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BOOK REVIEW

The Mechanics and Mathematics of Biological Growth, by Alain Goriely, Springer, New York, xxii+646 pp, ISBN 978-0-387-87709-9.

This extensive monograph concerns the mathematical and mechanical theory for biological growth and provides a comprehensive and extensive survey of research activities devoted to this field since the pioneering work of Galilei Galileo more than three centuries ago. The book grasps the conceptual and technical aspects underpinning the role of mechanics in the growth of biological tissues. It is the first major modern monograph on the subject, which synthesizes the research activity in this vivid field of the mathematics and mechanics of growth since now more than two decades. The presented theories and methods are illustrated by examples and applications, which makes the book accessible even for non specialists.

The monograph is divided into five parts of increasing complexity, starting with the one dimensional treatment and then extending to two and three space dimensions. After a comprehensive introduction devoted to the problem of growth treated from a historical and phenomenological perspective (Part I), basic concepts are introduced in Part II for the growth of filamentary structures, successively handling line growth and growing rods (conned morphoelastic rods) based on a general treatment of the mechanics of helical structures. The problem of growth-induced instabilities is addressed with application to growing vines. The third part is devoted to surface growth for two dimensional objects, focusing the discussion to axisymmetric membranes and shells, which proves useful for modeling the growth of seashells, horns, antlers, leaves, flowers, or pollen tubes. An important distinction is made here between hard and soft surfaces, the first ones being hard to deform and growing by accretion of mass only locally, whereas the deformable surfaces develop a fair amount of elastic deformations so that growth is distributed over the entire surface. Historically, two distinct approaches of surface growth have been employed, considering either boundary accretion in a mechanistic approach or a parametric description. These two distinct approaches are illustrated for the situation of seashell growth. The more general problem of developing a theory of the growth of shells lies outside the scope of the book. A theory of axisymmetric membranes and shells modeled as soft two-dimensional structures is presented in Chapter 8, under the assumption that the growth preserves the axisymmetric shape. This is further generalized in Chapter 9 by accounting for the internal reconfiguration occurring in biological membranes, due e.g. to local addition of material, membrane unfurling, remodeling of fibers. The removal of applied loads will then lead to a shape that differs from the initial configuration. Expanding along these ideas, the author develops a so-called theory of shell morphoelasticity that extends the similar theory developed in Chapters 4 and 6 for one-dimensional elastic rods, basing on the notion of evolving virtual reference configuration.

Part IV expands on the theory of volumetric growth whereby a full three-dimensional viewpoint of the problem of growth is taken. The exposition starts by a review of the equations of nonlinear elastic for isotropic and fiber reinforced (anisotropic) solid bodies, including the consideration of instability and bifurcations. The kinematics of volumetric growth is treated in Chapter 12 in relation to the notion of incompatibility - here adopting the perspective of differential geometry - and generation of internal stresses. The balance laws for growing bodies are exposed in Chapter 13, while evolution laws for growth and remodeling deserve Chapter 14, including consideration of growth-induced instability. The growth of spherical and cylindrical objects is of special interest at different scales (cellular level and tissue level) and is of special interest for many categories of biological structures and phenomena (cell growth, tumor growth, hollow stems, blood vessels). It is accompanied by a modification of the effective mechanical properties due to residual stresses, buckling phenomena, or singular behaviors all triggered by growth. The book emphasizes on the important role of residual stress as a regulation mechanism in arteries and blood vessels, the generation of bending in plants to the existence of differential growth leading to a state of tension and tension induced buckling in arteries. Residual stresses also results from fiber contraction in soft biological tissues.

The book concludes with an overview and discussion of ten open problems and challenges for future developments in the vivid and expanding field of growth, including multiphysical couplings, multiscale aspects, and consideration of accretion and bifurcation aspects.

The monograph is overall well structured and rich in illustrations and will be accessible and appealing to readers with different interest and background, including life scientists, applied mathematicians, researchers in mechanics, or in biomedical engineering. It will be appealing to both confirmed researchers in the field of growth and researchers willing to start activities in this field.

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