On infinitesimal CL-transformations of compact normal contact metric spaces

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Y. Tashiro and S. Tachibana have introduced a transformation in a normal contact space which carries C-loxodromes. In this place a C-loxodrome means a loxodrome cutting geodesic trajectories of ξ^i with constant angle. The transformation thus defined is called a CL-transformation [6].

They have shown some relations between a normal contact space and a C-Fibinian space under a CL-transformation. Recently S. Kôto and M. Nagao have obtained invariant tensors under a CL-transformation [1].

The main purpose of the present paper is to show that in a compact normal contact metric space an infinitesimal CL-transformation is necessarily projective. In §1 we state the fundamental identities of normal contact spaces. In §2 we shall deal with a C-loxodrom and a CL-transformation. In §3 we shall prepare some lemmas used to prove the main theorem. The last §4 is devoted to the proof of the main theorem.

1. Normal contact metric space.

Let M be an $n(=2m+1)$-dimensional contact manifold with (φ,ξ,η,g) structure. We know the relations between a tensor field $\varphi_j{}^i$, contravariant vector field ξ^i, covariant vector field η_i and a positive definit metric tensor field g_{ji} such that

(1. 1) $$\xi^i\eta_i = 1,$$

(1. 2) $$rank|\varphi_j{}^i| = n-1,$$

(1. 3) $$\varphi_j{}^i\xi^j = 0,$$

(1. 4) $$\varphi_j{}^i\eta_i = 0,$$

(1. 5) $$\varphi_j{}^i\varphi_k{}^j = -\delta_k{}^i+\eta_k\xi^i,$$

(1. 6) $$g_{ji}\xi^j = \eta_i,$$

(1. 7) $$g_{ji}\varphi_k{}^j\varphi_h{}^j = g_{hk}-\eta_h\eta_k.$$

A space is called a normal contact metric space, if the tensor $N_{jk}{}^{i}$[1] vanishes. In normal cotact metric space, the structure satisfies the equations

(1. 8) $$\nabla_j \eta_i = \varphi_{ji},$$

(1. 9) $$\nabla_k \varphi_{ji} = \eta_j g_{ki} - \eta_i g_{kj}$$

where ∇_j denotes the covariant derivative with respect to the Riemannian connection of g_{ji} and $\varphi_{ji} = \varphi^l g_{li}$.

Moreover in the normal contact metric space we have the identities[2]

(1. 10) $$\eta_r R_{kji}{}^{r} = \eta_k g_{ji} - \eta_j g_{ki},$$

(1. 11) $$\varphi_{jr} R_l{}^{r} + \frac{1}{2} \varphi^{rk} R_{rkjl} = (n-2)\varphi_{jl},$$

(1. 12) $$\eta_r R_j{}^{r} = (n-1)\eta_j$$

where $R_{kji}{}^{r}$ and R_{lr} are Riemannian curvature tensor and Ricci's tensor respectively. In the following paragraph we use a notation η^i instead of ξ^i.

2. C-loxodromes and CL-transformations.

In a normal contact metric space, we consider a parameterized curve $u(s)$ which satisfies differential equation

(2. 1) $$\frac{\delta^2 u^h}{ds^2} = \alpha \eta_j \varphi_i{}^{h} \frac{du^j}{ds} \frac{du^i}{ds}$$

where s indicates arc-length and δ covariant differential along the curves $u(s)$ and α is a certain scalar. If α is constant, above equation shows that its integral curve is a C-loxodrome which was introduced by Y. Tashiro and S. Tachibana, that is, the curve is a loxodrome cutting trajectories of ξ^i with constant angle.

Now let us consider a relation between symmetric affine connections in an almost contact manifold. If it carries C-loxodromes to C-loxodromes, then it will be called a CL-transformation. By standard arguments, we can express it by the following relation:

$$'\Gamma_{ji}{}^{h} - \Gamma_{ji}{}^{h} = \rho_j \delta_i{}^{h} + \rho_i \delta_j{}^{h} + \alpha(\eta_j \varphi_i{}^{h} + \eta_i \varphi_j{}^{h})$$

where ρ_i is a vector field and α is a cretain scalar.

In a normal contact metric space, a vector v^i is called an infinitesimal CL-transformation if it satisfies

1) The tensor was defined by S. Sasaki and Y. Hatakeyama [5].
2) M. Okamura [3].

$$\underset{v}{\pounds}\left\{ {h \atop ji} \right\} = \rho_j\delta_i^h + \rho_i\delta_j^h + \alpha(\eta_j\varphi_i^h + \eta_i\varphi_j^h) \tag{2. 2}$$

where $\underset{v}{\pounds}$ is the operator of Lie drivative and $\left\{ {h \atop ji} \right\}$ is Riemannian connection. Contracting h and j in (2. 2), we see that ρ_i is a gradient.

From (2. 2) we have

$$\nabla_k \underset{v}{\pounds}\left\{ {h \atop ji} \right\} = \nabla_k\rho_j\delta_i^h + \nabla_k\rho_i\delta_j^h + \alpha(\varphi_i^h\nabla_k\eta_j + \eta_j\nabla_k\varphi_i^h + \varphi_j^h\nabla_k\eta_i + \eta_i\nabla_k\varphi_j^h) + \nabla_k\alpha(\eta_j\varphi_i^h + \eta_i\varphi_j^h) \tag{2. 3}$$

and

$$\nabla_j \underset{v}{\pounds}\left\{ {h \atop ki} \right\} = \nabla_j\rho_k\delta_i^h + \nabla_j\rho_i\delta_k^h + \alpha(\varphi_i^h\nabla_j\eta_k + \eta_k\nabla_j\varphi_i^h + \varphi_k^h\nabla_j\eta_i + \eta_i\nabla_j\varphi_k^h) + \nabla_j\alpha(\eta_k\varphi_i^h + \eta_i\varphi_k^h). \tag{2. 4}$$

Substituting (2. 3) and (2. 4) into the following identity of Lie derivation[3)]

$$\underset{v}{\pounds} R_{kji}{}^h = \nabla_k \underset{v}{\pounds}\left\{ {h \atop ji} \right\} - \nabla_j \underset{v}{\pounds}\left\{ {h \atop ki} \right\} \tag{2. 5}$$

and using (1. 9) and (1. 10), we get

$$\underset{v}{\pounds} R_{kji}{}^h = \delta_j^h\nabla_k\rho_i - \delta_k^h\nabla_j\rho_i + \alpha\{2\varphi_{kj}\varphi_i^h + \eta_i(\eta_j\delta_k^h - \eta_k\delta_j^h) + \eta_j(\eta_i\delta_k^h - \eta^h g_{ki}) - \eta_k(\eta_i\delta_j^h - \eta^h g_{ji}) + \varphi_{ki}\varphi_j^h - \varphi_{ji}\varphi_k^h\} + \varphi_i^h(\nabla_k\alpha\eta_j - \nabla_j\alpha\eta_k) + \eta_i(\nabla_k\alpha\varphi_j^h - \nabla_j\alpha\varphi_k^h). \tag{2. 6}$$

Contracting k and h in (2. 6), we have

$$\underset{v}{\pounds} R_{ji} = -(n-1)\nabla_j\rho_i + 2\alpha(n\eta_i\eta_j - g_{ij}) + \eta_j\varphi_i^r\nabla_r\alpha + \eta_i\varphi_j^r\nabla_r\alpha. \tag{2. 7}$$

Transvecting (2. 6) with η_h, we have

$$\eta_h \underset{v}{\pounds} R_{kji}{}^h = \eta_j\nabla_k\rho_i - \eta_k\nabla_j\rho_i + \alpha(\eta_k g_{ji} - \eta_j g_{ki}). \tag{2. 8}$$

3. Some Lemmas.

In this section we shall prepare some lemmas which are useful to prove the main theorem.

LEMMA. 3. 1. *In a normal contact metric space, if v^i is an infinitesimal CL-trans-*

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3) K. Yano [7].

formation, then the following relation holds good:

(3. 1) $$\underset{v}{\pounds} g_{ji} = -\nabla_j \rho_i + \beta \eta_j \eta_i + \alpha(g_{ji} - \eta_j \eta_i)$$

where α and β are certain scalars.

PROOF. Taking the Lie derivative of the both sides of (1. 10) and substituting (2. 8) into the equation thus obtained, we get

(3. 2) $$R_{kji}{}^h \underset{v}{\pounds} \eta_h = g_{ji} \underset{v}{\pounds} \eta_k + \eta_k \underset{v}{\pounds} g_{ji} - g_{ki} \underset{v}{\pounds} \eta_j - \eta_j \underset{v}{\pounds} g_{ki} - \eta_j \nabla_k \rho_i + \eta_k \nabla_j \rho_i + \alpha(\eta_j g_{ki} - \eta_k g_{ji}).$$

Transvecting (3. 2) with g^{ji}, we have

(3. 3) $$R_k{}^h \underset{v}{\pounds} \eta_h = (n-1) \underset{v}{\pounds} \eta_k + \eta_k (g^{ji} \underset{v}{\pounds} g_{ji} + \nabla_r \rho^r) - \eta^r (\underset{v}{\pounds} g_{kr} + \nabla_k \rho_r) + \alpha \eta_k (1-n).$$

Similarly transvecting (3. 2) with η^k and using (1. 10), we get

(3. 4) $$\underset{v}{\pounds} g_{ji} = -\nabla_j \rho_i + \eta_j (\eta^r \underset{v}{\pounds} g_{ri} + \eta^r \nabla_i \rho_r) + \alpha(g_{ji} - \eta_j \eta_i).$$

On the other hand, transvecting (3. 2) with φ^{kj} we have

(3. 5) $$(\varphi^{kj} R_{kji}{}^h + 2\varphi_i{}^h) \underset{v}{\pounds} \eta_h = 0.$$

Transvecting (3. 3) with $\varphi_j{}^k$, we have

(3. 6) $$\varphi_j{}^k R_k{}^h \underset{v}{\pounds} \eta_h = \{(n-1) \underset{v}{\pounds} \eta_k - \eta^r (\underset{v}{\pounds} g_{kr} + \nabla_k \rho_r)\} \varphi_j{}^k$$

from which

(3. 7) $$(\varphi^{rk} R_{rkj}{}^h + 2\varphi_j{}^h) \underset{v}{\pounds} \eta_h = 2\varphi_j{}^k (\eta^r \underset{v}{\pounds} g_{kr} + \eta^r \nabla_k \rho_r).$$

If we put $\beta = \eta^r \eta^s (\underset{v}{\pounds} g_{rs} + \nabla_r \rho_s)$, taking account of (1. 2), from (3. 5) and (3. 7) we get

(3. 8) $$\eta^r (\underset{v}{\pounds} g_{ir} + \nabla_i \rho_r) = \beta \eta_i.$$

Substituting (3. 8) into (3. 4), we can get (3. 1)

LEMMA. 3. 2. *Between α and β in (3. 1), the following relation holds good:*

$$2\alpha = \beta.$$

PROOF. Taking the Lie derivative of the both sides of (1. 12) we have

$$R_k{}^h \underset{v}{\pounds} \eta_h + \eta_h \underset{v}{\pounds} R_k{}^h = (n-1) \underset{v}{\pounds} \eta_k.$$

This equation can be written as

(3. 9) $$\eta_h R_{kr} \underset{v}{\pounds} g^{hr} + \eta^r \underset{v}{\pounds} R_{kr} = (n-1) \underset{v}{\pounds} \eta_k - R_k{}^h \underset{v}{\pounds} \eta_h.$$

Substituting (3. 3) and (2. 7) into (3. 9), we have

(3. 10) $$\eta_h R_{kr} \underset{v}{\pounds} g^{hr} + \eta^r \{(1-n)\nabla_k \rho_r + 2\alpha(n\eta_k\eta_r - g_{kr}) + (\eta_k\varphi_r{}^h + \eta_r\varphi_k{}^h)\nabla_h\alpha\}$$
$$= -\eta_k(g^{ji} \underset{v}{\pounds} g_{ji} + \nabla_r\rho^r) + \eta^r(\underset{v}{\pounds} g_{kr} + \nabla_k\rho_r) + \alpha(n-1)\eta_k.$$

Substituting (3. 1) into (3. 10) and using (1. 1) and (1. 12), we get

(3. 11) $$\eta_h R_{kr} \nabla^h \rho^r - \beta(n-1)\eta_k + \eta^r(1-n)\nabla_k\rho_r + 2\alpha(n-1)\eta_k + \varphi_k{}^r\nabla_r\alpha$$
$$+ \eta_k(\beta - \alpha + n\alpha) - \beta\eta_k - \alpha(n-1)\eta_k = 0.$$

Transvecting (3. 11) with η^k and using (1. 12) we get

$$(n-1)(2\alpha - \beta) = 0.$$

Thus we have $2\alpha = \beta$. q. e. d.

LEMMA. 3. 3. *In an $n(n>3)$ dimensional compact normal contact metric space, if an infinitesimal transformation v^i satisfies*

(3. 12) $$\underset{v}{\pounds} g_{ji} = \lambda(g_{ji} + \eta_j\eta_i)$$

where λ is a certain scalar, then we have $\lambda = 0$.

PROOF. Substituting (3. 12) into the identity[4)]

$$\underset{v}{\pounds}\left\{ {h \atop ji} \right\} = \frac{1}{2} g^{hr}(\nabla_j \underset{v}{\pounds} g_{ri} + \nabla_i \underset{v}{\pounds} g_{rj} - \nabla_r \underset{v}{\pounds} g_{ji}),$$

we get

(3. 13) $$\underset{v}{\pounds}\left\{ {h \atop ji} \right\} = \frac{1}{2}\{\lambda_j(\delta_i{}^h + \eta^h\eta_i) + \lambda_i(\delta_j{}^h + \eta^h\eta_i) - \lambda^h(g_{ji} + \eta_j\eta_i)$$
$$+ \lambda(\varphi_j{}^h\eta_i + \varphi_i{}^h\eta_j)\}$$

where $\lambda_j = \nabla_j\lambda$.

Operating ∇_k to (3. 13), using (1. 8), we get

(3. 13) $$\nabla_k \underset{v}{\pounds}\left\{ {h \atop ji} \right\} = \frac{1}{2}\{\nabla_k\lambda_j(\delta_i{}^h + \eta^h\eta_i) + \lambda_j\eta_i\varphi_k{}^h + \lambda_j\eta^h\varphi_k{}^i + \nabla_k\lambda_i(\delta_i{}^h + \eta^h\eta_j)$$
$$+ \lambda_i\varphi_k{}^h\eta_j + \lambda_i\varphi_{kj}\eta^h - \nabla_k\lambda^h(g_{ji} + \eta_j\eta_i) - \lambda^h\eta_i\varphi_{kj}$$
$$- \lambda^h\eta_j\varphi_{ki} + \lambda_k(\eta_i\varphi_j{}^h + \eta_j\varphi_i{}^h) + \lambda(\eta_i\nabla_k\varphi_j{}^h + \varphi_j{}^h\varphi_{ki}$$
$$+ \eta_j\nabla_k\varphi_i{}^h + \varphi_i{}^h\varphi_{ki})\}$$

Interchanging j and k in (3. 14) and substituting the equation thus obtained

4) K. Yano [7].

and (3. 14) into (2. 5), we get

$$\underset{v}{\pounds} R_{kji}{}^h = \frac{1}{2}\{\varphi_k{}^h\lambda_i\eta_j - \varphi_j{}^h\lambda_i\eta_k + \varphi_i{}^h(\lambda_k\eta_j - \lambda_j\eta_k)$$
$$+\nabla_k\lambda_i(\delta_j^h + \eta^h\eta_j) - \nabla_j\lambda_i(\delta_k^h + \eta^h\eta_k) - \nabla_k\lambda^h(g_{ji} + \eta_j\eta_i)$$
$$+\nabla_j\lambda^h(g_{ki} + \eta_k\eta_i) + \lambda^h(\eta_k g_{ji} - \eta_j\varphi_{ki} - 2\eta_i\varphi_{kj})$$
$$+\eta^h(\lambda_j\varphi_{ki} - \lambda_k\varphi_{ji} + 2\lambda_i\varphi_{kj}) + \lambda_k\eta_j\varphi_i{}^h - \lambda_j\eta_k\varphi_i{}^h$$
$$+4\lambda\eta_i(\eta_j\delta_k^h - \eta_k\delta_j^h) + 2\lambda\eta^h(\eta_k g_{ji} - \eta_j g_{ki})\}. \tag{3. 15}$$

Transvecting (3. 15) with η^h and using (1. 4), we get

$$\eta_h \underset{v}{\pounds} R_{kji}{}^h = \frac{1}{2}\{2\eta_j\nabla_k\lambda_i - 2\eta_k\nabla_j\lambda_i - \eta_h\nabla_k\lambda^h(g_{ji} + \eta_j\eta_i)$$
$$+\eta_h\nabla_j\lambda^h(g_{ki} + \eta_k\eta_i) + \lambda_j\varphi_{ki} - \lambda_k\varphi_{ji} + 2\lambda_i\varphi_{kj}$$
$$-\eta_h\lambda^h(\eta_j\varphi_{ki} - \eta_k\varphi_{ji} + 2\eta_i\varphi_{kj}) + 2\lambda(\eta_k g_{ji} - \eta_j g_{ki})\}. \tag{3. 16}$$

Now taking the Lie derivative of both sides of (1. 10), we have

$$R_{kji}{}^h \underset{v}{\pounds}\eta_h + \eta_h \underset{v}{\pounds} R_{kji}{}^h = g_{ji}\underset{v}{\pounds}\eta_k - g_{ki}\underset{v}{\pounds}\eta_j + \eta_k\underset{v}{\pounds} g_{ji} - \eta_j\underset{v}{\pounds} g_{ki}. \tag{3. 17}$$

Substituting (3. 16) and (3. 12) into (3. 17), we have

$$R_{kji}{}^r \underset{v}{\pounds}\eta_r = g_{ji}\underset{v}{\pounds}\eta_k - g_{ki}\underset{v}{\pounds}\eta_j - \frac{1}{2}\{2\eta_j\nabla_k\lambda_i - 2\eta_k\nabla_j\lambda_i$$
$$-\eta_r\nabla_k\lambda^r(g_{ji} + \eta_j\eta_i) + \eta_r\nabla_j\lambda^r(g_{ki} + \eta_k\eta_i) + \varphi_{ki}\lambda_j - \varphi_{ji}\lambda_k$$
$$+2\varphi_{kj}\lambda_i - \eta_r\lambda^r(\varphi_{ki}\eta_j - \varphi_{ji}\eta_k + 2\varphi_{kj}\eta_i)\}. \tag{3. 18}$$

Transvecting (3. 18) with φ^{kj} and $\varphi_l{}^k g^{ji}$, we find respectively

$$\frac{1}{2}\varphi^{kj}R_{kji}{}^r \underset{v}{\pounds}\eta_r = -\varphi_i{}^r \underset{v}{\pounds}\eta_r + \frac{1}{2}\{-\varphi_i{}^r\eta_s\nabla_r\lambda^s - n(\lambda_i - \eta_r\lambda^r\eta_i)\} \tag{3. 19}$$

and

$$\varphi_i{}^k R_k{}^r \underset{v}{\pounds}\eta_r = (n-1)\varphi_i{}^r \underset{v}{\pounds}\eta_r + \frac{1}{2}\{(n-2)\varphi_i{}^r\eta_s\nabla_r\lambda^s + 3(\lambda_i - \eta_r\lambda^r\eta_i)\}. \tag{3. 20}$$

Adding (3. 19) and (3. 20), and taking account of (1. 11), we get

$$\lambda_i - \eta_r\lambda^r\eta_i - \varphi_i{}^r\eta_s\nabla_r\lambda^s = 0, \quad (n>3) \tag{3. 21}$$

Tranvecting (3. 21) with $\varphi_j{}^r$, we have

$$\eta_r\nabla_i\lambda^r = \mu\eta_i - \varphi_j{}^r\lambda_r \tag{3. 22}$$

where $\mu = \eta^r\eta^s\nabla_r\lambda_s$.

Moreover, transvecting (3. 18) with η^k and using (1. 10) and (3. 22), we get

(3. 22) $$2\nabla_j\lambda_i = \mu(-g_{ji}+3\eta_j\eta_i)-2(\eta_i\varphi_j{}^r\lambda_r+\eta_j\varphi_i{}^r\lambda_r).$$

Operating ∇_k to (3. 22) and transvecting the equation thus obtained with φ^{kj}, we get

(3. 23) $$2\varphi^{kj}\nabla_k\nabla_j\lambda_i = \varphi_i{}^r\nabla_r\mu+(3n-5)\mu\eta_i+2\eta_i\nabla_r\lambda^r.$$

By (1. 11), this equation can be written as

(3. 24) $$\varphi_i{}^r\{2(n-2)\lambda_r-R_{rs}\lambda^s+\nabla_r\mu\}+(3n-5)\mu\eta_i+2\eta_i\nabla_r\lambda^r=0.$$

Transvecting (3. 24) with η^i, we have

(3. 25) $$2\nabla_r\lambda^r+(3n-5)\mu = 0.$$

On the other hand, transvecting (3. 23) with g^{ji}, we have

(3. 26) $$2\nabla_r\lambda^r+(n-3)\mu = 0.$$

Comparing (3. 25) and (3. 26), it follows that $\nabla_r\lambda^r = 0$.

Consequently by Green's theorem we have $\lambda =$ constant.

Again applying Green's theorem to

$$\nabla_r v^r = \frac{n+1}{2}\lambda$$

which is obtained from (3.12), we have $\lambda = 0$. q. e. d.

4. Infinitesimal CL-transformation in compact normal contact metric spaces.

Let v^i be an infinitesimal CL-transformation in compact normal contact metric space. Then from lemmas 3. 1 and 3. 2, we have

(4. 1) $$\underset{v}{\pounds} g_{ji} = -\nabla_j\rho_i+\alpha(g_{ji}+\eta_j\eta_i).$$

Above equation can be written as

(4. 2) $$\nabla_j v_i+\nabla_i v_j+\nabla_j\rho_i = \alpha(g_{ji}+\eta_j\eta_i).$$

In (4. 2), since $\nabla_j\rho_i = \nabla_i\rho_j$, putting $w_i = v_i+\frac{1}{2}\rho_i$, then we have

(4. 3) $$\underset{w}{\pounds} g_{ji} = \nabla_j w_i+\nabla_i w_j = \alpha(g_{ji}+\eta_j\eta_i).$$

By lemma 3. 3, we have $\alpha = 0$.

Thus we have following

THEOREM. 4. 1. *In an $n(n>3)$ dimensional compact normal contact metric space, an infinitesimal CL-transformation is necessarily projective.*

By theorem 4. 1, we have immediately the following

COLORARY 4. 1. *In an $n(n>3)$ dimensional compact η-Einstein space, an infinitesimal CL-transformation is necessarily an isometry.*

An η-Einstein space is a normal contact metric space with Ricci's tensor satisfying $R_{ji} = ag_{ji}+b\eta_j\eta_i$, where a and b are constsnts and its example was given by M. Okumura [4].

COLOLLARY 4. 2[6]. *Let M be a compact normal contact netric space of consant scalar curvature* $R \neq n(n-1)$ *and* ρ_i *be an associated vector of an infinitesimal CL-transorfmation, then* $\eta^r \rho_r = 0$.

Next, let M be an Einstein normal contact metric space, then

$$R_{ji} = kg_{ji} \tag{4. 4}$$

where k is constant.

Transvecting (4. 4) with η_i, we have

$$\eta^i R_{ji} = k\eta_j. \tag{4. 5}$$

Comparing (1. 12) and (4. 5), we have

$$k = n-1. \tag{4. 6}$$

Taking the Lie derivative of the both sides of (4. 4), we get

$$\mathop{\pounds}_{v} R_{ji} = (n-1) \mathop{\pounds}_{v} g_{ji} \tag{4. 7}$$

Now, let v^i be an infinitesimal CL-transformation, then substituting (2. 7) and (4. 1) into (4. 7), we have

$$\begin{aligned}(1-n)\nabla_j\rho_i+2\alpha(n\eta_i\eta_j-g_{ij})+\eta_j\varphi_i{}^r\nabla_r\alpha+\eta_i\varphi_j{}^r\nabla_r\alpha \\ = (n-1)\{-\nabla_j\rho_i+\alpha(g_{ji}+\eta_j\eta_i)\}.\end{aligned} \tag{4. 8}$$

Transvecting (4. 8) with g^{ji}, we have

$$(n^2-1)\alpha = 0.$$

Hence, we have

$$\alpha = 0.$$

Thus we have the following

THEOREM 4. 2. *In an Einstein normal contact metric spac.., an infinitesimal CL-transformation is necessarily projective.*

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