BOOK REVIEW

Lectures on Clifford (Geometric) Algebras and Applications, by Rafal Ablamowicz and Garret Sobczyk (Eds), Birkhäuser, Boston 2004, xvii + 221 pp., 38.52 €, ISBN 0-8176-3257-3

The book under review contains the series of lectures on Clifford Geometric Algebras presented at the 6th International Conference on Clifford Algebras and their Applications in Mathematical Physics, held in May, 2002, at Tennessee Technological University in Cookeville, Tennessee.

Chapter 1 – Introduction to Clifford Algebras by Pertti Lounesto is devoted to the basic state of the author, namely "Clifford algebra is by definition the minimal construction designed to control the geometry in question". A concise, but coherent introduction to geometric algebras by thoroughly examining the Clifford product of two vectors, the definition of a bivector, and how these concepts are used to represent reflections and rotations in the plane and in higher dimensional spaces is given. The geometric significance of quaternions is explained first in three and then in four dimensions. It is shown how the more advanced concepts of spinors, exterior algebra and contraction, the Grassmann-Cayley algebra and shuffle product naturally evolve from the more basic concepts and also how the lower dimensional Clifford algebras can be represented in terms of the familiar algebra of square matrices. At the end several categorical definitions of Clifford algebras of a quadratic form are given, and their deformations to Clifford algebras of an arbitrary bilinear form.

In *Chapter 2 – Mathematical Structure of Clifford Algebras* – the author Ian Porteous claims that the "control of the geometry" which is a property of each Clifford algebra follows directly from the close relationship to the corresponding classical group. It is shown how the Clifford algebras can be constructed from matrix algebras over the real numbers, complex numbers, or quaternions with famous periodicity of eight. Useful tables of their most important features, such as tables of the spinor groups, groups of motion, conjugation types, the general linear groups, and the corresponding dimensions of the associated Lie algebras. A brief history

and discussion of Vahlem matrices and conformal transformations, which have found at recent time applications in pattern recognition and image processing is given.

John Ryan in *Chapter 3 – Clifford Analysis* – introduces the basic concepts of Clifford analysis in which the complex analysis of the plane is extended to three and higher dimensions. It is also introduced a generalized Cauchy-Riemann operator, often called a Dirac operator, and the analogues of holomorphic functions which are called Clifford holomorphic functions or monogenic functions. A generalization of Cauchy's theorem and Cauchy's integral formula. Using Cauchy's theorem, the Möbius invariance of monogenic functions can be established.

In *Chapter 4 – Applications of Clifford Algebras in Physics* – William E. Baylis explores some of extensive applications of Clifford algebras that have been made to physics. One of the earliest applications is to electromagnetism and the efficient formulation of Maxwell's equation as a single equation when expressed in the Pauli algebra, which is the geometric algebra of the physical space, or in the even more powerful Dirac algebra, which is the algebra of Minkowski space-time. The both geometric algebras which relationships and interplay are clearly explained, can be applied to the study of special relativity because of the ease of representing not only the rotations in 3-dimensinal space, but the more general Lorentz transformations of space and time. Charge dynamics in uniform fields, directed plane waves, polarization and phase shifters, standing waves, and the potential of a moving charge are some of the many explored topics.

Jon Selig, in *Chapter 5 – Cliffor Algebras in Engineering* applies Clifford algebras to robotics and computer vision. Because of the fact that Clifford algebras is a very efficient instrument for the problems of computer graphics they are well suited to modern microprocessor architecture. Using quaternions to express rotations, the author tackles the problem of satellite navigation. This problem may be considered also through vector-parameterization of the 3-dimensional rotation group [2]. Further he uses biquaternions to write down the kinematic equations of the Stewart platform, which is used in aircraft simulations and in novel machine tools. Analogically, in [3] dual vector-parameters are introduced. Both the quaternions and biquaternions are examples of different Clifford algebras (see also [1] and [4]). Through another Clfford algebra, the author represents points, lines and planes in three dimensions.

Chapter 6 – Clifford Bumdles and Clifford Algebras, by Tom Branson, explains some of the deepest applications of Clifford algebras that have been made in differential geometry. As it is claimed in the book, the Clifford algebras are as fundamental to mathematics. In this aspect, one may expect that using them should

make possible new insights in all areas of mathematics which have their roots in geometry. The author introduces to the reader all kinds of new geometrical objects constructed in Clifford algebra. Starting with the the concept of the Dirac operator and the structure group of orthogonal transformations on Riemannian geometry introduced in previous chapters, further the author discusses spin geometry and objects like Stein-Weiss gradients, the Bochner-Witzenbock formulas, and the Hijazi inequality. Discussing all this, Branson has made a nice remark: "Spin geometry is a fairly deep subject, and is not a subject where you can completely master one line before going on the next line. At some point the reader has to believe in these objects and let the understanding build."

The end of the book contains a very valuable *Appendix* where the editors have tried, for the first time, to present an up-to-date summary of the existing Clifford (geometric) algebra software that is accessible on the web. The software provides tools for generating examples, checking conjectures, finding counterexamples, and advancing understanding via experimentation.

After reading this book the reader will be convinced that the principal aim of the book to provide beginning graduate students in mathematics and physics and other who are new-interested with no prior knowledge in the secrets of Clifford algebras has been achieved. Nevertheless, the presented material subsumes research activities and some of the recent scientific advances of this theory, and it is an useful learning tool for scientists and engineers from academia and industry.

References

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