕ. By the Hermitian property of J, it is seen that ϕ_{ji} is skew-symmetric, and that

$$\phi_i{}^r \phi_r{}^h = -\delta_i{}^h + \xi_i \xi^h, \quad \xi^r \phi_r{}^h = 0, \quad \xi_r \phi_i{}^r = 0,$$
$$g_{rs} \phi_j{}^r \phi_i{}^s = g_{ji} - \xi_j \xi_i, \quad \xi_r \xi^r = 1,$$

namely, the aggregate (ϕ, ξ, g) defines ***almost contact metric structure.***

Denoting by ∇_j the operator of van der Waerden-Bortolotti covariant differentiation with respect to the induced Riemannian metric tensor g, the equation of Gauss for M of $\tilde{M}$ is obtained:

$$\nabla_j B_i = A_{ji}C + K_{ji}D + L_{ji}E, \tag{1.2}$$

where A_{ji}, K_{ji} and L_{ji} are components of the second fundamental forms in the direction of normals C, D, E respectively. Equations of Weingarten are also given by

$$\begin{cases} \nabla_j C = -A_j{}^h B_h + l_j D + m_j E, \\ \nabla_j D = -K_j{}^h B_h - l_j C + n_j E, \\ \nabla_j E = -L_j{}^h B_h - m_j C - n_j D, \end{cases} \tag{1.3}$$

where $A = (A_j{}^h), A_{(2)} = (K_j{}^h)$ and $A_{(3)} = (L_j{}^h)$, which are related by $A_{ji} = A_j{}^r g_{ir}, K_{ji} = K_j{}^r g_{ir}$ and $L_{ji} = L_j{}^r g_{ir}$ respectively, and l_j, m_j and n_j being components of the third fundamental forms.

In the sequel, we denote the normal components of $\nabla_j C$ by $\nabla_j^{\perp} C$. The distinguished normal C is said to *parallel* in the normal bundle if we have $\nabla_j^{\perp} C = 0$, that is, l_j and m_j vanish identically.

Since J is parallel, by differentiating (1.1) covariantly along M and using (1.1), (1.2) and (1.3), and by comparing the tangential and normal parts, we find (see [18])

$$\nabla_j \phi_i{}^h = -A_{ji}\xi^h + A_j{}^h \xi_i, \tag{1.4}$$

$$\nabla_j \xi_i = -A_{jr}\phi_i{}^r, \tag{1.5}$$

$$K_{ji} = -L_{jr}\phi_i{}^r - m_j \xi_i, \tag{1.6}$$

$$L_{ji} = K_{jr}\phi_i{}^r + l_j \xi_i. \tag{1.7}$$

Now we put $U_j = \xi^r \nabla_r \xi_j$. Then U is orthogonal to the structure vector ξ. Because of (1.5) and properties of the almost contact metric structure, it follows that

$$\phi_{jr} U^r = A_{jr}\xi^r - \alpha \xi_j, \tag{1.8}$$

$$U^r \nabla_j \xi_r = A_{jr}^2 \xi^r - \alpha A_{jr}\xi^r, \tag{1.9}$$

where we have put $\alpha = A_{ji}\xi^j \xi^i$.

Remark. In what follows, to write our formulas in convention forms, we denote by $\beta = A_{ji}{}^2\xi^j\xi^i$, $\mu = n_t\xi^t, T_r A_{(2)} = k$ and $\nu = (\nabla_t k)\xi^t$.

From (1.8), we get $g(U,U) = \beta - \alpha^2$. Thus we easily see that $A\xi = \alpha\xi$ if and only if $\beta - \alpha^2 = 0$.

Differentiating (1.8) covariantly along M and making use of (1.4) and (1.5), we find

$$\xi_j(A_{kr}U^r + \nabla_k\alpha) + \phi_{jr}\nabla_k U^r = \xi^r\nabla_k A_{jr} - A_{jr}A_{ks}\phi^{rs} + \alpha A_{kr}\phi_j{}^r, \tag{1.10}$$

which shows that

$$(\nabla_k A_{rs})\xi^r\xi^s = 2A_{kr}U^r + \nabla_k\alpha. \tag{1.11}$$

In the rest of this paper we shall suppose that $\tilde{M}$ is a Kaehlerian manifold of constant holomorphic sectional curvature c, which is called a *complex space form.* Then equations of Gauss and Codazzi are given by

$$\begin{aligned} R_{kjih} =& \frac{c}{4}(g_{kh}g_{ji} - g_{jh}g_{ki} + \phi_{kh}\phi_{ji} - \phi_{jh}\phi_{ki} - 2\phi_{kj}\phi_{ih}) \\ &+ A_{kh}A_{ji} - A_{jh}A_{ki} + K_{kh}K_{ji} - K_{jh}K_{ki} + L_{kh}L_{ji} - L_{jh}L_{ki}, \end{aligned} \tag{1.12}$$

$$\begin{aligned} &\nabla_k A_{ji} - \nabla_j A_{ki} - l_k K_{ji} + l_j K_{ki} - m_k L_{ji} + m_j L_{ki} \\ &= \frac{c}{4}(\xi_k\phi_{ji} - \xi_j\phi_{ki} - 2\xi_i\phi_{kj}), \end{aligned} \tag{1.13}$$

$$\nabla_k K_{ji} - \nabla_j K_{ki} = l_j A_{ki} - l_k A_{ji} + n_k L_{ji} - n_j L_{ki}, \tag{1.14}$$

$$\nabla_k L_{ji} - \nabla_j L_{ki} = m_j A_{ki} - m_k A_{ji} - n_k K_{ji} + n_j K_{ki}, \tag{1.15}$$

where R_{kjih} are covariant components of the Riemann-Christoffel curvature tensor of M, and those of the Ricci by

$$\nabla_k l_j - \nabla_j l_k = A_{jr}K_k{}^r - A_{kr}K_j{}^r + m_j n_k - m_k n_j, \tag{1.16}$$

$$\nabla_k m_j - \nabla_j m_k = A_{jr}L_k{}^r - A_{kr}L_j{}^r + n_j l_k - n_k l_j, \tag{1.17}$$

$$\nabla_k n_j - \nabla_j n_k = K_{jr}L_k{}^r - K_{kr}L_j{}^r + l_j m_k - l_k m_j + \frac{c}{2}\phi_{kj}. \tag{1.18}$$

2. Semi-invariant submanifolds satisfying $dn = 2\theta\omega$

In this section we shall suppose that M is a semi-invariant submanifold of codimension 3 in a complex projective space $\mathbb{C}P^{n+1}$ and that the third fundamental form n satisfies $dn = 2\theta\omega$ for a certain scalar θ on M, namely,

$$\nabla_j n_i - \nabla_i n_j = 2\theta\phi_{ji}. \tag{2.1}$$

Then we see that $dn = 2\theta\omega$ is independent of the choice of D and E.

There is no loss of generality such that we may assume $T_r A_{(3)} = 0$ (p.61, [8]).

From (1.6) and (1.7), we have

$$K_{jr}\xi^r = -m_j, \quad L_{jr}\xi^r = l_j, \tag{2.2}$$

$$m_r\xi^r = -k, \quad l_r\xi^r = 0. \tag{2.3}$$

Further we obtain

$$\phi_{ir}l^r = m_i + k\xi_i, \quad \phi_{ir}m^r = -l_i, \tag{2.4}$$

$$K_{jr}L_i{}^r + K_{ir}L_j{}^r + l_jm_i + l_im_j = 0. \tag{2.5}$$

From (1.12) we have

$$\begin{aligned} -(R_\xi)_{ji} =& \frac{c}{4}(g_{ji} - \xi_j\xi_i) + \alpha A_{ji} - (A_{jr}\xi^r)(A_{is}\xi^s) \\ &+ kK_{ji} - m_jm_i - l_jl_i \end{aligned} \tag{2.6}$$

because of (2.2) and (2.3), where we denote by $(R_\xi)_{ji} = R_{jkih}\xi^k\xi^h$.

From (1.18) and (2.1) we have

$$K_{jr}L_i{}^r - K_{ir}L_j{}^r + l_jm_i - l_im_j = 2(\theta - \frac{c}{4})\phi_{ij},$$

which together with (2.5) yields

$$K_{jr}L_i{}^r + l_jm_i = (\theta - \frac{c}{4})\phi_{ij}. \tag{2.7}$$

We notice here that θ is constant if $n > 2$ (see [8]).

In the previous paper [8], we proved the following:

Lemma 2.1 [8]. *Let M be a semi-invariant submanifold of codimension 3 in $\mathbb{C}P^{n+1}$ satisfying (2.1). If $\theta \neq \frac{c}{2}$, then we have $\nabla_j{}^\perp C = -k\xi_j E$ on M. Further if $A\xi = \alpha\xi$, then the distinguished normal is parallel in the normal bundle.*

In what follows, we assume that M satisfies (2.1) with $\theta \neq \frac{c}{2}$ and $n > 2$. Then by Lemma 2.1 and (1.3), we have

$$l_j = 0, \quad m_j = -k\xi_j \tag{2.8}$$

Thus (1.6), (1.7) and (2.7) turn out respectively to

$$L_{jr}\phi_i{}^r = -K_{ji} + k\xi_j\xi_i, \tag{2.9}$$

(2.10) $$K_{jr}\phi_i{}^r = L_{ji},$$

(2.11) $$K_{jr}L_i{}^r = (\theta - \frac{c}{4})\phi_{ij}.$$

From the last two equations, it follows that

(2.12) $$L_{ji}{}^2 = (\theta - \frac{c}{4})(g_{ji} - \xi_j\xi_i).$$

Furthermore, if we make use of (2.8), then the other structure equations (1.13) $\sim$ (1.17) are reduced respectively to

(2.13) $$\nabla_k A_{ji} - \nabla_j A_{ki} = k(\xi_j L_{ki} - \xi_k L_{ji}) + \frac{c}{4}(\xi_k\phi_{ji} - \xi_j\phi_{ki} - 2\xi_i\phi_{kj}),$$

(2.14) $$\nabla_k K_{ji} - \nabla_j K_{ki} = n_k L_{ji} - n_j L_{ki},$$

(2.15) $$\nabla_k L_{ji} - \nabla_j L_{ki} = k(\xi_k A_{ji} - \xi_j A_{ki}) - n_k K_{ji} + n_j K_{ki},$$

(2.16) $$A_{jr}K_k{}^r - A_{kr}K_j{}^r = k(n_k\xi_j - n_j\xi_k),$$

(2.17) $$A_{jr}L_k{}^r - A_{kr}L_j{}^r = \xi_k\nabla_j k - \xi_j\nabla_k k + k(A_{kr}\phi_j{}^r - A_{jr}\phi_k{}^r),$$

where we have used (1.5). Because of (2.2) and (2.8), it is clear that

(2.18) $$K_{jr}\xi^r = k\xi_j, \quad L_{jr}\xi^r = 0.$$

Multiplying (2.16) and (2.17) with ξ^k and summing for the index k, we have respectively

(2.19) $$\xi^s A_{sr}K_j{}^r = kA_{jr}\xi^r + k(n_j - \mu\xi_j),$$

(2.20) $$\xi^s A_{sr}L_j{}^r = \nu\xi_j - \nabla_j k + kU_j$$

by virtue of (1.5) and (2.18).

Transforming (2.19) by $\phi_k{}^j$ and taking account of (2.10), we find

(2.21) $$\xi^s A_{sr}L_k{}^r = k(\phi_{kr}n^r - U_k),$$

which together with (2.20) implies that

(2.22) $$\nabla_j k = \nu\xi_j - k(\phi_{jr}n^r - 2U_j).$$

If we transform (2.17) by $\phi_i{}^k$ and make use of (2.9) and (2.22), then we obtain

$$\begin{aligned} A_{sr}L_j{}^r\phi_i{}^s + A_{jr}K_i{}^r =& k\{(n_i - \mu\xi_i)\xi_j + 2\xi_j(A_{ir}\xi^r - \alpha\xi_i) \\ &+ 2\xi_i A_{jr}\xi^r - A_{ji} - A_{sr}\phi_j{}^r\phi_i{}^s\}, \end{aligned}$$

or, use (2.16)

$$A_{sr}L_j{}^r\phi_i{}^s = A_{sr}L_i{}^r\phi_j{}^s. \tag{2.23}$$

Since θ is constant if $n > 2$, by differentiation (2.12) covariantly gives

$$L_{jr}\nabla_k L_i{}^r + L_{ir}\nabla_k L_j{}^r = (\theta - \frac{c}{4})(\xi_j A_{kr}\phi_i{}^r + \xi_i A_{kr}\phi_j{}^r), \tag{2.24}$$

from which, taking the skew-symmetric part with respect to indices k and j and making use of (2.11) and (2.15),

$$\begin{aligned}&L_{jr}\nabla_k L_i{}^r - L_{kr}\nabla_j L_i{}^r + k(\xi_k A_{jr}L_i{}^r - \xi_j A_{kr}L_i{}^r)\\ &= (\theta - \frac{c}{4})\{n_j\phi_{ki} - n_k\phi_{ji} + \xi_j A_{kr}\phi_i{}^r - \xi_k A_{jr}\phi_i{}^r + \xi_i(A_{kr}\phi_j{}^r - A_{jr}\phi_k{}^r)\}\end{aligned}$$

for any indices k, j and i. Thus, interchanging indices k and i, we get

$$\begin{aligned}&L_{jr}\nabla_i L_k{}^r - L_{ir}\nabla_j L_k{}^r + k(\xi_i A_{jr}L_k{}^r - \xi_j A_{ir}L_k{}^r)\\ &= (\theta - \frac{c}{4})\{n_j\phi_{ik} - n_i\phi_{jk} + \xi_j A_{ir}\phi_k{}^r - \xi_i A_{jr}\phi_k{}^r + \xi_k(A_{ir}\phi_j{}^r - A_{jr}\phi_i{}^r)\}.\end{aligned}$$

Hence, if we use (2.11) and (2.15), then we obtain

$$\begin{aligned}&L_{jr}\nabla_k L_i{}^r - L_{ir}\nabla_k L_j{}^r\\ &= (\theta - \frac{c}{4})\{2n_k\phi_{ij} + \xi_j A_{ir}\phi_k{}^r - \xi_i A_{jr}\phi_k{}^r + \xi_k(A_{ir}\phi_j{}^r - A_{jr}\phi_i{}^r)\}\\ &\quad + k\{\xi_j(A_{kr}L_i{}^r + A_{ir}L_k{}^r) - \xi_i(A_{kr}L_j{}^r + A_{jr}L_k{}^r) + \xi_k(A_{ir}L_j{}^r - A_{jr}L_i{}^r)\},\end{aligned}$$

which together with (2.24) yields

$$\begin{aligned}&2L_{jr}\nabla_k L_i{}^r\\ &= (\theta - \frac{c}{4})\{2n_k\phi_{ij} + \xi_j(A_{ir}\phi_k{}^r + A_{kr}\phi_i{}^r) + \xi_i(A_{kr}\phi_j{}^r - A_{jr}\phi_k{}^r)\\ &\qquad\qquad + \xi_k(A_{ir}\phi_j{}^r - A_{jr}\phi_i{}^r)\}\\ &\quad + k\{\xi_j(A_{kr}L_i{}^r + A_{ir}L_k{}^r) - \xi_i(A_{kr}L_j{}^r + A_{jr}L_k{}^r) + \xi_k(A_{ir}L_j{}^r - A_{jr}L_i{}^r)\}.\end{aligned}$$

Multiplying ξ^j to the last equation and summing for j, and taking account of (2.18) and (2.21), we find

$$\begin{aligned}&(\theta - \frac{c}{4})(A_{ir}\phi_k{}^r + A_{kr}\phi_i{}^r) + (k^2 + \theta - \frac{c}{4})(U_k\xi_i + U_i\xi_k)\\ &+ k\{A_{kr}L_i{}^r + A_{ir}L_k{}^r - k(\xi_i\phi_{kr}n^r + \xi_k\phi_{ir}n^r)\} = 0.\end{aligned} \tag{2.25}$$

3. The Jacobi operator satisfying $R_\xi\phi A = A\phi R_\xi$

We continue now, our arguments under the same hypotheses $dn = 2\theta\omega$ for a scalar $\theta(\neq \frac{c}{2})$ as in section 2. Furthermore suppose, throughout this paper, that $R_\xi \phi A = A\phi R_\xi$. Then from (2.6) we obtain

$$\frac{c}{4}(A_{jr}\phi_i{}^r + A_{ir}\phi_j{}^r) - (A_{jr}\xi^r)(A_{is}U^s) - (A_{ir}\xi^r)(A_{js}U^s) = k(A_{jr}L_i{}^r + A_{ir}L_j{}^r) \tag{3.1}$$

where we have used (1.5), (1.7) and (2.8). If we multiply ξ^j to (3.1) and sum for j, and make use of (1.8) and (2.18), then we obtain

$$k\xi^s A_{sr}L_i{}^r = -\alpha A_{ir}U^r - \frac{c}{4}U_i, \tag{3.2}$$

which together with (2.21) gives,

$$k^2\phi_{jr}n^r = (k^2 - \frac{c}{4})U_j - \alpha A_{jr}U^r.$$

Thus, by applying $A_t{}^j\xi^t$ and using (1.8), it is seen that $k^2 n_t U^t = 0$.

We set $\Omega = \{p \in M : k(p) \neq 0\}$, and suppose that Ω is nonempty. From now on, we discuss our arguments on the open subset Ω of M. Then by the discussion above we have

$$n(U) = 0. \tag{3.3}$$

Lemma 3.1. *$\theta \neq \frac{c}{4}$ on Ω.*

proof. Suppose that $\theta = \frac{c}{4}$ on Ω. Then from (2.12) it follows that

$$L_{ji} = 0,$$

which together with (2.9) gives

$$K_{ji} = k\xi_j\xi_i.$$

In this case (2.15) turns out to be

$$k(\xi_k A_{ji} - \xi_j A_{ki} + n_j\xi_k\xi_i - n_k\xi_j\xi_i) = 0,$$

which shows

$$k\{n_k + A_{kr}\xi^r - (\alpha + \mu)\xi_k\} = 0.$$

Then the last two equations imply

$$A_{ji} = \xi_j A_{ir}\xi^r + \xi_i A_{jr}\xi^r - \alpha\xi_j\xi_i$$

on Ω. Since U is orthogonal to ξ, it is seen that

$$AU = 0,$$

which together with (3.1) and $L_{ji} = 0$ implies that $A\phi = \phi A$ and hence $A\xi = \alpha\xi$ on Ω. Therefore by Lemma 2.1 we have $k = 0$, a contradiction. This completes the proof.

Applying (3.1) by $L_k^{\ i}$ and using (2.9), (2.12) and (3.2), we find

$$\frac{c}{4}k(A_{jr}K_k^{\ r} + A_{sr}L_j^{\ r}\phi_k^{\ s}) - k(A_{jr}\xi^r)(L_{kt}A_s^{\ t}U^s) + (A_{js}U^s)(\alpha A_{kr}U^r + \frac{c}{4}U_k)$$
$$= k^2\{(\theta - \frac{c}{4})A_{jk} + A_{sr}L_j^{\ s}L_k^{\ r} + (\frac{c}{2} - \theta)\xi_k A_{jr}\xi^r\},$$

from which, taking the skew-symmetric part and making use of (2.16) and (2.23),

$$k^2\{((\frac{c}{2} - \theta)A_{kr}\xi^r + \frac{c}{4}n_k)\xi_j - ((\frac{c}{2} - \theta)A_{jr}\xi^r + \frac{c}{4}n_j)\xi_k\}$$
$$= k\{(A_{jr}\xi^r)(L_{kt}A_s^{\ t}U^s) - (A_{kr}\xi^r)(L_{jt}A_s^{\ t}U^s)\} + \frac{c}{4}(U_j A_{kr}U^r - U_k A_{jr}U^r).$$

Applying U^k to this and using (3.3), we have

$$\frac{c}{4}\{(\beta - \alpha^2)A_{jr}U^r - (A_{rs}U^rU^s)U_j\} = k(U^k L_{kt}A_s^{\ t}U^s)A_{jr}\xi^r.$$

If we multiply $A_m^{\ j}\xi^m$ and summing for j, we find

$$\beta U^k L_{kt}A_s^{\ t}U^s = 0.$$

From the last two equations, it follows that

$$\beta\{(\beta - \alpha^2)A_{jr}U^r - (A_{rs}U^rU^s)U_j\} = 0.$$

Since $\beta(\beta - \alpha^2) = 0$ is impossible because of the second assertion of Lemma 2.1 and (2.3), it is seen that

$$A_{jr}U^r = \lambda U_j, \tag{3.4}$$

where we have defined the function λ by

$$(\beta - \alpha^2)\lambda = A_{rs}U^rU^s.$$

Therefore (3.2) is reduced to

$$k\xi^s A_{sr}L_j^{\ r} = -(\alpha\lambda + \frac{c}{4})U_j,$$

which together with (1.8) and (2.12) yields

$$k(\theta - \frac{c}{4})\phi_{jr}U^r = -(\alpha\lambda + \frac{c}{4})L_{jr}U^r.$$

Then from Lemma 3.1 and the equation above we have

$$L_{jr}U^r = x\phi_{jr}U^r, \tag{3.5}$$

where we have defined

$$(\alpha\lambda + \frac{c}{4})x = -k(\theta - \frac{c}{4}). \tag{3.6}$$

Transforming (3.5) by $\phi_i^{\ j}$ and using (2.9), we find

$$K_{ir}U^r = xU_i. \tag{3.7}$$

Because of (2.11), (3.5) and (3.7), it is clear that

$$(x^2 - \theta + \frac{c}{4})\phi_{ir}U^r = 0.$$

As is already seen that $\beta - \alpha^2 \neq 0$ on Ω, we have $x^2 = \theta - \frac{c}{4}$. Thus, by Lemma 3.1, we verify that x is nonzero constant if $n > 2$. Hence (3.6) implies

$$-\alpha\lambda = kx + \frac{c}{4}. \tag{3.8}$$

Thus, using (2.21), (2.22), (3.2) and (3.4), we have

$$\nabla_j k = \nu\xi_j + (k - x)U_j. \tag{3.9}$$

Remark. $\alpha\lambda \neq 0$ on Ω. In fact, if not, then we have from (3.8), $kx + \frac{c}{4} = 0$. Since x is nonzero constant, it follows that k is constant. Thus, (3.9) means $k = x$, that is $\theta = \frac{c}{4}$, a contradiction.

Applying (3.1) by U^i and making use of (1.8) and (3.4), (3.5) and (3.8), we find

$$(kx + \frac{c}{4})A_{jr}{}^2\xi^r = \lambda(\frac{c}{2} - \beta + \alpha\lambda)A_{jr}\xi^r + (kx + \frac{c}{4})(\alpha\lambda + \frac{c}{2})\xi_j.$$

Hence, it is verified that

$$A_{jr}{}^2\xi^r = \varepsilon A_{jr}\xi^r + (\alpha\lambda + \frac{c}{2})\xi_j, \tag{3.10}$$

where the function ε is defined by

$$\alpha\varepsilon = \beta - \alpha\lambda - \frac{c}{2} \tag{3.11}$$

by virtue of the fact that $\alpha\lambda \neq 0$ on Ω.

Using (3.9), the equation (2.17) turns out to be

$$A_{jr}L_k^{\ r} - A_{kr}L_j^{\ r} = (k - x)(U_j\xi_k - U_k\xi_j) + k(A_{kr}\phi_j^{\ r} - A_{jr}\phi_k^{\ r}).$$

Multiplying U^k to this and summing for k, and making use of (1.8), (3.4), (3.5) and (3.10), we find

$$\{(k - x)(\varepsilon - \alpha) + \lambda(k + x)\}(A_{jr}\xi^r - \alpha\xi_j) = 0.$$

Thus, by Lemma 2.1, we have

$$(k - x)(\varepsilon - \alpha) + \lambda(k + x) = 0. \tag{3.12}$$

Now, we are going to prove that Ω is empty.

Lemma 3.2. *$\beta - \alpha^2 = \frac{2k\theta}{k-x}$ on Ω.*

Proof. By (3.9), (2.22) implies

$$\phi_{jr}n^r = (1 + \frac{x}{k})U_j. \tag{3.13}$$

Thus, by the property of almost contact metric structure, it is clear that

$$n_j = \mu\xi_j - (1 + \frac{x}{k})\phi_{jr}U^r. \tag{3.14}$$

Combining (2.25) to (3.1), we have

$$\theta(A_{jr}\phi_i^{\ r} + A_{ir}\phi_j^{\ r} + U_i\xi_j + U_j\xi_i) = \lambda(U_j\phi_{ir}U^r + U_i\phi_{jr}U^r), \tag{3.15}$$

where we have used (1.8), (3.4), (3.8) and (3.13). Since $\beta - \alpha^2 \neq 0$, by multiplying this with U^i and summing for i, and using (1.8) and (3.10), we have

$$\theta(\alpha - \varepsilon + \lambda) = \lambda(\beta - \alpha^2).$$

From this and (3.11), we get $(\theta+\alpha\lambda)(\beta-\alpha^2) = (2\alpha\lambda+\frac{c}{2})\theta$. Since we have $\theta = x^2+\frac{c}{4}$ and (3.8), it follows that $(k-x)(\beta-\alpha^2) = 2k\theta$, which proves the lemma.

From Lemma 3.2, we have

$$\nabla_j\beta - 2\alpha\nabla_j\alpha = -\frac{2\theta x}{(k-x)^2}\nabla_j k,$$

which together with (3.9) implies that

$$U^r\nabla_j U r = \frac{\theta x}{x-k}U_j - \frac{\theta x\nu}{(k-x)^2}\xi_j \tag{3.16}$$

because of $g(U,U) = \beta-\alpha^2$.

Next, we put $A\xi = \alpha\xi + \rho W$, where ρ is a function on M which is not vanish on Ω and W is a unit vector field orthogonal to ξ. Then we have $\phi U = \rho W$ and $\rho^2 = \beta-\alpha^2$ because of (1.8). Thus W is also orthogonal to U. Further with (3.10) and (3.11) we get

$$A_{jr}W^r = \rho\xi_j + (\varepsilon-\alpha)W_j. \tag{3.17}$$

by virtue of $\rho \neq 0$ on Ω. We have from (3.16)

$$W^j U^r\nabla_j U r = 0. \tag{3.18}$$

Using (1.8), (2.18) and (2.19), we obtain

$$\rho K_{jr}W^r = \rho k W_j + k\{n_j - (\alpha+\mu)\xi_j\},$$

which together with (1.8) and (3.14) yields

$$K_{jr}W^r = -xW_j. \tag{3.19}$$

Lemma 3.3. *$\nabla k = (k-x)U$ on Ω.*

Proof. Differentiation (3.9) covariantly gives

$$\nabla_k\nabla_j k = (\nabla_k\nu)\xi_j + \{\nu\xi_k + (k-x)U_k\}U_j - \nu A_{kr}\phi_j{}^r + (k-x)\nabla_k U_j,$$

which shows

$$\begin{aligned}&\xi_j\nabla_k\nu - \xi_k\nabla_j\nu + \nu(\xi_k U_j - \xi_j U_k + A_{jr}\phi_k{}^r - A_{kr}\phi_j{}^r)\\ &= (k-x)(\nabla_j U_k - \nabla_k U_j).\end{aligned} \tag{3.20}$$

On the other hand, differentiating (3.7) covariantly, we find

$$(\nabla_k K_{jr})U^r + K_{jr}\nabla_k U^r = x\nabla_k U_j, \tag{3.21}$$

which together with (3.7) implies that $(\nabla_k K_{ji})U^j U^i = 0$. If we take account of (2.14), (3.3), (3.5) and the last equation, then we get

$$U^r U^s(\nabla_r K_{js}) = 0.$$

Applying (3.21) by U^k and using this, we obtain

$$K_{jr}(U^s\nabla_s U^r) = xU^s\nabla_s U_j.$$

From this and (3.19), it follows that

$$W^r U^s\nabla_s U_r = 0.$$

Multiplying $U^j W^k$ to (3.20) and summing for j and k and making use of (3.4), (3.17), (3.18) and the last equation, we obtain $\rho\nu(\lambda+\varepsilon-\alpha) = 0$ and hence $\nu(\lambda+\varepsilon-\alpha) = 0$. From this and (3.12) we verify that $\nu\lambda = 0$. So we have $\nu(kx+\frac{c}{4}) = 0$ because of (3.8). Thus, it is, using (3.9), seen that ν vanishes on Ω. This completes the proof.

Lemma 3.4. *$du = 0$ on Ω, where the 1-form u is defined by $u(X) = g(U, X)$ for any vector X on M.*

Proof. Since $\nu = 0$, (3.20) becomes

$$(k - x)(\nabla_j U_i - \nabla_i U_j) = 0.$$

If $du \neq 0$, then we have $k = x$ and hence $k^2 = \theta - \frac{c}{4}$. Thus (3.8) implies $\alpha\lambda + \theta = 0$. From this and (3.12), it follows that $\theta = 0$. Thus $k^2 + \frac{c}{4} = 0$, a contradiction. Hence we have

$$\nabla_j U_i - \nabla_i U_j = 0.$$

This completes the proof of the lemma.

Lemma 3.5. *$\nabla\alpha = (\varepsilon - 3\lambda)U$ on Ω.*

Proof. Differentiating (3.4) covariantly, we find

$$(\nabla_k A_{jr})U^r + A_{jr}\nabla_k U^r = U_j \nabla_k \lambda + \lambda \nabla_k U_j,$$

from which, taking the skew-symmetric part and making use of Lemma 3.4,

$$\begin{aligned}&(kx - \frac{c}{4})(\xi_j A_{kr}\xi^r - \xi_k A_{jr}\xi^r) + A_j{}^r \nabla_r U_k - A_{kr}\nabla_j U^r \\ &= U_j \nabla_k \lambda - U_k \nabla_j \lambda,\end{aligned}$$

where we have used (1.8), (2.13) and (3.5). Hence, by applying U^k and remembering (3.4) and (3.16) with $\nu = 0$, we obtain

$$(\beta - \alpha^2)\nabla_j \lambda = (U^t \nabla_t \lambda)U_j, \tag{3.22}$$

which unable us to obtain $\xi^t \nabla_t \lambda = 0$. From this and (3.8), we verify that $\lambda \xi^t \nabla_t \alpha = 0$ and hence

$$\xi^t \nabla_t \alpha = 0. \tag{3.23}$$

If we take the inner product (1.10) with ξ^k, and use (3.23), then we get

$$\phi_j{}^r \xi^k \nabla_r U_k = (3\lambda - \alpha)U_j + \nabla_j \alpha,$$

where we have used (1.5), (1.11), (2.13), (2.18) and (3.4), which together with (1.9) and (3.10) yields $\nabla_j \alpha = (\varepsilon - 3\lambda)U_j$. Hence Lemma 3.5 is proved.

Lemma 3.6. *$d\mu(\xi) = 0$ and $x\mu = \frac{\lambda(k+x)^2}{k-x}$ on Ω.*

Proof. Using (2.14), (3.5) and Lemma 3.4, the equation (3.21) implies

$$x(n_k \phi_{jr} U^r - n_j \phi_{kr} U^r) + K_{jr}\nabla_k U^r - K_{kr}\nabla_j U^r = 0.$$

Since U is orthogonal to the structure vector ξ, by applying ξ^k and using (1.8), (2.18) and Lemma 3.4, we get

$$x\mu(A_{jr}\xi^r - \alpha\xi_j) - K_j{}^r(U^k \nabla_r \xi_k) + kU^r \nabla_j \xi_r = 0.$$

On the other hand, we have from (1.8), (2.19) and (3.14)

$$\xi^s A_{sr} K_j{}^r = -xA_{jr}\xi^r + \alpha(k + x)U_j.$$

Therefore, if we take account of (1.9) and (3.10), then the last two equations implies

$$\{x\mu + (\varepsilon - \alpha)(k + x)\}(A_{jr}\xi^r - \alpha\xi_j) = 0.$$

From this and (3.12) we see that $x(k - x)\mu - \lambda(k + x)^2 = 0$, which together with (3.22) and Lemma 3.3 gives $\xi^t \nabla_t \mu = 0$. Therefore, Lemma 3.6 is proved.

Lemma 3.7. *Ω is empty set, that is, k vanishes identically on whole space M.*

Proof. Differentiating (3.14) covariantly and taking account of (1.4), (1.5), (3.4) and Lemma 3.3, we find

$$\begin{aligned}\nabla_k n_j =& \xi_j \nabla_k \mu - \mu A_{kr}\phi_j{}^r + \frac{x}{k^2}(k-x)U_k\phi_{jr}U^r \\ &- (1+\frac{x}{k})(\lambda U_k \xi_j + \phi_{jr}\nabla_k U^r),\end{aligned}$$

from which, taking the skew-symmetric part and using (2.1),

$$\begin{aligned}&2\theta\phi_{kj} + \frac{x}{k^2}(k-x)\{U_j\phi_{kr}U^r - U_k\phi_{jr}U^r\} \\ &= \xi_j\nabla_k\mu - \xi_k\nabla_j\mu - \mu(A_{kr}\phi_j{}^r - A_{jr}\phi_k{}^r) \\ &\quad - (1+\frac{x}{k})\{\lambda(U_k\xi_j - U_j\xi_k) + \phi_{jr}\nabla_k U^r - \phi_{kr}\nabla_j U^r\}.\end{aligned} \tag{3.24}$$

Applying this by ξ^j and using Lemma 3.3 and Lemma 3.5, we find

$$\nabla_k\mu = \mu U_k + (1+\frac{x}{k})(\lambda U_k + \phi_k{}^r U^j \nabla_r \xi_j),$$

or, using (1.9), (3.10) and (3.12),

$$\nabla_k\mu = (\mu+\lambda)(1+\frac{x}{k})U_k. \tag{3.25}$$

On the other hand, the skew-symmetric part of (1.10) gives

$$\begin{aligned}\phi_{kr}\nabla_j U^r - \phi_{jr}\nabla_k U^r = \frac{c}{2}\phi_{kj} + 2A_{jr}A_{ks}\phi^{rs} + \alpha(A_{jr}\phi_k{}^r - A_{kr}\phi_j{}^r) \\ + (\varepsilon - 2\lambda)(U_k\xi_j - U_j\xi_k),\end{aligned}$$

where we have used (2.13), (2.18), (3.4) and Lemma 3.5.

If we substitute (3.25) and this into (3.24), and make use of Lemma 3.6, then we obtain

$$\begin{aligned}&2\theta\phi_{kj} + \frac{x}{k^2}(k-x)(U_j\phi_{kr}U^r - U_k\phi_{jr}U^r) \\ &= (1+\frac{x}{k})\{(\mu+\varepsilon-2\lambda)(U_k\xi_j - U_j\xi_k) + \frac{c}{2}\phi_{kj} + 2A_{jr}A_{ks}\phi^{rs} \\ &\quad + (\mu+\varepsilon)(A_{jr}\phi_k{}^r - A_{kr}\phi_j{}^r)\}.\end{aligned}$$

Multiplying this equation with U^j and summing for j, and taking account of (1.8), (3.4), (3.10) and Lemma 3.2, we find

$$\begin{aligned}&2(\theta - \frac{c}{4})(1+\frac{x}{k})(A_{kr}\xi^r - \alpha\xi_k) \\ &= (1+\frac{x}{k})\{-(\mu+\varepsilon-2\lambda)(\beta-\alpha^2)\xi_k - 2\lambda(\varepsilon-\alpha)A_{kr}\xi^r + 2\lambda(2kx+\alpha\lambda)\xi_k \\ &\quad + (\mu+\varepsilon)(\lambda+\varepsilon-\alpha)A_{kr}\xi^r - 2(\mu+\varepsilon)(kx+\alpha\lambda)\xi_k\}.\end{aligned}$$

Thus, it follows that

$$2(\theta - \frac{c}{4}) = -2\lambda(\varepsilon - \alpha) + (\mu + \varepsilon)(\lambda + \varepsilon - \alpha)$$

because of $\beta - \alpha^2 \neq 0$ on Ω, or using (3.8), (3.12) and Lemma 3.6,

$$\lambda^2(k + x) = \theta(k - x).$$

Differentiating this covariantly and using Lemma 3.3, we get

$$(k + x)\nabla_j\lambda = x\lambda U_j.$$

By the way, we have from (3.8)

$$\alpha\nabla_j\lambda = (3\lambda^2 - \lambda\varepsilon + x^2 - kx)U_j,$$

where we have used Lemma 3.3 and Lemma 3.5. Combining the last three equations, we verify that

$$x\alpha\lambda = 3\theta(k - x) - \lambda(k + x)\varepsilon - x(k^2 - x^2),$$

or using (3.8) and (3.12)

$$(6x^2 + \frac{5}{4}c)k = x^3,$$

which shows that $(6x^2 + \frac{5}{4}c)(k - x) = 0$, a contradiction because of $\alpha\lambda \neq 0$ on Ω. Therefore Ω is empty set. This completes the proof.

4. The proof of Theorem

Proof of Theorem. Let M be a connected real $(2n-1)$-dimensional $(n > 2)$ semi-invariant submanifold of codimension 3 satisfying $dn = 2\theta\omega$ for a certain scalar $\theta < \frac{c}{2}$ in $\mathbb{C}P^{n+1}$. Suppose that $R_\xi\phi A = A\phi R_\xi$. Then by Lemma 3.6 we have $k = 0$ on M. Thus, (2.8) tells us that the distinguished normal C is parallel in the normal bundle. Hence, by Lemma 4.1 of [8], we have $A_{(2)} = A_{(3)} = 0$. Therefore, by the reduction theorem in [5], [12], M is a real hypersurface in a complex projective space $\mathbb{C}P^n$. Since we have $\nabla^\perp C = 0$, equations (1.13) and (3.1) are reduced respectively to

$$\nabla_k A_{ji} - \nabla_j A_{ki} = \frac{c}{4}(\xi_k\phi_{ji} - \xi_j\phi_{ki} - 2\xi_i\phi_{kj}),$$

$$\frac{c}{4}(A_{jr}\phi_i{}^r + A_{ir}\phi_j{}^r) - (A_{jr}\xi^r)(A_{is}U^s) - (A_{ir}\xi^r)(A_{js}U^s) = 0.$$

Using (1.4), (1.5) and above two equations, it is proved in [4] that $g(U, U) = 0$. Hence we have $A\phi = \phi A$. Thus, by Theorem O we have our Theorem.

In the case where $\theta = \frac{c}{4}$, that is, M is a semi-invariant submanifold with $dn = \frac{c}{2}\omega$, then from Theorem we have

Corollary 4.1. *Let M be a semi-invariant submanifold of codimension 3 with $dn = \frac{c}{2}\omega$ in $\mathbb{C}P^{n+1}$. If M satisfies $R_\xi\phi A = A\phi R_\xi$, then M is locally congruent to one of the following spaces in $\mathbb{C}P^n$:*

(A_1) *a geodesic hypersphere (that is, a tube of radius r over a hyperplane $\mathbb{C}P^{n-1}$, where $0 < r < \frac{\pi}{2}$),*

(A_2) *a tube of radius r over a totally geodesic $\mathbb{C}P^k$ $(1 \leq k \leq n-2)$, where $0 < r < \frac{\pi}{2}$.*

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