

BACKWARD FORMALISM TO DERIVE THE SIZE OF SECONDARY EJECTED DROPLETS PRODUCED BY CROWN SPLASHING OF DROPS IMPINGING ON A SOLID WALL*

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Abstract. Crown splashing, produced by high speed impact of a droplet on a rough or wet wall, is physically very complicated. It is impossible to determine the size of secondary ejected droplets by literally solving the full set of nonlinear Navier-Stokes equations supplemented by complex initial and boundary conditions. In order to get useful impact laws and, most importantly, to propose a general concept of deriving useful results without going through the complex mathematical details, we propose a backward formalism in which we determine the size of the secondary ejected droplets by tracing the past event whenever it is required and just what is required. This procedure allows us to discard those complex details of negligible importance. Such a formalism, conceptually very simple and possibly meaningful for other complex problems, leads to a reasonably correct formula for the most probable diameter of secondary ejected droplets, compared to known experimental data.

Key words. Backward formalism, most probable diameter, crown splashing, droplet impact

1. Introduction

The impact of droplets on a solid wall involves complex physics and has many important engineering applications such as spray coating, erosion of soils, spray cooling, spray injection in engines, etc. When the impact velocity is high enough, splashing occurs (see FIG. 1.1 and the explanation in Appendix A). A number of experimental studies have been made to measure the disintegration criterion (splashing threshold)[1, 3, 6, 7, 8, 16, 19]. The size of the secondary ejected droplets [1, 3, 4, 6, 16, 20] is also of primary importance for physicists and engineers.

The detailed flow of the crown splashing is governed by the nonlinear Navier-Stokes equations, supplemented by very complex boundary conditions since the gas-liquid boundary breaks up to produce secondary droplets.

Conventionally, we solve a physical problem by following the forward procedure:

1. specify or determine the initial value and boundary conditions;
2. build up the governing equations;
3. simplify the mathematical model whenever it is possible;
4. solve the initial boundary value problem numerically or analytically when it is simple enough;
5. put the solution into physically interpretable forms, draw the results in figures to show the dependence of solutions on input conditions, or integrate the solution to find global product such as the lift coefficient in aerodynamics.

Such a conventional procedure is impossible to be used to obtain the size of secondary ejected droplets by splashing, since we are facing very complicated nonlinear

* Received: March 15, 2002; Accepted (in revised version): June 20, 2002.

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partial differential equations, though one can get some flow pictures just by solving the problem numerically for a small time [11].

A physically complicated problem may involve a final physical law which does not depend on all the complex intermediate details. If we follow a conventional solution procedure, we have to go through all the details. For purpose of obtaining a useful law which may not depend on the intermediate details, we propose a backward formalism. Precisely, for the present problem, we determine the size of the secondary ejected droplets by tracing the past event whenever it is required and just what is required. This procedure allows us to discard those complex details which turn out to be not needed. Such a formalism, conceptually very simple and possibly meaningful for other complex problems, leads to a reasonably correct formula for the most probable diameter of secondary ejected droplets, compared to known experimental data.

The backward formalism is a mathematical procedure. However, it must be combined with the most fundamental physics of the problem under consideration. In this paper, we avoid mixing up the formalism with the specific physics. It turns out that some common knowledge of basic physics is enough to understand the physics of impact relevant to the derivation of the model, actually zeroth order.

One would readily be disappointed with the fact that only algebraic equations are involved in the formalism. How could the basic physics of impact be revealed since the impact involves flow that should be described by nonlinear partial differential equations? The trick is that, using the backward formalism to build up the formula for the droplet size, we just need algebraic conservation laws at each step when going back to the initial stage of impact.

The next section is devoted to schematic description of the backward formalism and to the derivation of the most probable diameter of the secondary ejected droplets using the backward formalism.

In Section 3, we propose to simplify the model, and then we make some comparison with known experimental results.

In Appendix A, the main physics of crown splashing is described and the basic notations are detailed.

More details emphasizing the physical aspects will be provided elsewhere.

2. The Most Probable Diameter by Backward Formalism

2.1. Principle of Backward Formalism. The principle of backward formalism can be explained by considering the tree structure of FIG. 2.1. Starting from the initial time $t = 0$, the initial data affect the subsequent events through the various branches (which could represent information pipelines, flow pipes, etc.). Imagine that we want to determine what will happen at the exit 1 at the final time. If we follow a conventional forward formalism, then we have to take into account all the initial inlets at time $t = 0$ and follow all the pipelines. The small details in the pipe trees may become very complicated to follow. Even though one can get through all the pipelines, much work is useless since the information at the exit 1 has nothing to do with the information going through a large number of small pipes connected to the exits 2 and 3. However, if we use the backward formalism, then we just need to trace straight back to the initial time $t = 0$ by starting from the exit 1. The information

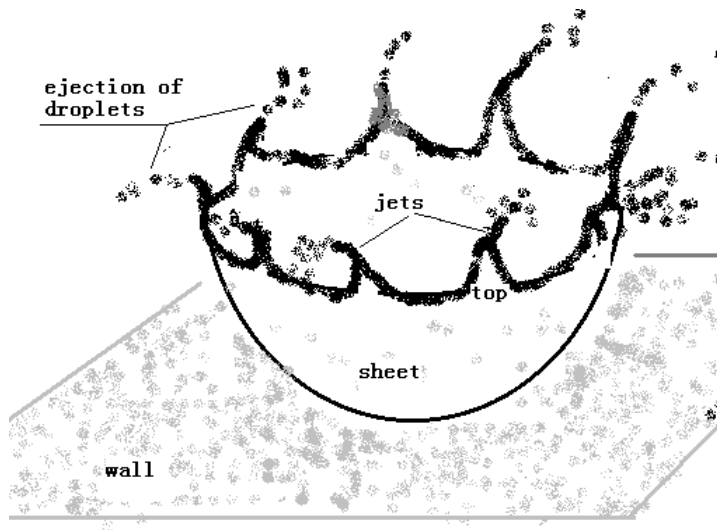


FIG. 1.1. *Crown-type splashing. The crown is in the shape of a conic sheet. At the top of the crown, the sheet disintegrates into jets which further break up to produce droplets. See also FIG. 1 of Cossali et al. [3].*

at exit 1 does not depend on the details which happen through the small and curved pipes.

The above illustration is only informative. It simply means that if we use the backward formalism, then we just need to determine what are useful at the previous time levels. This could avoid a large number of unuseful, sometimes untractible, details.

2.2. Most Probable Diameter. According to Appendix A, the real sequential events are: a) first contact of the impact droplet with the wall, b) spreading of the liquid underneath the droplet in the form of a disc, c) creation of lamella by surface tension force and by minimum potential energy requirement, d) formation of cusps on the top of the lamella, e) breakup of cusps to form fingers (jets), f) breakup of jets to produce secondary ejected droplets. See the Appendix A and the cited references for details. To determine the most probable diameter of the secondary ejected droplets, we consider the inverse of the above events. This is the essence of the backward formalism, to be detailed below.

Step 1. We are looking for the most probable diameter of the secondary ejected droplets. According to the above-mentioned argument, the ejected droplets are produced by the breakup of fingers according to the minimal surface energy principle. This has been investigated by Rayleigh and is known to be the Rayleigh jet breakup mechanism [9]. Assume that the finger diameter is d_j , then, by the Rayleigh theory, the diameter of the secondary droplets is given by

$$d_s = 1.889d_j \quad (2.1)$$

Step 2. By (2.1), we need to evaluate the diameter d_j of the fingers (jets). Since

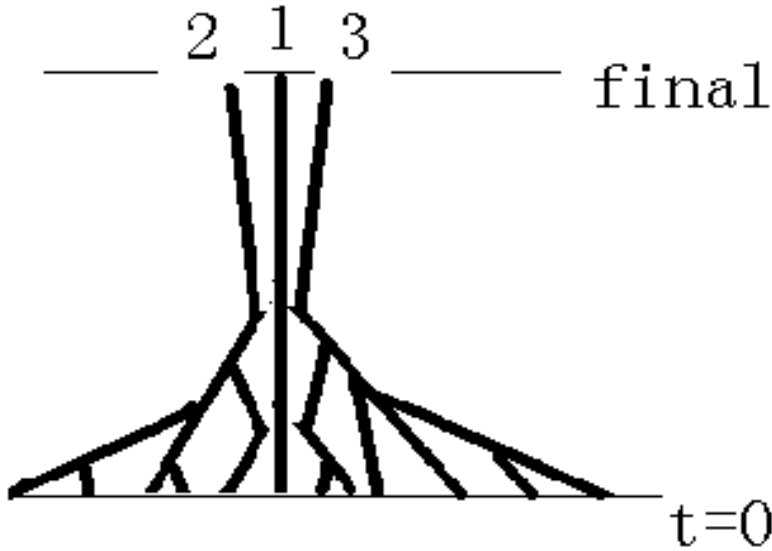


FIG. 2.1. Schematic representation of the backward formalism.

the fingers form from the lamella, it should be determined by the thickness of the lamella h_{sh} . The formation of fingers from the lamella can be considered to take place at a negligible distance (zeroth distance).

Let the magnitude of the liquid velocity in the lamella be V_{sh} and let this velocity inside the fingers be V_j . Assume that each finger comes from a sheet element of width l and thickness of h_{sh} . The mass flow rates at both sides of the finger/jet interface must be equal; i.e.,

$$lh_{sh}V_{sh} = \frac{\pi}{4}d_j^2V_j, \quad (2.2)$$

where the left-hand side is the mass flow rate toward the interface from the lamella and the right-hand side is toward the finger.

The surface energy of the sheet element of width l and length $V_{sh}dt$ (where dt is a time interval) is given by

$$S_{sh} = \sigma 2lV_{sh}dt, \quad (2.3)$$

and the surface energy of a finger of diameter d_j and of length V_jdt is given by

$$S_j = \sigma \pi d_j V_j dt. \quad (2.4)$$

If the surface energy is assumed to be conserved during the transformation, then we have, by equating (2.3) and (2.4),

$$2lV_{sh} = \pi d_j V_j$$

which, after eliminating l from (2.2), reduces to

$$d_j = 2h_{sh}. \quad (2.5)$$

If we conserve the total energy (kinetic energy plus surface energy), then we obtain $d_j = 2.24h_{sh}$. But the physical arguments used to arrive at this are much more complicated and will be presented elsewhere. We are satisfied with (2.5) since it is essentially the same as a more complicated model. It is surprising that the relation (2.5) does not depend on the velocity. The more complicated model, using conservation of total energy, also leads to a relation that does not depend on the velocity.

Hence by (2.1) and (2.5), we have

$$d_s = 3.778h_{sh}. \quad (2.6)$$

Step 3. By (2.6), it remains to find a relation for h_{sh} . We know that if the top edge of the lamella does not break up, then the surface tension force will drive the edge to move into the liquid at the velocity [18]

$$V_e = \sqrt{\frac{2\sigma}{\rho_l h_{sh}}}.$$

The edge most possibly breaks up at the point that this velocity is equal to the liquid velocity since at such a point there is enough time for the transformation to take place. By writing

$$V_{sh} = \sqrt{\frac{2\sigma}{\rho_l h_{sh}}}, \quad (2.7)$$

we obtain

$$h_{sh} = \frac{2\sigma}{\rho_l V_{sh}^2}. \quad (2.8)$$

Note that the balance condition (2.7) can be more elegantly written as

$$W_L = 2, \quad (2.9)$$

where $W_L = \frac{\rho_l h_{sh} V_{sh}^2}{\sigma}$ is the local Weber number.

By (2.6) and (2.8), we obtain

$$d_s = \frac{7.556}{W_{sh}} d_l \quad (2.10)$$

where

$$W_{sh} = \frac{\rho_l d_l V_{sh}^2}{\sigma}$$

is the lamella Weber number.

Step 4. By (2.10), we finally need to know the velocity V_{sh} of the liquid inside the lamella or the lamella Weber number W_{sh} . Note that we are working on a zeroth order model, so that only an averaged value of V_{sh} is needed. The detailed knowledge of the velocity, which must be a complicated three-dimensional spatially varying function, is

not needed here. This is obtained here by using the global energy conservation law. Before impact, the total energy of the impact droplet is

$$E_{imp} = \pi d_l^2 \sigma + \frac{1}{2} m V_l^2,$$

where $m = \frac{\pi}{6} d_l^3 \rho_l$ is the mass of the droplet. Within the framework of a zeroth order model, the entire droplet is considered to be transformed into a lamella of thickness h_{sh} and having a total surface area $2A$ where A is obtained by mass conservation

$$\rho_l A h_{sh} = m,$$

which yields

$$A = \frac{\pi}{6} d_l^3 \frac{1}{h_{sh}}.$$

Assume that the energy loss due to viscous dissipation is E_d . Then the total energy of the lamella

$$E_{lamella} = 2\sigma A + \frac{1}{2} m V_{sh}^2$$

is equal to $E_{imp} - E_d$; i.e.,

$$A(2\sigma + \frac{1}{2} h_{sh} \rho_l V_{sh}^2) + E_d = \pi d_l^2 \sigma + \frac{1}{2} m V_l^2. \quad (2.11)$$

The energy lost in the creation of lamella against the viscosity can be estimated by

$$E_d = \int_0^{t_s} \int_{\Omega} \phi d\Omega dt \approx \frac{1}{6} \pi d_l^3 t_s \bar{\phi},$$

where Ω is the volume of the initial droplet, t_s is the characteristic time scale, and ϕ (the averaged value over the droplet is denoted as $\bar{\phi}$) is the local dissipation rate. The characteristic time should be approximately d_l/V_l , and the averaged dissipation rate should be approximately $\mu_l (V_l/h_{sh})^2$. Hence,

$$E_d \approx \frac{1}{6} \pi \mu_l V_l \frac{d_l^4}{h_{sh}^2} = \frac{1}{24} \frac{W}{Re} W_{sh}^2 \pi d_l^2 \sigma. \quad (2.12)$$

Physically, one may consider that the estimate (2.12) is crude. But it is still within the framework of a zeroth order model. Using (2.12) and (2.9), the final form for the conservation of energy (2.11) becomes

$$\frac{1}{4} W_{sh} + \frac{1}{24} \frac{W}{Re} W_{sh}^2 = 1 + \frac{W}{12}. \quad (2.13)$$

The equation (2.13) can be solved to yield

$$W_{sh} = \frac{2(W+12)}{\sqrt{9 + \frac{2W(W+12)}{Re}} + 3}. \quad (2.14)$$

Step 5. Finally, by combining (2.14) and (2.10), we obtain the formula

$$\frac{d_s}{d_l} = 3.778 \left(\sqrt{9 + \frac{2W(W+12)}{R_e}} + 3 \right) (W+12)^{-1} \quad (2.15)$$

For W large enough, we can replace $W+12$ by W , so that $\frac{2W(W+12)}{R_e} \approx \frac{2W^2}{R_e}$ and the above formula can be simplified as

$$\frac{d_s}{d_l} = 3.778 \frac{\left(\sqrt{9 + \frac{2W^2}{R_e}} + 3 \right)}{W}. \quad (2.16)$$

3. Mathematical Simplification and Comparison with Experiments

3.1. Mathematical Simplification. The formula (2.16) is not mathematically elegant. Universal physical laws generally have a very simple form. Fortunately, we have the following theorem:

THEOREM 3.1. *Let k and K (\sqrt{K} is called the impact number) be respectively defined by*

$$k = \frac{W^2}{R_e} \text{ and } K = W \sqrt{R_e}.$$

Then we have

$$\frac{d_s}{d_l} = \frac{3.778}{K^{0.5}} \varphi(k),$$

where

$$\varphi(k) = \frac{\sqrt{9 + 2k} + 3}{k^{\frac{1}{4}}}$$

satisfies

$$\varphi(k) = 4.70 + 1.2 \times 10^{-3} \left(k - \frac{27}{2} \right)^2 - 9.3 \times 10^{-5} \left(k - \frac{27}{2} \right)^3 + O \left(k - \frac{27}{2} \right)^4.$$

Proof. First remark that $\frac{d\varphi(k)}{dk} = 0$ at $k = \frac{27}{2}$. Then one can expand $\varphi(k)$ at $k = \frac{27}{2}$ to Taylor series to prove the conclusion. \square

The above theorem means that, for $k = \frac{W^2}{R_e}$ satisfying

$$k = \frac{27}{2} + O(1) \quad (3.1)$$

(see FIG. 3.1 for the dependence of $\varphi(k)$ on k), then one can replace $\varphi(k)$ by 4.70 so that (2.16) reduces to

$$\frac{d_s}{d_l} = \frac{18}{\sqrt{K}}. \quad (3.2)$$

This means that for a large range of impact conditions (that is, for $k = \frac{W^2}{R_e}$ sufficiently close to $\frac{27}{2}$), the most probable diameter d_s is determined by the very simple law (3.2), which simply says that the most probable diameter is inversely proportional to the impact number.

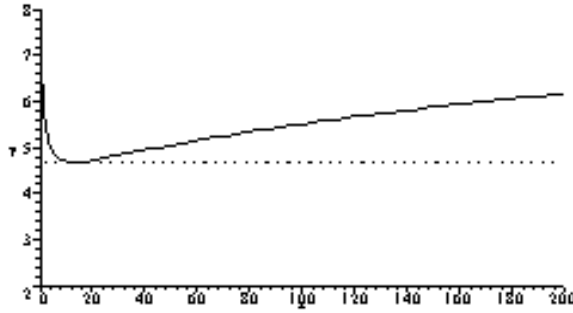


FIG. 3.1. Function $y = \varphi(k)$. The dot line corresponds to $y = 4.7$.

TABLE 3.1. Comparison between the theory and the data of Cossali et al. [3] for $\frac{d_s}{d_l}$.

Impact conditions	Data	Theory
$\frac{W=320}{R_e=8131}$	0.08~0.25	0.105
$\frac{W=643}{R_e=11526}$	0.06~0.15	0.07
$\frac{W=749}{R_e=12440}$	0.10~0.14	0.066

3.2. Comparison with Experiments. The experimental data are rather incomplete. Here we demonstrate some typical comparisons for which the data are complete.

Levin and Hobbs [4] measured the diameters of ejected droplets from the impact of a water drop 2.5 mm in diameter at an impact speed of 4.2 m/s. The Weber number W is near 600 if we assume $\sigma = 0.071 N/m$. The Reynolds number is $R_e = 10500$. In the data of Levin and Hobbs, there are two maximums: one near $d_s = 120 \mu m$, and another near $d_s = 235 \mu m$. The theoretical formula (2.16) yields $d_s \approx 187 \mu m$ which is in the middle of two maximums.

Cossali et al. [3] measured the diameter of the ejected droplets as a function of time and displayed a time-dependent diameter in their FIG. 13, with $W = 320, 643$, and 749. The Reynolds numbers are 8131, 11526, and 12440, respectively. The impact conditions can be expressed for $K = W\sqrt{R_e}$ as $K = 28855, 69032$, and 83540, for $W = 320, 643$, and 749, respectively. The corresponding values for $\frac{W^2}{R_e}$ are 12.6, 35.9, and 45, which indeed satisfy the order of magnitude relation (3.1). The comparison between theory (2.16) and data is shown in Table 3.1. The data of Cossali et al. are not the most probable values but are a range of values taking into account the unsteadiness. The theoretical results lie within the range of the experimental data for the first two cases. But the third case does not work well. In fact, the data are incomplete in the figure of Cossali et al. [3] for the third case.

Samenfink et al. [12] measured the size distribution of a droplet of diameter $180 \mu m$ at an impact velocity $15.2 m/s$. The corresponding Weber number and Reynolds number are $W = 585, R_e = 2736$. Using (2.16), we obtain $d_s = 23 \mu m$. The measured value is around $26 \mu m$.

Appendix A. Recall of Known Physics of Crown Splashing.

Experimentally, it was found that the dynamics of impact is mainly characterized by the impact Weber number and Reynolds number (or their combinations). The impact Weber number is defined as $W = \frac{\rho_l d_l V_l^2}{\sigma}$ and the impact Reynolds number as $Re = \frac{\rho_l d_l V_l}{\mu_l}$. Here σ is the surface tension, ρ_l is the liquid density, μ_l is the viscosity, d_l is the diameter of the impact droplet, and V_l is the impact velocity. Another common parameter used to study splashing is defined by $K = R_e^\xi W^\eta$ where ξ and η are some constants. The parameter K or \sqrt{K} was called splashing parameter by Aziz and Chandra [1]. The splashing condition is often expressed as $K > \xi(r_a, h)$ where $\xi(r_a, h)$ is a function of the roughness (r_a) and the thickness of the initial film (h). For instance, Stow and Hadfeld [16] expressed K as $K = R_e^{0.31} W^{0.69}$, Cossali et al. [3] used $K = \left(R_e^{\frac{1}{2}} W\right)^{\frac{4}{5}}$, Mundo et al. [6] defined $K = \left(R_e^{\frac{1}{2}} W\right)^{\frac{1}{2}}$. Under their impact conditions, Range and Feuillebois [7, 8] correlated the following condition for splashing¹

$$W > a \ln^b \frac{2r_a}{d_l}$$

where a and b are two constants which should be problem dependent.

Under suitable impact conditions, especially when the wall is wet or rough, crown splashing occurs [3, 4, 6, 16, 20]: the droplet is first transformed into a crown and the crown then disintegrates to produce much smaller secondary droplets (see FIG. 1.1, see also FIG. 1 of Cossali et al. [3]).

To the zeroth order physics, the detailed mechanisms of crown splashing have been reviewed by Rieber and Frohn [11] who showed numerically that the splashing is composed of (a) lamella formation, (b) formation of a free rim at the top of the lamella, (c) breakup of the rim (due to Rayleigh instability) to produce cusps, (d) formation of fingers (jets), (e) breakup of fingers to produce secondary droplets.

In fact, lamella formation is due to the surface tension effect at the edge of the liquid flowing on the wall during the initial state of impact. If there is no surface tension, the liquid will flow on the wall in the form of a disc. For the same volume of liquid, a flat disc has an surface area much larger than the corresponding lamella. Hence the formation of lamella leads to a decrease in surface (potential) energy and is consistent with the minimum potential energy principle.

The formation of a free rim at the top of the lamella also reduces surface energy. The breakup of the rim, the formation of fingers, and the breakup of fingers all involve a decrease of surface energy.

Hence, the physics of splashing can be simply interpreted as a cascade of surface energy decreasing. First of all, the wall stops the impinging droplet so that the liquid tends to flow on the wall, which tends to transform the total energy into surface energy if the liquid forms a large flat disc. Then the flow follows the principle of minimal potential energy in such a way that the deformation follows a cascade of surface energy decreasing: from flat disc to lamella, from lamella to free rim, from free rim to cusps, from cusps to fingers, and finally from fingers to droplets.

¹This formula was based on the work of the present author (Ph.D. Thesis of University of Paris VI, 1992, unpublished).

This is the simplest mechanism that we adopt in the analysis.

Appendix B. Size Distribution Functions.

In a real impact, coalescence, unsteadiness, and nonlinear effect make the size of the droplets polyvalued, so that there is a well-defined distribution function. Experimentally, it has been found that crown splashing leads to droplets of various sizes. It has been discovered that the size distribution function obeys the log-normal law [4, 12, 17]:

$$f(d) = \frac{1}{\sqrt{2\pi}\gamma d} \exp\left(-\frac{(\ln \frac{d}{d_s} - \gamma^2)^2}{2\gamma^2}\right), \quad (\text{B.1})$$

where $f(x)$ is defined such that $f(x) dx$ is the probability of a droplet having its diameter lying in $(x, x + dx)$.

There are two free parameters in the log-normal distribution function: d_s and γ . Here d_s is the most probable value of the diameter of the secondary ejected droplets (alternatively d_s can be replaced by the mean diameter), and γ is the standard coefficient (characterizing the width of the distribution curve) in the log-normal distribution function (alternatively this can be replaced by the variance). Then it lacks the parameter γ . We discovered that we can determine γ by using the principle of maximum rate of entropy production [21]. Details are not given here. The Shannon entropy is found to be given by

$$S(\gamma) = \frac{1}{2} + 3(\gamma^2 + \ln d_s) - \frac{1}{2} \ln \frac{2}{\pi^3 \gamma^2}, \quad (\text{B.2})$$

and the maximum rate of entropy production occurs at $\frac{d^2 S(\gamma)}{d\gamma^2} = 0$ which corresponds to $\gamma = \frac{\sqrt{6}}{6} \approx 0.41$. The experimental value of Stow and Stainer [17] for γ varies from 0.348 to 0.699. Hence the estimated value $\gamma = 0.41$ lies within the experimental range.

Acknowledgement. The author is grateful to Professor Tang T. for his helpful comments on this paper.

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