Erratum

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Lemma 5.1 in [1] (see the notation of that paper) makes the following claim.

LEMMA 1.1. *The set* $\tilde{F}$ *is analytic in* $V \times U_1'$, *and* $\dim \tilde{F} \le n$.

Although this statement is correct, we have since realized that the proof of the analyticity part must be corrected. However, by a nice coincidence we had already given a correct argument in our more recent paper [2] before noticing the difficulty in the original proof. Because the analyticity appears in [2] in a different context, we repeat the proof here in the form in which it belongs in [1], using all notation and definitions as introduced in that paper.

Proof of Lemma 1.1. We need to show that $\tilde{F}$ is an analytic set in $V \times U_1'$.

First, we do this for the case where $\hat{F}$ is a (single-valued) holomorphic map. Here it is obvious. Indeed, (5.1) of [1] can be written as

$$\tilde{F} = \{(z, z') \in V \times U_1' : \rho'(\hat{F}('w, h('w, \bar{z})), \bar{z}') = 0 \ \forall\, 'w \in {}'\Omega\}, \tag{2.1}$$

where $w_n = h('w, \bar{z})$ is the equation of Q_z. This, however, is a family of (anti)holomorphic equations for z, z'.

Consider now the general case where $\hat{F} \colon \Omega \to U_1'$ is a holomorphic correspondence with sheet number $m \ge 2$. There exists an analytic set $\sigma_1 \subset \Omega$ of dimension $\le n-1$ such that, for any $w^0 \in \Omega \setminus \sigma_1$, there exists a neighborhood $U(w_0) = {}'U(w^0) \times U_n(w^0)$ of w^0 in which $\hat{F}$ consists exactly of m separate holomorphic branches $f^1, \dots, f^m$. If $(z, z') \in \tilde{F}$ and $(Q_z \cap U(w^0)) \ni w = ('w, 'w_n)$, then the functions $\alpha_j = \alpha_j('w, \bar{z}, \bar{z}') := \rho'(f^j('w, h('w, \bar{z})), \bar{z}')$, $j = 1, \dots, m$, satisfy the equations

$$\alpha_j = 0 \tag{2.2}$$

if $(z, z') \in \tilde{F}$ and $Q_z \cap U(w^0) \ni w$.

Conversely, if (2.2) holds for all $j = 1, \dots, m$, $'w \in {}'U(w^0)$, then

$$\rho'(\hat{F}('w, h('w, \bar{z})), \bar{z}') = 0$$

by analyticity for all $'w \in {}'\Omega$ and all branches of $\hat{F}$.

Consider now the polynomial $P('w, \bar{z}) = t^m + a_1 t^{m-1} + \dots + a_m$, whose roots are exactly the α_j for $j = 1, \dots, m$. As symmetric polynomials of the α_j, the coefficients $a_k('w, \bar{z}, \bar{z}')$, $k = 1, \dots, m$, are functions that are well-defined on $('\Omega \setminus \sigma_1) \times (V \setminus \sigma_2) \times U_1'$, where $\sigma_2 := \{z \in V : Q_z \cap \Omega \subset \sigma_1\}$. Notice that they are holomorphic in $'w$ and antiholomorphic in z, z'. Furthermore, the set σ_2 is analytic of dimension $\leq n-1$ (it is, in fact, even discrete). Thus the coefficients a_k may be continued to $'\Omega \times V \times U_1'$ in such a way that they remain holomorphic in $'w$ and antiholomorphic in z, z'. The set $\tilde{F}$ is now defined as

$$\{(z, z') \in V \times U_1' : a_k('w, \bar{z}, \bar{z}') = 0 \ \forall k = 1, \ldots, m, \ \forall 'w \in 'U\}.$$

Therefore, it is analytic. $\square$

References

[1] K. Diederich and S. Pinchuk, *Regularity of continuous CR maps in arbitrary dimension,* Michigan Math. J. 51 (2003), 111–140.

[2] ———, *Analytic sets extending the graphs of holomorphic mappings,* preprint, 2003.