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RING HOMOMORPHISMS ON COMMUTATIVE REGULAR BANACH ALGEBRAS

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ABSTRACT. We give a partial representation of a ring homomorphism, which need not be continuous nor surjective, from a semisimple commutative regular Banach algebra into a semisimple commutative Banach algebra. As a corollary to our main theorem, we prove that there are no surjective ring homomorphism from $C_0(\mathbb{R})$ onto $C_0(\mathbb{D})$.

1. Introduction and the statement of results

Let $\mathcal{A}$ and $\mathcal{B}$ be algebras over the complex number field $\mathbb{C}$. A mapping $\rho\colon \mathcal{A} \to \mathcal{B}$ is a ring homomorphism provided that

$$\rho(f+g) = \rho(f) + \rho(g) \qquad (f, g \in \mathcal{A})$$
$$\rho(fg) = \rho(f)\rho(g) \qquad (f, g \in \mathcal{A}).$$

If, in addition, ρ preserves scalar multiplication, that is, $\rho(\lambda f) = \lambda\rho(f)$ for every $f \in \mathcal{A}$ and $\lambda \in \mathbb{C}$, then ρ is an ordinary homomorphism. The zero mapping $\rho(z) = 0$ ($z \in \mathbb{C}$), the identity mapping $\rho(z) = z$ ($z \in \mathbb{C}$) and the complex conjugate $\rho(z) = \bar{z}$ ($z \in \mathbb{C}$) are typical examples of ring homomorphisms on $\mathbb{C}$. These are called trivial ring homomorphisms on $\mathbb{C}$, or in short trivial. It is obvious that the trivial ring homomorphisms on $\mathbb{C}$ are continuous. The converse is also valid, that is, a continuous ring homomorphism is trivial. Moreover, the following is well-known, so we omit a proof (For a proof, see, for example [9, Proposition 2.1]).

Proposition A. *If ρ is a ring homomorphism on $\mathbb{C}$, each of the following two statements implies the other:*

(a) *ρ is trivial.*
(b) *There exist $\alpha_0, \beta_0 > 0$ such that $|z| < \alpha_0$ implies $|\rho(z)| \le \beta_0$.*

One might expect that ring homomorphisms on $\mathbb{C}$ are necessarily trivial. Unfortunately, this is not true. In fact, there exists a non-trivial ring homomorphism on

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$\mathbb{C}$. By Proposition A, we see that a ring homomorphism τ on $\mathbb{C}$ is non-trivial if and only if the following are satisfied:

(*) for each $\alpha, \beta > 0$, there exists $z \in \mathbb{C}$ with $|z| < \alpha$ but $|\tau(z)| > \beta$.

We shall use (*) in Lemma 3.3. It seems that the existence of a non-trivial ring homomorphism had been investigated by C. Segre [14] and M. H. Lebesgue [6] (see [5]). H. Kestelman [5] had given many different ways to construct a non-trivial ring homomorphism under the axiom of choice, or one of some equivalent propositions, say the well-ordering theorem of Zermelo, or Zorn's lemma. By its construction, we see that there are infinitely many non-trivial ring homomorphisms on $\mathbb{C}$. More explicitly, M. Charnow [2] proved that if G is the set of all ring automorphisms on an algebraically closed field k, then $\sharp G = \sharp 2^k$, where $\sharp S$ denotes the cardinal number of a set S. In particular, there are $\sharp 2^{\mathbb{C}}$ ring automorphisms on $\mathbb{C}$. Moreover, ring homomorphic image is very complicated. Let $H(\Omega)$ be the algebra of all holomorphic functions on a region $\Omega \subset \mathbb{C}$. In [9], it is proved that there exists an injective ring homomorphism from $H(\Omega)$ into $\mathbb{C}$, that is, we may regard $H(\Omega)$ as a subring of $\mathbb{C}$. Ring homomorphisms are studied by many authors (cf. [1, 3, 4, 7, 8, 9, 10, 11, 12, 13, 15, 17]).

In this paper, we will consider a ring homomorphism ρ from a semisimple commutative regular complex Banach algebra A into a semisimple commutative complex Banach algebra B; Neither the continuity nor the surjectivity of ρ are assumed. The maximal ideal spaces of A and B are denoted by M_A and M_B, respectively. We will give a representation of such a ring homomorphism. For simplicity, we will denote the Gelfand transform of a by the same letter a; This will cause no confusion. Recall that A is *regular* if and only if for each pair (F, K) of closed subset F and compact subset K of M_A with $F \cap K = \emptyset$, there exists $a \in A$ such that $a(F) = 0$ and $a(K) = 1$ (cf. [16, Theorem 27.2]). Note that we do not assume that A and B are with unit. We will denote by A_e the commutative Banach algebra obtained by adjunction of a unit element e to A. Recall that P is a prime ideal of A if P is a proper ideal satisfying that $fg \in P$ implies $f \in P$ or $g \in P$. In particular, every maximal modular ideal is a prime ideal. Although we are concerned with *ring* homomorphisms, by an ideal we mean an *algebra* ideal.

Now we are ready to state our main result.

Theorem 1.1. *Let A be a semisimple commutative regular complex Banach algebra and B a semisimple commutative complex Banach algebra with maximal ideal spaces M_A and M_B, respectively. If $\rho\colon A \to B$ is a ring homomorphism, then there exist a decomposition $\{M_{-1}, M_0, M_1, M_d\}$ of M_B and a continuous mapping $\varphi\colon M_B \setminus M_0 \to$*

M_{A_e} *such that*

$$\rho(f)(y) = \begin{cases} \overline{f(\varphi(y))} & y \in M_{-1} \\ 0 & y \in M_0 \\ f(\varphi(y)) & y \in M_1 \\ \tau_y(q_y(f)) & y \in M_d \end{cases} \tag{1.1}$$

for every $f \in A$, *where* q_y *is the quotient mapping from a prime ideal* P_y *of* A *onto* A/P_y *and* τ_y *is a nonzero field homomorphism from the quotient field of* P_y *into* $\mathbb{C}$.

For a subset S of B, we say that S is *separating* if to each $y_1, y_2 \in M_B$ with $y_1 \neq y_2$ there corresponds $b_1 \in S$ such that $b_1(y_1) \neq b_1(y_2)$. If, for every $y \in M_B$, there exists $b_2 \in S$ such that $b_2(y) \neq 0$, then we say that S *vanishes nowhere.*

Corollary 1.2. *Let* $\rho\colon A \to B$ *be a ring homomorphism. If the range* $\rho(A)$ *contains a subalgebra* B_0 *of* B *such that* B_0 *is separating and vanishes nowhere, then there exist a decomposition* $\{M_{-1}, M_1, M_d\}$ *of* M_B *and an injective, continuous and closed mapping* $\varphi\colon M_B \to M_A$ *with the following property: To each* $y \in M_d$ *there corresponds a non-trivial ring automorphism* τ_y *from* $\mathbb{C}$ *onto itself such that*

$$\rho(f)(y) = \begin{cases} \overline{f(\varphi(y))} & y \in M_{-1} \\ f(\varphi(y)) & y \in M_1 \\ \tau_y(f(\varphi(y))) & y \in M_d \end{cases} \tag{1.2}$$

for all $f \in A$. *In particular,* B *is necessarily regular.*

2. Construction of the mapping φ

Recall that we never assume that A and B are with unit. Let $A_e = \{f + \lambda e : f \in A, \lambda \in \mathbb{C}\}$ be the Banach algebra obtained by adjunction of a unit element e to A. Note that A_e is well-defined even for unital A. The maximal ideal space M_{A_e} of A_e is the one-point compactification of M_A. We see that A_e is regular since so is A. If $\{x_\infty\} = M_{A_e} \setminus M_A$, then $f(x_\infty) = 0$ for every $f \in A$.

In this section, ρ will be a ring homomorphism from A into B, and ρ_y for $y \in M_B$ will be the induced ring homomorphism from A into $\mathbb{C}$ defined by

$$\rho_y(f) = \rho(f)(y) \qquad (f \in A).$$

We define a subset M_0 of M_B by

$$M_0 = \{y \in M_B : \rho_y \text{ is identically } 0\}.$$

Lemma 2.1. *Let* $y \in M_B \setminus M_0$.

(a) ρ_y *can be extended to a unique ring homomorphism* $\tilde{\rho}_y$ *of* A_e.

(b) *Let* $f+\lambda e \in A_e$. *Then* $f+\lambda e \in \ker\tilde{\rho}_y$ *if and only if* $fa+\lambda a \in \ker\rho_y$ *for every* $a \in A$ *with* $\rho_y(a) \neq 0$.

Proof. (a) Take $a \in A$ with $\rho_y(a) \neq 0$. If we define $\tilde{\rho}_y \colon A_e \to \mathbb{C}$ by

$$\tilde{\rho}_y(f+\lambda e) = \rho_y(f) + \frac{\rho_y(\lambda a)}{\rho_y(a)} \qquad (f+\lambda e \in A_e), \tag{2.1}$$

then it is obvious to verify that $\tilde{\rho}_y$ is additive and $\tilde{\rho}_y|_A = \rho_y$. By the equations

$$\rho_y(\nu h) = \rho_y(h)\,\frac{\rho_y(\nu a)}{\rho_y(a)} \qquad (\nu \in \mathbb{C}, h \in A) \tag{2.2}$$

and

$$\frac{\rho_y(\lambda\mu a)}{\rho_y(a)} = \frac{\rho_y(\lambda a)}{\rho_y(a)}\,\frac{\rho_y(\mu a)}{\rho_y(a)} \qquad (\lambda, \mu \in \mathbb{C}), \tag{2.3}$$

we have, for each $f+\lambda e$, $g+\mu e \in A_e$, that

$$\begin{aligned}
\tilde{\rho}_y((f+\lambda e)(g+\mu e)) &= \tilde{\rho}_y(fg+\mu f+\lambda g+\lambda\mu e) \\
&= \rho_y(fg+\mu f+\lambda g) + \frac{\rho_y(\lambda\mu a)}{\rho_y(a)} && \text{(by (2.1))} \\
&= \rho_y(f)\rho_y(g) + \rho_y(f)\frac{\rho_y(\mu a)}{\rho_y(a)} + \rho_y(g)\frac{\rho_y(\lambda a)}{\rho_y(a)} \\
&\qquad + \frac{\rho_y(\lambda a)}{\rho_y(a)}\,\frac{\rho_y(\mu a)}{\rho_y(a)} && \text{(by (2.2) and (2.3))} \\
&= \left\{\rho_y(f) + \frac{\rho_y(\lambda a)}{\rho_y(a)}\right\}\left\{\rho_y(g) + \frac{\rho_y(\mu a)}{\rho_y(a)}\right\} \\
&= \tilde{\rho}_y(f+\lambda e)\,\tilde{\rho}_y(g+\mu e),
\end{aligned}$$

which proves that $\tilde{\rho}_y$ is multiplicative. We have now proved that there exists a ring homomorphism $\tilde{\rho}_y$ from A_e into $\mathbb{C}$ such that $\tilde{\rho}_y|_A = \rho_y$.

It remains to be proved that $\tilde{\rho}_y = {\rho_y}^*$, whenever ${\rho_y}^*$ is another ring homomorphism with ${\rho_y}^*|_A = \rho_y$. So, take $f+\lambda e \in A_e$ arbitrarily. Since

$$\rho_y(\lambda a) = {\rho_y}^*(\lambda a) = {\rho_y}^*(\lambda e)\rho_y(a), \tag{2.4}$$

it follows from (2.1) and (2.4) that

$${\rho_y}^*(f+\lambda e) = {\rho_y}^*(f) + {\rho_y}^*(\lambda e) = \rho_y(f) + \frac{\rho_y(\lambda a)}{\rho_y(a)} = \tilde{\rho}_y(f+\lambda e),$$

and the uniqueness is proved. In particular, $\tilde{\rho}$ does not depend on a choice of $a \in A$ with $\rho_y(a) \neq 0$.

(b) By the uniqueness, $\tilde{\rho}_y$ is of the form (2.1) for any $a \in A$ with $\rho_y(a) \neq 0$. Now it is obvious that $f+\lambda e \in \ker\tilde{\rho}_y$ if and only if $fa+\lambda a \in \ker\rho_y$ for every $a \in A$ with $\rho_y(a) \neq 0$. The proof is complete. □

From now on, the letter $\tilde{\rho}_y$ will denote the unique ring homomorphism from A_e to $\mathbb{C}$ with $\tilde{\rho}_y|_A = \rho_y$ for $y \in M_B \setminus M_0$.

Definition 2.1. For $y \in M_B \setminus M_0$, we define a nonzero ring homomorphism $\sigma_y\colon \mathbb{C} \to \mathbb{C}$ by

$$\sigma_y(\lambda) = \tilde{\rho}_y(\lambda e) \qquad (\lambda \in \mathbb{C}).$$

By a simple calculation, we see that $\sigma_y(r) = r$ for every $y \in M_B \setminus M_0$ and rational real number r. It follows from the equation

$$\rho_y(\lambda f) = \tilde{\rho}_y(\lambda f) = \tilde{\rho}_y(\lambda e)\rho_y(f) \qquad (\lambda \in \mathbb{C}, f \in A)$$

that

$$\rho_y(\lambda f) = \sigma_y(\lambda)\rho_y(f) \qquad (\lambda \in \mathbb{C}, f \in A) \tag{2.5}$$

for every $y \in M_B \setminus M_0$. Thus (2.1) and (2.5) give

$$\tilde{\rho}_y(f + \lambda e) = \rho_y(f) + \sigma_y(\lambda) \qquad (f + \lambda e \in A_e) \tag{2.6}$$

for all $y \in M_B \setminus M_0$.

Lemma 2.2. *Let $y \in M_B \setminus M_0$. Then*

(a) *the kernel* $\ker \rho_y$ *is a prime ideal of A, which is contained in at most one maximal modular ideal of A,*

(b) $\ker \tilde{\rho}_y$ *is contained in a unique maximal ideal of A_e, and*

(c) *if* $\ker \rho_y$ *is a maximal modular ideal of A, then* $\ker \tilde{\rho}_y$ *is a maximal ideal of A_e.*

Proof. (a) By (2.5), we see that $\ker \rho_y$ is an *algebra* ideal of A. Now it is obvious that $\ker \rho_y$ is a prime ideal.

Suppose that $\ker \rho_y$ is contained in a maximal modular ideal of A, that is, there exists $x_1 \in M_A$ such that

$$\ker \rho_y \subset \{f \in A : f(x_1) = 0\}. \tag{2.7}$$

Take $x_2 \in M_A \setminus \{x_1\}$ arbitrarily. We will show that $\ker \rho_y$ is not contained in the maximal modular ideal $\{f \in A : f(x_2) = 0\}$. Choose an open neighborhood V_j of x_j, for $j = 1, 2$, so that $V_1 \cap V_2 = \emptyset$. The regularity of A therefore shows the existence of $f_j \in A$ such that

$$f_j(x_j) = 1 \quad \text{and} \quad f_j(M_A \setminus V_j) = 0 \qquad (j = 1, 2). \tag{2.8}$$

Hence $f_1 f_2 = 0$ on M_A, and so $\rho_y(f_1)\rho_y(f_2) = 0$. It follows from (2.7) and (2.8) that $\rho_y(f_1) \neq 0$, and hence $f_2 \in \ker \rho_y \setminus \{f \in A : f(x_2) = 0\}$. Since $x_2 \in M_A \setminus \{x_1\}$ was arbitrary, $\ker \rho_y$ is contained in at most one maximal modular ideal.

(b) Note that $\ker \tilde{\rho}_y$ is a proper ideal of a unital commutative Banach algebra A_e since $\tilde{\rho}_y|_A = \rho_y$ is nonzero. Thus $\ker \tilde{\rho}_y$ is contained in at least one maximal ideal of A_e. We see that the proof of (a) can be applied to A_e and $\ker \tilde{\rho}_y$, and so $\ker \tilde{\rho}_y$ is contained in at most one maximal ideal of A_e. We thus conclude that $\ker \tilde{\rho}_y$ is contained in a unique maximal ideal of A_e.

(c) Suppose that

$$\ker \rho_y = \{f \in A : f(x_0) = 0\} \tag{2.9}$$

for some $x_0 \in M_A$. Take $a \in A$ with $a(x_0) \neq 0$. Then $\rho_y(a) \neq 0$ by (2.9). If $g + \mu e \in \ker \tilde{\rho}_y$, then $ga + \mu a \in \ker \rho_y$ by (b) of Lemma 2.1, and hence (2.9) implies $(g(x_0) + \mu)a(x_0) = 0$. Since $a(x_0) \neq 0$, we have $g + \mu e \in \{\tilde{f} \in A_e : \tilde{f}(x_0) = 0\}$. Thus $\ker \tilde{\rho}_y \subset \{\tilde{f} \in A_e : \tilde{f}(x_0) = 0\}$.

Take $a' \in A$ with $\rho_y(a') \neq 0$. Since $(g(x_0) + \mu)a'(x_0) = 0$ for every $g + \mu e \in \{\tilde{f} \in A_e : \tilde{f}(x_0) = 0\}$, we have $ga' + \mu a' \in \ker \rho_y$ by hypothesis. Thus (b) of Lemma 2.1 shows $g + \mu e \in \ker \tilde{\rho}_y$, which implies that $\{\tilde{f} \in A_e : \tilde{f}(x_0) = 0\} = \ker \tilde{\rho}_y$. We thus conclude that $\ker \tilde{\rho}_y$ is a maximal ideal whenever $\ker \rho_y$ is a maximal modular ideal. □

Definition 2.2. By (b) of Lemma 2.2, for each $y \in M_B \setminus M_0$, $\ker \tilde{\rho}_y$ is contained in a unique maximal ideal of A_e. So, there exists a mapping φ: $M_B \setminus M_0 \to M_{A_e}$ such that $\ker \tilde{\rho}_y \subset \{\tilde{f} \in A_e : \tilde{f}(\varphi(y)) = 0\}$ for every $y \in M_B \setminus M_0$.

Lemma 2.3. *Let $y \in M_B \setminus M_0$ and let r be a rational real number. If $\tilde{h} \in A_e$ satisfies $\tilde{h}(\tilde{G}) = r$ for some open neighborhood $\tilde{G} \subset M_{A_e}$ of $\varphi(y)$, then $\tilde{\rho}_y(\tilde{h}) = r$.*

Proof. Put $\tilde{h}_r = \tilde{h} - re \in A_e$, and so $\tilde{h}_r = 0$ on $\tilde{G}$. By the regularity of A_e, there exists $\tilde{g} \in A_e$ such that $\tilde{g}(\varphi(y)) = 1$ and $\tilde{g}(M_{A_e} \setminus \tilde{G}) = 0$. Then $\tilde{g}\tilde{h}_r = 0$ on M_{A_e}, and hence $\tilde{\rho}_y(\tilde{g})\tilde{\rho}_y(\tilde{h}_r) = 0$. Since $\tilde{g}(\varphi(y)) = 1$, we have by the definition of φ that $\tilde{\rho}_y(\tilde{g}) \neq 0$, and hence $\tilde{\rho}_y(\tilde{h}_r) = 0$. Since $\tilde{\rho}_y(re) = \sigma_y(r) = r$, we have $\tilde{\rho}_y(\tilde{h}) = r$. □

Definition 2.3. We introduce the following notation

$$M_{-1} = \{y \in M_B \setminus M_0 : \sigma_y(\lambda) = \bar{\lambda}, \quad (\lambda \in \mathbb{C})\}$$
$$M_1 = \{y \in M_B \setminus M_0 : \sigma_y(\lambda) = \lambda \quad (\lambda \in \mathbb{C})\},$$

where σ_y is as in Definition 2.1. Put

$$M_d = \{y \in M_B \setminus M_0 : \sigma_y \text{ is non-trivial}\}.$$

Then M_{-1}, M_0, M_1 and M_d are (possibly empty) pairwise disjoint subsets of M_B with $M_B = M_{-1} \cup M_0 \cup M_1 \cup M_d$.

It should be mentioned that we can define the quotient field of an integral domain R, commutative ring which has no zero divisor, even if R has no unit: For if $a \in$

$R\backslash\{0\}$, then the equivalence class a/a, with respect to the usual equivalence relation, is a unit. Moreover, we can identify $b \in R$ with ba/a.

Lemma 2.4. *Let $y \in M_B\backslash M_0$ and $q_y \colon A \to A/\ker\rho_y$ be the quotient mapping. If F_y is the quotient field of $q_y(A)$, then there exists a unique nonzero field homomorphism $\tau_y \colon F_y \to \mathbb{C}$ such that*

(a) $\rho_y = \tau_y \circ q_y$,

(b) $\tau_y|_{\mathbb{C}} = \sigma_y$, *and*

(c) $\tau_y = \sigma_y$ *whenever* $\ker\rho_y$ *is a maximal modular ideal of* A.

Proof. Note first that the quotient field F_y of $q_y(A)$ is well-defined since $q_y(A)$ is an integral domain by (a) of Lemma 2.2. We define a mapping $\tau_y \colon F_y \to \mathbb{C}$ by

$$\tau_y(q_y(f)/q_y(g)) = \frac{\rho_y(f)}{\rho_y(g)} \qquad (q_y(f)/q_y(g) \in F_y). \tag{2.10}$$

A simple calculation shows that τ_y is a well-defined nonzero field homomorphism on F_y. Take $a \in A$ with $\rho_y(a) \neq 0$.

(a) As noted above, we may identify $q_y(f)$ with $q_y(fa)/q_y(a)$ for every $f \in A$. Under this identification, we have by (2.10) that

$$\rho_y(f) = \frac{\rho_y(fa)}{\rho_y(a)} = \tau_y(q_y(fa)/q_y(a)) = \tau_y(q_y(f))$$

for every $f \in A$, proving $\rho_y = \tau_y \circ q_y$.

(b) The identification $\lambda \in \mathbb{C}$ with $q_y(\lambda a)/q_y(a) \in F_y$ shows that

$$\tau_y(\lambda) = \tau_y(q_y(\lambda a)/q_y(a)) = \frac{\rho_y(\lambda a)}{\rho_y(a)} \qquad (\lambda \in \mathbb{C}).$$

From (2.5) it follows that $\rho_y(\lambda a)/\rho_y(a) = \sigma_y(\lambda)$ for every $\lambda \in \mathbb{C}$, and hence $\tau_y|_{\mathbb{C}} = \sigma_y$.

(c) Suppose that $\ker\rho_y$ is a maximal modular ideal of A. Then $q_y(A) = A/\ker\rho_y$ is isomorphic to $\mathbb{C}$. Thus we may assume $F_y = \mathbb{C}$, and hence $\tau_y = \sigma_y$ by (b). □

3. Topological properties of the decomposition of M_B

In this section, $\{M_{-1}, M_0, M_1, M_d\}$ will stand for the decomposition of M_B as in Definition 2.3.

Lemma 3.1. *M_0 is a closed subset of M_B.*

Proof. Let $\{y_\alpha\} \subset M_0$ be a net converging to a point $y_0 \in M_B$. Take $f \in A$ arbitrarily. Then $\rho(f)(y_\alpha) = \rho_{y_\alpha}(f) = 0$ by definition. Since $\rho(f)$ is a continuous function on M_B, we have $\rho_{y_0}(f) = 0$. Since $f \in A$ was arbitrary, we have $y_0 \in M_0$, proving M_0 closed. □

Lemma 3.2. *Both $M_{-1} \cup M_0$ and $M_0 \cup M_1$ are closed subsets of M_B.*

Proof. Since M_0 is closed, it is enough to show that $\mathrm{cl}(M_j) \subset M_0 \cup M_j$ for $j = \pm 1$. Here and after, $\mathrm{cl}(S)$ denotes the closure of a set S. It will cause no confusion if we use the same letter to designate a closure in M_A and M_B.

For $j = \pm 1$, take $y_0 \in \mathrm{cl}(M_j)$ and a net $\{y_\alpha\} \subset M_j$ converging to y_0. If $y_0 \notin M_0$, then there exists $a \in A$ such that $\rho_{y_0}(a) \neq 0$. Since $\rho_{y_\alpha}(a) = \rho(a)(y_\alpha)$ converges to $\rho_{y_0}(a) \neq 0$, without loss of generality we may assume $\rho_{y_\alpha}(a) \neq 0$ for all α. It follows from (2.5) that

$$\sigma_{y_\alpha}(\lambda) = \frac{\rho_{y_\alpha}(\lambda a)}{\rho_{y_\alpha}(a)} \to \frac{\rho_{y_0}(\lambda a)}{\rho_{y_0}(a)} = \sigma_{y_0}(\lambda) \qquad (\lambda \in \mathbb{C}).$$

On the other hand, since $\{y_\alpha\} \subset M_j$, we have $\sigma_{y_0}(\lambda) = \bar{\lambda}$ if $j = -1$, and $\sigma_{y_0}(\lambda) = \lambda$ if $j = 1$. Thus $y_0 \in M_j$ for $j = \pm 1$. We thus obtain $\mathrm{cl}(M_j) \subset M_0 \cup M_j$, and the proof is complete. □

Lemma 3.3. *The range $\varphi(M_d) \subset M_{A_e}$ is at most finite.*

Proof. Assume, to get a contradiction, that $\varphi(M_d)$ contains a countable subset $\{w_n\}_{n\in\mathbb{N}}$. We may assume that $\{w_n\}_{n\in\mathbb{N}} \subset M_A$. We first assert that there exists a subset $\{x_k\}_{k\in\mathbb{N}}$ of $\{w_n\}_{n\in\mathbb{N}}$ with the following property: To each $k \in \mathbb{N}$ there corresponds an open neighborhood U_k of x_k such that $\{\mathrm{cl}(U_k)\}_{k\in\mathbb{N}}$ is a pairwise disjoint family; If each w_k is an isolated point of $\{w_n\}_{n\in\mathbb{N}}$, then it is obvious that there is such an open neighborhood U_k of w_k, and so we will consider the case where there is a limit point, say w_1, in $\{w_n\}_{n\in\mathbb{N}}$. Take $x_1 \in \{w_n\}_{n\in\mathbb{N}}$ with $x_1 \neq w_1$ arbitrarily. There exists an open neighborhood U_1 of x_1 such that $w_1 \notin \mathrm{cl}(U_1)$. Since w_1 is assumed to be a limit point in $\{w_n\}_{n\in\mathbb{N}}$, there exists $x_2 \in (M_A \setminus \mathrm{cl}(U_1)) \cap \{w_n\}_{n\in\mathbb{N}}$ such that $x_2 \neq w_1$. Choose an open neighborhood U_2 of x_2 so that $\mathrm{cl}(U_2) \cap (\mathrm{cl}(U_1) \cup \{w_1\}) = \emptyset$. Inductively, for each $k \in \mathbb{N}$ with $k \geq 2$ there exists $x_k \in \{w_n\}_{n\in\mathbb{N}}$ and an open neighborhood U_k of x_k such that

$$\mathrm{cl}(U_k) \cap (\cup_{n=1}^{k-1} \mathrm{cl}(U_n) \cup \{w_1\}) = \emptyset. \tag{3.1}$$

From (3.1) it is obvious that each U_k is an open neighborhood of x_k such that $\{\mathrm{cl}(U_k)\}_{k\in\mathbb{N}}$ is a pairwise disjoint family.

For each $k \in \mathbb{N}$, take an open neighborhood V_k of x_k, with compact closure $\mathrm{cl}(V_k)$, such that $\mathrm{cl}(V_k) \subset U_k$. The regularity of A shows that there exists $g_k \in A$ such that $g_k(\mathrm{cl}(V_k)) = 1$ and $g_k(M_A \setminus U_k) = 0$. Take $y_k \in M_d$ with $x_k = \varphi(y_k)$. Since σ_{y_k} is non-trivial, it follows from $(*)$ (see Proposition A) that there exists $\lambda_k \in \mathbb{C}$ such that

$$|\lambda_k| < \frac{1}{2^k \|g_k\|} \quad \text{and} \quad |\sigma_{y_k}(\lambda_k)| > 2^k. \tag{3.2}$$

Set $f_k = \lambda_k g_k \in A$. It follows from (3.2) that $\|f_k\| < 2^{-k}$, and so the series $\sum_{k=1}^{\infty} f_k$ converges in the norm of A, say f_0. Then $f_0 = f_k$ on U_k since $g_m(M_A \setminus U_m) = 0$ for every $m \in \mathbb{N}$. By Lemma 2.3, applied to an open $U_k \subset M_{A_e}$ and $f_0 - f_k \in A_e$, we have $\rho_{y_k}(f_0 - f_k) = 0$, and so $\rho_{y_k}(f_0) = \rho_{y_k}(f_k)$. Another application of Lemma 2.3 yields $\rho_{y_k}(g_k) = 1$ since $g_k(V_k) = 1$. By (2.5), we have

$$\rho_{y_k}(f_k) = \sigma_{y_k}(\lambda_k)\rho_{y_k}(g_k) = \sigma_{y_k}(\lambda_k).$$

It follows from (3.2) that

$$|\rho(f_0)(y_k)| = |\rho_{y_k}(f_0)| = |\rho_{y_k}(f_k)| = |\sigma_{y_k}(\lambda_k)| > 2^k.$$

We now arrived at a contradiction since $\rho(f_0)$ is bounded on M_B, and hence we have proved that the range $\varphi(M_d)$ is at most finite. □

Lemma 3.4. *Set* $M_d(x) \stackrel{\text{def}}{=} \{y \in M_d : \varphi(y) = x\}$ *for* $x \in M_{A_e}$.

(a) *Each* $y_0 \in M_j$ *is an interior point of* $M_j \cup M_d(\varphi(y_0))$ *for* $j = \pm 1$.

(b) *Each* $y_0 \in M_d$ *is an interior point of* $M_d(\varphi(y_0))$. *In particular,* $M_d(\varphi(y_0))$ *is an open subset of* M_B.

Proof. For $j = \pm 1$, take $y_0 \in M_j \cup M_d$ and set $x_0 = \varphi(y_0)$. There exist open neighborhoods $\tilde{U}, \tilde{V} \subset M_{A_e}$ of x_0 such that $\mathrm{cl}(\tilde{V}) \subset \tilde{U}$, $\mathrm{cl}(\tilde{V})$ compact and

$$\varphi(M_d) \setminus \{x_0\} \subset M_{A_e} \setminus \mathrm{cl}(\tilde{U}). \tag{3.3}$$

This would be possible since $\varphi(M_d)$ is at most finite by Lemma 3.3. Since A_e is regular, there exists $\tilde{f} \in A_e$ such that

$$\tilde{f}(\mathrm{cl}(\tilde{V})) = 1 \quad \text{and} \quad \tilde{f}(M_{A_e} \setminus \tilde{U}) = 0. \tag{3.4}$$

By Lemma 2.3, applied to $\tilde{f}$ and $\tilde{V}$, we have

$$\tilde{\rho}_{y_0}(\tilde{f}) = 1. \tag{3.5}$$

Since $\tilde{f}(M_{A_e} \setminus \mathrm{cl}(\tilde{U})) = 0$ by (3.4), another application of Lemma 2.3 shows that

$$\tilde{\rho}_y(\tilde{f}) = 0 \quad \text{for every } y \in \varphi^{-1}(M_{A_e} \setminus \mathrm{cl}(\tilde{U})). \tag{3.6}$$

Recall that $\tilde{f} \in A_e$ is of the form $\tilde{f} = f + \lambda e$ for some $f \in A$ and $\lambda \in \mathbb{C}$. Let $\{x_\infty\} = M_{A_e} \setminus M_A$. If $x_0 = x_\infty$, then by (3.4) we have that $1 = \tilde{f}(x_0) = f(x_0) + \lambda$, and hence $\lambda = 1$ since $f \in A$ vanishes at infinity. If $x_0 \neq x_\infty$, assume, without loss of generality, that $\tilde{U}, \tilde{V} \subset M_A$ and $\mathrm{cl}(\tilde{U})$ compact in M_A. It follows from (3.4), with $x_\infty \in M_{A_e} \setminus \mathrm{cl}(\tilde{U})$, that $0 = \tilde{f}(x_\infty) = \lambda$. In each case, $\tilde{f}$ is of the form $f + re$, where $r = 0$ or $r = 1$. Thus (2.6) gives

$$\tilde{\rho}_y(\tilde{f}) = \rho_y(f) + r \qquad (y \in M_B \setminus M_0).$$

It follows from (3.5) and (3.6) that $\rho_{y_0}(f) = 1 - r$ and that

$$\rho_y(f) = -r \quad \text{for every } y \in \varphi^{-1}(M_{A_e} \setminus \mathrm{cl}(\tilde{U})). \tag{3.7}$$

Since $\rho(f)$ is continuous, there exists an open neighborhood $O \subset M_B$ of y_0 such that

$$|\rho_y(f) - 1 + r| < \frac{1}{2} \qquad (y \in O). \tag{3.8}$$

Since $M_j \cup M_d$ is open by Lemma 3.2, we may assume $O \subset M_j \cup M_d$. It follows from (3.7) and (3.8) that $\varphi(y') \in \mathrm{cl}(\tilde{U})$ for every $y' \in O \cap M_d$, and so $\varphi(y') = x_0$ by (3.3). This implies that $O \subset M_j \cup M_d(x_0)$, that is, if $y_0 \in M_j$, then $y_0 \in O \subset M_j \cup M_d(x_0)$, proving (a); If $y_0 \in M_d$, then $y_0 \in O \cap M_d \subset M_d(x_0)$ as proved above, which proves (b) since M_d is open by Lemma 3.2. This completes the proof. □

4. A proof of results and remarks

Proof of Theorem 1.1. Let $\rho\colon A \to B$ be a ring homomorphism and $\{M_{-1}, M_0, M_1, M_d\}$ the decomposition of M_B as in Definition 2.3. Let q_y be the quotient mapping of A onto $A/\ker \rho_y$ for every $y \in M_B \setminus M_0$. By (a) of Lemma 2.4, for every $y \in M_B \setminus M_0$ there exists a nonzero field homomorphism τ_y from the quotient field F_y of $A/\ker \rho_y$ into $\mathbb{C}$ such that $\rho_y = \tau_y \circ q_y$: If $f \in A$, then $\rho(f)(y) = 0$ for every $y \in M_0$, and $\rho(f)(y) = \tau_y(q_y(f))$ for every $y \in M_B \setminus M_0$.

Let $y \in M_B \setminus M_0$ and φ the mapping as in Definition 2.2. Suppose that $\ker \rho_y$ is a maximal modular ideal of A. Then (c) of Lemma 2.2 shows that $\ker \tilde{\rho}_y$ is a maximal ideal of A_e. By the definition of φ, we have $\ker \tilde{\rho}_y = \{\tilde{f} \in A_e : \tilde{f}(\varphi(y)) = 0\}$, which implies that $f - f(\varphi(y))e \in \ker \tilde{\rho}_y$ for every $f \in A$. It follows from (2.6) that

$$\rho_y(f) = \sigma_y(f(\varphi(y))) \qquad (f \in A) \tag{4.1}$$

whenever $\ker \rho_y$ is a maximal modular ideal. By (2.5), if $y \in M_{-1}$ ($y \in M_1$), then $\overline{\rho_y}$ (resp. ρ_y) is a *nonzero complex homomorphism* on A. So, $\ker \rho_y$ is a maximal modular ideal of A for every $y \in M_{-1} \cup M_1$. By the definition of M_{-1} and M_1 with (4.1), we have for each $f \in A$ that $\rho(f)(y) = \overline{f(\varphi(y))}$ for $y \in M_{-1}$ and $\rho(f)(y) = f(\varphi(y))$ for $y \in M_1$. We thus conclude that ρ is of the form (1.1).

Finally, we shall prove the continuity of $\varphi\colon M_B \setminus M_0 \to M_{A_e}$. Take $y_0 \in M_B \setminus M_0$, and set

$$M_d(\varphi(y_0)) = \{y \in M_d : \varphi(y) = \varphi(y_0)\}.$$

If $y_0 \in M_d$, it follows from (b) of Lemma 3.4 that $M_d(\varphi(y_0))$ is open, and hence φ is continuous on M_d. So, we need consider only the case where $y_0 \in M_{-1} \cup M_1$. Suppose that $y_0 \in M_1$ and choose a net $\{y_\alpha\} \subset M_B \setminus M_0$ converging to y_0. By (a) of Lemma 3.4, y_0 is an interior point of $M_1 \cup M_d(\varphi(y_0))$. Thus, we may assume that $y_\alpha \in M_1 \cup M_d(\varphi(y_0))$ for every α. By (4.1) and the definition of $M_d(\varphi(y_0))$, we have

$$f(\varphi(y_\alpha)) = \begin{cases} \rho_{y_\alpha}(f) & y_\alpha \in M_1 \\ f(\varphi(y_0)) & y_\alpha \in M_d(\varphi(y_0)) \end{cases}$$

for every $f \in A$. Since $\rho_{y_0}(f) = f(\varphi(y_0))$, it follows that

$$|f(\varphi(y_\alpha)) - f(\varphi(y_0))| \leq |\rho_{y_\alpha}(f) - \rho_{y_0}(f)| \qquad (f \in A)$$

for every α. Since $\rho(f)$ is continuous, $\rho_{y_\alpha}(f) = \rho(f)(y_\alpha)$ converges to $\rho_{y_0}(f)$. Hence $f(\varphi(y_\alpha))$ converges to $f(\varphi(y_0))$ for every $f \in A$, that is, $\tilde{f}(\varphi(y_\alpha))$ converges to $\tilde{f}(\varphi(y_0))$ for every $\tilde{f} \in A_e$. By the definition of the Gelfand topology, we see that $\varphi(y_\alpha)$ converges to $\varphi(y_0)$. This implies the continuity of φ on M_1. In the same way, we see that φ is continuous on M_{-1}. The proof is complete. □

Proof of Corollary 1.2. Under the notation of Theorem 1.1, $M_0 = \emptyset$ since B_0 vanishes nowhere on M_B, and hence $\varphi\colon M_B \to M_{A_e}$ is a continuous mapping. We first show that $\ker \rho_y$ is a maximal modular ideal of A for every $y \in M_B$. To prove this, take $y \in M_B$ and $f \notin \ker \rho_y$ arbitrarily. Since the subalgebra B_0 of B vanishes nowhere, there exists $b \in B_0$ such that $b(y) = 1/\rho_y(f)$. Because $B_0 \subset \rho(A)$, there exists $a \in A$ such that $\rho(a) = b$, and so $\rho_y(f)\rho_y(a) = 1$. It follows from (2.6) that $fa - e \in \ker \tilde{\rho}_y$. By the definition of φ, we have $f(\varphi(y))a(\varphi(y)) - 1 = 0$, and so $f(\varphi(y)) \neq 0$. This implies that $\{f \in A : f(\varphi(y)) = 0\} \subset \ker \rho_y$. Hence, $\ker \rho_y = \{f \in A : f(\varphi(y)) = 0\}$ for every $y \in M_B$. Since $M_0 = \emptyset$, we have $\varphi(M_B) \subset M_A$. So, we may regard φ as a mapping from M_B into M_A.

Since $\ker \rho_y$ is a maximal modular ideal of A for every $y \in M_B$, (4.1) holds for every $y \in M_B$. If $y \in M_d$, then (c) of Lemma 2.4 and the definition of M_d imply that $\tau_y = \sigma_y$ is non-trivial. We thus conclude that ρ is of the form (1.2) for all $f \in A$. Since $\rho(A)$ contains a subalgebra B_0 which vanishes nowhere, we see that τ_y is surjective: For if $y \in M_d$ and $\lambda \in \mathbb{C}$, then there exists $a' \in A$ such that $\rho_y(a') = \lambda$, and so by (1.2) we have that $\tau_y(a'(\varphi(y))) = \lambda$, proving τ_y surjective.

We next show that φ is injective. Let $y_1, y_2 \in M_B$ with $y_1 \neq y_2$. Since B_0 is a separating subalgebra, there exists $b_0 \in B_0$ such that $b_0(y_1) = 0$ and $b_0(y_2) = 1$. Choose $a_0 \in A$ so that $\rho(a_0) = b_0$. Then $\rho(a_0)(y_1) = 0$ and $\rho(a_0)(y_2) = 1$. Thus (1.2) gives $a_0(\varphi(y_1)) = 0$ and $a_0(\varphi(y_2)) = 1$, proving φ injective.

In the following step, we show that φ is a closed mapping. If B is unital then φ is a closed mapping since φ is a continuous mapping from a compact space into a Hausdorff space. We thus consider the case where B is without unit. In this case, A is also without unit: For if A has a unit e, it follows from (1.2) that $\rho(e)(y) = 1$ for every $y \in M_B$, and hence $\rho(e)$ is a unit of B because B is assumed to be semisimple. We define a mapping $\tilde{\varphi}\colon M_{B_e} \to M_{A_e}$ by

$$\tilde{\varphi}(y) = \begin{cases} \varphi(y) & y \in M_B \\ x_\infty & y = y_\infty \end{cases}$$

where $\{x_\infty\} = M_{A_e} \setminus M_A$ and $\{y_\infty\} = M_{B_e} \setminus M_B$. Then $\tilde{\varphi}$ is continuous: In fact, it is enough to show the continuity of $\tilde{\varphi}$ at y_∞. Let $\{y_\alpha\} \subset M_{B_e}$ be a net converging to

y_∞. Lemma 3.3 with the injectivity of φ implies that M_d is at most finite, and hence $M_{B_e} \setminus M_d$ is an open neighborhood of y_∞. Thus we may assume $\{y_\alpha\} \subset M_{B_e} \setminus M_d$. Pick $f \in A$ arbitrarily. Note that

$$f(\tilde{\varphi}(y_\alpha)) = \begin{cases} f(\varphi(y_\alpha)) & y_\alpha \in M_B \setminus M_d \\ f(x_\infty) = 0 & y_\alpha = y_\infty. \end{cases} \tag{4.2}$$

It follows from (1.2) and (4.2) that $|f(\tilde{\varphi}(y_\alpha))| = |\rho(f)(y_\alpha)|$ for each α. Since $\rho(f)$ is continuous on M_{B_e}, $\rho(f)(y_\alpha)$ converges to $\rho(f)(y_\infty) = 0$. This implies that $f(\tilde{\varphi}(y_\alpha))$ converges to $0 = f(\tilde{\varphi}(y_\infty))$. Since $f \in A$ was arbitrary, we thus obtain $\tilde{f}(\tilde{\varphi}(y_\alpha))$ converges to $\tilde{f}(\varphi(y_\infty))$ for every $\tilde{f} \in A_e$. By the definition of the Gelfand topology, we see that $\tilde{\varphi}(y_\alpha)$ converges to $x_\infty = \tilde{\varphi}(y_\infty)$, proving the continuity of $\tilde{\varphi}$. Now it is easy to see that φ is a closed mapping. In fact, let F be a closed subset of M_B. Then $F \cup \{y_\infty\} \subset M_{B_e}$ is compact. Since $\tilde{\varphi}$ is continuous on M_{B_e}, $\tilde{\varphi}(F \cup \{y_\infty\}) = \varphi(F) \cup \{x_\infty\}$ is compact in M_{A_e}, and so $\varphi(F) \subset M_A$ is closed. This proves that φ is a closed mapping.

Finally, we show that B is regular. To do this, let F and K be a closed subset and a compact subset of M_B with $F \cap K = \emptyset$. Since φ is an injective, continuous and closed mapping as proved above, $\varphi(F)$ is closed and $\varphi(K)$ is compact in M_A with $\varphi(F) \cap \varphi(K) = \emptyset$. Since A is regular, there exists $a_1 \in A$ such that $a_1(\varphi(K)) = 1$ and $a_1(\varphi(F)) = 0$. By (1.2), we have that $\rho(a_1)(K) = 1$ and $\rho(a_1)(F) = 0$, and so the regularity of B is proved. □

Example 4.1. Let $\mathbb{D}$ and $\bar{\mathbb{D}}$ be the open and the closed unit discs respectively. Let $A(\bar{\mathbb{D}})$ be the disc algebra, that is, the uniform algebra of all complex-valued continuous functions on $\bar{\mathbb{D}}$, which are holomorphic in $\mathbb{D}$. Let $H^\infty(\mathbb{D})$ be the commutative Banach algebra of all bounded holomorphic functions on $\mathbb{D}$. Neither $A(\bar{\mathbb{D}})$ nor $H^\infty(\mathbb{D})$ are regular. Let $B = A(\bar{\mathbb{D}})$ or $H^\infty(\mathbb{D})$. By Corollary 1.2, there are no ring homomorphism ρ from a semisimple regular commutative Banach algebra A to B such that $\rho(A)$ contains a separating and vanishes nowhere subalgebra of B. In particular, both $A(\bar{\mathbb{D}})$ and $H^\infty(\mathbb{D})$ can not be the ring homomorphic images of any semisimple regular commutative Banach algebra A (cf. [11, Example 1]). The case where $A = C_0(X)$, the regular commutative Banach algebra of all complex-valued continuous functions on a locally compact Hausdorff space X, which vanish at infinity, was proved by Molnár [12, Corollary].

Example 4.2. Let X and Y be locally compact Hausdorff spaces such that Y can not be embedded into X. By Corollary 1.2, there are no surjective ring homomorphism from $C_0(X)$ onto $C_0(Y)$.

Remark 4.1. Let X be the closure of $\{1/n : n \in \mathbb{N}\}$ in $\mathbb{R}$ with its usual topology. P. Šemrl [15, Example 5.4] constructed a ring homomorphism $\rho\colon C(X) \to \mathbb{C}$ such that $\ker\rho$ is a nonmaximal prime ideal of $C(X)$, where $C(X)$ denotes the commutative regular Banach algebra of all complex-valued continuous functions on X. There do exist infinitely many such mappings. In fact, let $\mathcal{A}$ be a uniform algebra on an infinite compact metric space and G the set of all ring homomorphisms of $\mathcal{A}$ into $\mathbb{C}$, whose kernels are nonmaximal prime ideals. In [10, Corollary 1.2], it is proved that $\sharp G = \sharp 2^{\mathbb{C}}$, where $\sharp S$ denotes the cardinal number of a set S.

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