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Erratum

Statistical Mechanics of Nonlinear Wave Equations (4): Cubic Schrödinger

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K. Vaninsky has kindly informed me that the application of the Kolmogorov–Čentsov criterion to the tightness of the mean-values M_d is not right. The criterion states that if $\mathbf{x}\colon t \in [0,1] \to x(t) \in \mathrm{R}$ is a random process with $E|\Delta x|^\alpha \leqq (\Delta t)^\beta$ for some $\beta > 1$, then $|\Delta x| \leqq c(\mathbf{x})|\Delta t|^\gamma$ with probability 1 for any $\gamma > (\beta - 1)/\alpha$. The same is true for a 2-dimensional time $t \in [0,1]^2$, provided $\beta > 2$ and $\gamma < (\beta - 2)/\alpha$. The criterion was started for $d = 1$ but applied to $d = 2$; also the conclusion (mid-p. 485) has its exponents 1/4 and 1/12 inexplicably reversed. The correction is to improve the estimates pp. 483–485 and 489–490. Happily, this is easy. The Gaussian character of the reference measure M_0 leads, in the style of the original text, to the improved bounds:

$$M_d|e^{tX_d}Z(x+h) - e^{tX_d}Z(x)|^{2n} \leqq c_1(n)h^n,$$

$$M_d|(e^{hX_d} - 1)e^{tX_d}Z(x)|^{2n} \leqq c_2(n)h^{n/2}$$

for any whole number n. The correct application of Kolmogorov–Čentsov requires $n = 5$ or more. The use of $n \uparrow \infty$ and a refinement of the criterion produces the correct (and sharp) version of the display mid-p. 485: $|\Delta Z| \leqq c(Z)[(\Delta t)^{1/4-} + (\Delta x)^{1/2-}]$.

Communicated by A. Jaffe