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# BOOK REVIEW

*Geometric Control of Mechanical Systems: Modeling, Analysis, and Design for Simple Mechanical Control Systems*, by Francesco Bullo and Andrew D. Lewis, Springer Science+Buisiness Media Inc., 2005, xvi + 726pp, 64.15€, ISBN 0-387-22195-6

The book under review covers the theory and application of ideas in nonlinear control theory to mechanical systems, an area which has a great deal of progress during the past decade. The areas of application of control theory to mechanical systems include robotics and automation, autonomous vehicles in marine, aerospace, and other environments, flight control, problems in nuclear magnetic resonance, fluid mechanics, and etc. The authors say: *"Control theory for mechanics, and mechanics for control theory"*. In this aspect the book provides a background in geometric mechanics for researchers which are closed to geometric control theory and it is also a background in geometric nonlinear control for readers familiar with geometric mechanics. The book illustrates also to students a value of certain mathematical ideas in physical science and it helps them to pursue research in related field. For example, at the University of Illinois at Urbana-Champaign, this text has been used as an introduction to current research results on nonlinear control of mechanical systems.

The subtitle of the book points out that it consists of three parts: Modeling, Analysis and Design.

*Part I – Modeling of Mechanical Systems.*

This part begins with the words of Leonardo da Vinci:*"Mechanics is the paradise of the mathematical sciences, because by means of it one comes to the fruits of mathematics."* There are no other words which describe so fully and deeply the ideas of the authors. *Chapter 1* gives introductory examples and problems concerning rigid body systems, manipulators and multi-body systems, constrained mechanical systems. *Chapter 2* suggests basic concepts and notation in the field of linear and multilinear algebra. Here one can read about sets, maps, vector spaces, inner products and bilinear maps, tensors. And since algebra and geometry always go together, in *Chapter 3* very naturally comes the turn to differential geometry. After a prelude to differential geometry, one learns about manifolds, maps and submanifolds, tangent and vector bundles, vector fields, distributions and codistributions, affine differential geometry and some more advanced topics in differential geometry. Further on, providing a general framework for modeling of simple (but it does not mean easy) mechanical control systems on manifolds follows *Chapter 4*. In this chapter, the reader can get an idea of what parts of the preceding two chapters are most important for modeling mechanical systems. The steps which are described for turning a mechanical control system into a mathematical model roughly speaking are as follows: determine the configurational manifold of the system; choose convenient coordinates; construct the coordinate representative of the forward kinematic map; determine the kinetic energy of the system as a function on the tangent bundle of the configurational manifold, and then derive from this the Riemannian metric that defines this kinetic energy function; in case of external forces, determine these as cotangent bundle-valued maps on the tangent bundle; if the system is subject to linear velocity constraints, determine these as a distribution on the configuration manifold; model the control forces on the system as covector fields on the configuration manifold. *Chapter 5* in this part discusses an important class of manifold, called Lie groups, that arise naturally in rigid body kinematics, as well as the properties of mechanical systems defined on Lie groups or possessing Lie group symmetries. In more details, in this chapter the authors study rigid body kinematics in the language of matrix groups and matrix algebras, and characterize the groups of rotations and rigid displacements in two- and three-dimensional Euclidean space. Further on, in this chapter, the abstract notions of a Lie group and a Lie algebra are considered. As it is known, a Lie group is a smooth manifold equipped with a group operation and a Lie algebra is a vector space equipped with a bracket operation. An explicit construction of matrix Lie algebras are presented via the notion of a one-parameter subgroup generator. Since the differential geometric structure of Lie groups is extremely rich and it is connected to the notion of invariance under the canonical left action, the treatment of the authors discusses invariance notions for vector fields, Riemannian metrics, and affine connections. Next the equations of motion of rigid bodies are investigated and the properties of these equations are linked to the structure of the configuration Lie groups of these systems (see also Mladenova [3], [4]). The classic Euler and Kirchhoff equations for rigid body motion in space and in an ideal fluid, respectively, are presented. In the next section of this chapter the notions of group actions and of invariance under a group action are introduced. Mechanical systems invariant under a group action are said to have a symmetry and are known to satisfy the classic Noether Conservation Law and as an illustration of this the angular momentum conservation law for a rigid body with a fixed center of mass is derived. The chapter ends with a discussion of principle fiber bundles, their infinitesimal equivalence, and geometric treatment (in Riemannian sense) of the reduction procedure.

*Part II – Analysis of Mechanical Control System*

The analysis results in this part concern the stability *Chapter 6*, controllability *Chapters 7* and *8*, and perturbation theory *Chapter 9*. The authors work as much as possible with their intrinsic geometrical system models. The stability analysis of mechanical systems is a classic topic in mechanics and dynamics that has affected a number of engineering disciplines. For stability, a coordinate-free notion of exponential stability is given and the global structure of stability for mechanical systems via Lyapunov analysis is presented. The problems of relative equilibria and their stability are also treated. The presentation for controllability given here differs from some standard treatments in that the development of accessibility has as its basis the classic results of Sussmann and Jurdjevic [5]. Results of a related nature are those of Krener [2] and Hermann and Krener [1]. In more details this part makes an overview of controllability for control-affine systems, controllability definitions and results for mechanical control systems are given and examples illustrating these results are presented. Low-order controllability and kinematic reduction are considered. The relationship between controllability and kinematic controllability is shown. The perturbation analysis contains an overview of averaging theory for oscillatory control systems, averaging of affine connection systems subject to oscillatory controls, and a series expansion for a controlled trajectory from rest is presented.

*Part III – A Sampling of Design Methodologies*

The real value of the geometric formulation of mechanical systems is when it comes to control design. Of the four design chapters, i.e., *Chapters 10-13*, three deal with stabilization, two – with tracking, and one – with motion planning. This means that two chapters deal with both stabilization and tracking. The stabilization results are geometric in flavor, and they are based on the stability and averaging theory developed in the second part of the book. Similarly, the tracking theory emphasizes the geometry inherent in mechanical control systems. The consideration of the motion planning problems demonstrates the power of the differential-geometric formulation. By understanding geometry, one comes to a series of mathematical questions, which answers provide simple, explicit motion planning algorithms for systems for which it is not a priori clear that such algorithms should exist. *Chapter 10* considers the problems of linear and nonlinear potential shaping for stabilization, *Chapter 11* – stabilization and tracking for fully actuated systems, *Chapter 12* – stabilization and tracking using oscillatory controls, and the last *Chapter 13* – motion planning for underactuated systems, which contains very nice examples for motion planning for the planar rigid body, for the robotic leg and for the snakeboard, and in this way the systems with non-holonomic constraints are naturally involved.

At the end, it has to be noted that all chapters contain a lot of definitions, propositions and theorems (some of them proved at the end of the book – in *Appendix B*, other proofs are referred to other books and papers), and all chapters conclude with very nice examples, problems, questions, and exercises. It is very important to mention the rich bibliography (527 cited titles) given in the book as well as a full subject and symbol indices.

Another, very important for this book, and this differs it from many other books, is the fact, that the authors mention in every part and chapter what is not considered inside, and cite the authors and the titles where the reader may obtain an additional information.

As a conclusion, after reading the book the reader will be convinced that its aim is to provide an introduction for graduate students and researchers in the field of mathematics and mechanics to the basic problems of geometric control of mechanical systems. Written professionally, the book shows in a non-ambiguous way the deep connection between mathematics and mechanics in both directions. Subsuming research activities and some of the recent scientific advances of control theory, it is an useful learning tool for scientists and engineers from academia and industry.

## References

[1] Herrmann R. and Krener A., *Nonlinear Contollability and Observability*, IEEE Trans. Automatic Control **22** (1977) 728-740.

[2] Krener A. , *A Generalization of Chow's Theorem and the Bang-Bang Theorem to Non-linear Control Problems*, J. SIAM. Series A. Control **12** (1974) 43-52.

[3] Mladenova C., *An Approach to Description of a Rigid Body Motion*, CRAS (Sofia) **38** (1985) 1657-1660.

[4] Mladenova C., *Robot Problems over Configurational Manifold of Vector-Parameters and Dual Vector-Parameters*, J. Intell. Robotic Systems **11** (1994) 117-133.

[5] Sussmann H. and Jurdjevic V., *Controllability of Nonlinear Systems*, J. Diff. Eqs. **12** (1972) 95-116.

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