



BOOK REVIEW

Elasticity and Geometry: From Hair Curls to the Nonlinear Response of Shells, by Basile Audoly and Yves Pomeau, Oxford University Press, Oxford 2010, x + 586 pages, ISBN: 978-0-19-850625-6.

The authors, Basile Audoly (Research Fellow at CNRS, Paris) and Yves Pomeau (Senior Researcher Emeritus at CNRS, Professor Emeritus of Mathematics at the University of Arizona and Corresponding Member of the French Academy of Sciences) are well-known scientists working actively in the field of mechanics of solids and fluids. In the book under review, they have caught the renewed growing interest in the exploration of the mechanical behaviour of slender and thin elastic objects, such as rods, plates and shells, among the researchers from engineering, mathematics and physics communities.

The book combines the features of a graduate-level textbook and a research monograph.

As a textbook, it offers a self-contained introduction to the theory of elastic rods, plates and shells emphasising on the application of the classical differential geometry of curves and surfaces in the description of the mechanical response of these slender or thin solid bodies. In this capacity, the book is neither intended, nor can serve as a comprehensive treatise on the subject. For that purpose, the interested reader is referred to the excellent books [1, 3, 4]. As the authors notice in the Introduction, “no prior knowledge of elasticity theory is required”, but “basic understanding of Euclidean geometry and of calculus with real variables is assumed”. The reader can find all needed knowledge of calculus of variation and geometry of curves and surfaces either throughout the main text of the book or in the Appendices.

As a research monograph, the book provides a survey of a wide variety of recent developments concerning the mechanics of human hair, rippled leaves, uncoiled springs, end effects in plate buckling, finite amplitude buckling of strips, crumpled paper and fractal buckling near edges and many others. The majority of the reported results have appeared previously only in journal publications to which the authors have contributed significantly. Most of the presented solutions of the considered particular problems are obtained in explicit form by applying classical analysis, bifurcation theory and other techniques of nonlinear analysis. It should be

stressed, however, that the leading role is given to the physical phenomena and experimental observations rather than to the mathematical formalism and techniques. A large number of impressive illustrations help the reader to understand otherwise difficult concepts and approaches.

The book comprises of an Introduction, fourteen Chapters, four Appendices and a glossary. Chapters 2–14 are divided into three almost independent parts: Part I Rods, Part II Plates and Part III Shells, respectively.

After some preliminaries concerning the notations, conventions and common mathematical background, the exposition proceeds, in Chapter 2, with the fundamentals of three-dimensional Hookean elasticity (static theory, small strain approximation, linear constitutive laws). The material presented here is sufficient for the derivation of the theories for one and two-dimensional elastic bodies in the following parts of the book. Nevertheless, the interested reader can be referred to the excellent treatise [2] to enlarge his/her knowledge in this field.

Chapters 3–5 constitute the first part of the book (Part I Rods).

Chapter 3 is devoted to the derivation of the rod equilibrium equations from those of the three-dimensional elasticity taking into account that these slender structures are essentially one dimensional being much longer in one direction than in the two other directions. The kinematic assumptions include both bending and twist. A standard dimensional reduction procedure is applied, which leads to the well-known Kirchhoff rod equations describing three-dimensional deformations of a rod in equilibrium and “mix in a rather tangled way the geometry of three-dimensional curves and the conditions of mechanical equilibrium” as noticed by the authors.

Chapter 4 deals with an application of the Kirchhoff rod theory concerning the mechanical behaviour of the human hair. It is shown how the mechanics of a hair strand can be reduced to a system with two dimensionless parameters describing the effects of gravity and of permanent curling.

Another application of the theory of elastic rods is presented in Chapter 5. It concerns the spontaneous formation of ripples along the edge of a thin material sheet when it is stretched. Here, a simple rod model is suggested and employed to analyze this phenomenon.

The second part of the book (Part II Plates) consists of Chapters 6–10.

Chapter 6 is devoted to the derivation of the Föppl–von Kármán equations, which describe the equilibrium of homogeneous isotropic elastic plates in terms of the transversal displacement and Airy’s stress function. These equations serves to analyze the buckling of plates subject to transverse loading.

The buckling near threshold at length is examined in Chapter 7. Buckling and post-buckling behavior of a compressed clamped strip is studied in Chapter 8. These buckling phenomena are analyzed and compared with experiments.

Chapter 9 is devoted to the investigation of the mechanics of crumpled paper. The observed polyhedral shapes of crumpled paper are explained using solutions of the Föppl–von Kármán equations in the form of folds and d-cones, which emerge at large loads.

In Chapter 10, another kind of solutions of the Föppl–von Kármán equations are obtained and used to study the fractal buckling near plate edges. These solutions are self-similar and correspond to the buckling of an elastic plate subject to large edge loads.

The last Chapters 11 – 14 form the third part of the book (Part III Shells).

Chapter 11 exposes and analyzes the theory of shallow shells. Chapter 12 deals with the so-called membrane theory developed in the special case of shells of revolution. It is also discussed how this theory can be extended so as to take into account bending effects too. The mechanics of the elastic torus is the subject matter of Chapter 13. It is underlined that “the calculation of the elastic response of a torus requires an elaborate boundary layer analysis.” Chapter 14 focuses on spherical elastic balls and gives an answer to the question: “how does a ball, such as a tennis or ping pong ball, flatten when hit, and how long does it take for it to bounce?”

The target readers of this worth-reading book are graduate students and researchers in mechanical and civil engineering, biomechanics, theoretical physics and applied mathematics.

References

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