

Preface

This special issue contains the proceedings of an international conference “Quantum and classical interacting particles systems: stochastic and deterministic approach” held at the university Rennes 1 in May 26-28 2004. This event was sponsored by the CNRS research group GRIP (Groupement de Recherches sur les Interactions de Particules), together with the European network HYKE, the French ministry of research, and the city of Rennes.

C. Bardos, B. Ducomet, F. Golse, A. Gottlieb and N. Mauser discussed the mean-field limit of the quantum N -body problem in the case where the interaction is not only a 2-body potential, but includes also 3-body and higher order interaction terms — this type of model being used for instance in nuclear physics.

E. Caglioti and F. Rousset investigated the mean field limit for classical particles interacting via a smooth potential. This limit has been rigorously established in the early 70s (by Braun and Hepp), on finite time intervals. E. Caglioti and F. Rousset analyze this limit in the long time regime, and consider in particular two important examples in this class of problems, the vortex model and the piston problem, where a massive particle (the piston) interacts with a gas of light particles by elastic collisions.

J. Carrillo, M. di Francesco and M. Gualdani discussed the long time limit of various nonlinear parabolic equations — such models arising for instance in the hydrodynamics of particle systems with inelastic collisions. They study in detail the time-discretization of this type of equations, as well as the equations themselves, in terms of the Euclidian Wasserstein distance, obtaining in this way precise information on the long-time behavior of the solutions. The numerical schemes are also used to compute the relevant asymptotic profiles.

D. Benedetto, F. Castella, R. Esposito and M. Pulvirenti review the state of the art in the derivation of nonlinear, quantum Boltzmann models from the quantum N -body problem. At the time of this writing, this remains an outstanding problem of mathematical physics, in spite of many interesting partial results obtained over the past few years.

C. Mouhot presents new results concerning the spectral theory of the Boltzmann collision integral linearized at some Maxwellian density. In particular, he presents a striking new method for estimating the spectral gap in this linearized Boltzmann collision integral in the natural weighted L^2 space.

C. Pallard reviews the regularity theory for solutions of the (relativistic) Vlasov-Maxwell system, a model for non collisional, magnetized plasmas. His approach is focussed on some kind of representation theorem for the second order derivatives of the fundamental solution of the wave equation in terms of the successive powers of the advection operator.

V. Ricci discusses various issues related to the Boltzmann-Grad limit of the Lorentz gas — i.e. a gas of point particles interacting by elastic collisions with a system of fixed spherical scatterers. It has been known since the work of Gallavotti (1969) that, in the case of a Poisson distribution of scatterers, the limiting phase-space density of particles evolves according to a linear Boltzmann equation. Two additional results are discussed in the paper by V. Ricci: a) the case of scatterers centered at the vertices of some lattice, with some probability of presence that is adapted to the size of the scatterers, and b) the case of a Poisson distribution of scatterers with some additional external force field, that may lead to a non-Markovian limit.

We hope that this selection of survey papers on recent results about the dynamics of large particle systems will convince the reader of the variety, depth and richness of

this exciting field of mathematical physics.

Finally, we express our gratitude to the director of the IRMAR (mathematics institute at the universit  Rennes 1), Prof. N. Lerner, and to the staff of the IRMAR (especially C. Halet and C. Boschet) for the kind assistance in the organization of this conference and the warm hospitality enjoyed by all the participants.

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Rennes-Paris, December 2006