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Higher Spin BRS Cohomology of Supersymmetric Chiral Matter in $D = 4$

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Abstract: We examine the BRS cohomology of chiral matter in $N = 1, D = 4$ supersymmetry to determine a general form of composite superfield operators which can suffer from supersymmetry anomalies. Composite superfield operators $\Psi_{(a,b)}$ are products of the elementary chiral superfields S and $\bar{S}$ and the derivative operators $D_\alpha, \bar{D}_{\dot\beta}$ and $\partial_{\alpha\dot\beta}$. Such superfields $\Psi_{(a,b)}$ can be chosen to have "a" symmetrized undotted indices α_i and "b" symmetrized dotted indices $\dot\beta_j$. The result derived here is that each composite superfield $\Psi_{(a,b)}$ is subject to potential supersymmetry anomalies if $a - b$ is an odd number, which means that $\Psi_{(a,b)}$ is a fermionic superfield.

1. Introduction

The only known candidate for a unified theory of all matter and forces is superstring theory, but there are two major obstacles to making a comparison between this theory and experiment. The first problem is to discover how and why supersymmetry gets broken, preferably without generating a ridiculously huge cosmological constant. The second problem is to explain why our own universe is picked out from other possibilities. In a recent book written for the general public [15], Weinberg has expressed some doubt whether either of these questions has a mathematical answer – and suggested that the explanation may simply be that if our universe were not as it is, we wouldn't be here to ask the question.

But of course this "explanation" is a last resort. Our purpose here is to continue the search for supersymmetry anomalies. If these exist, their elimination would naturally be expected to impose restrictions on the possible superstring theories. In addition, it has been conjectured [5] that such anomalies might also provide a natural mechanism whereby "supersymmetry breaks itself," while at the same time retaining the cosmological constant at the zero value it naturally has in many unbroken supersymmetric theories.

The essential missing link in this program is that, as yet, there has been no calculation of a non-zero coefficient for any supersymmetry anomaly. Efforts in this direction will be reported elsewhere.

In this paper, we work out in detail the cohomology of the BRS operator defined by the supersymmetry invariance of chiral multiplets of rigid $N = 1, D = 4$ supersymmetry. The new result here is that this cohomology space contains potential anomalies in the renormalization of fermionic superfields with all half-integer spins. Formerly it had been shown that there were potential anomalies for fermionic superfields with spin $\frac{1}{2}$ only.

This may be very important for superstring theories, since such higher spin multiplets necessarily occur in all such theories. In addition, these higher spin potential anomalies may be of more immediate phenomenological interest [11] in relation to supermultiplets containing particles like the Δ which has spin $\frac{3}{2}$.

2. Summary of Previous Work

A systematic method for the calculation of local BRS cohomology spaces was described in [6]. The method starts with the definition of a grading operator which generates a "spectral" sequence of simpler nilpotent operators whose cohomology spaces are easily found. This sequence of spaces converges to a space isomorphic to the desired cohomology space. To facilitate computation of each of the cohomology spaces, we introduce a Fock space so that each successive cohomology space is the kernel of a "Laplacian" operator.

The cohomology space enables one to determine whether the theory can possess anomalies, either in the renormalization of the action itself or in the renormalization of higher dimensional composite operators formed by the fields in the theory. In most cases, it is quite arduous to analyze the cohomology space of a field theory in this general way, especially when the space itself is nontrivial to describe, as is frequently the case. The cohomology of Yang–Mills theory was examined in a specific case in [6]. An investigation was done of the simplest supersymmetric theory in four dimensions, the Wess–Zumino chiral theory, in [7]. The present work completes those results, as explained below. The results of [7] were generalized in [8] and [9] to include the case where chiral matter is coupled to supersymmetric Yang–Mills theory. Most recently, this method was used in a general study of the cohomology of the supertranslation operator [10]. The results of [10] and the results here are closely related, since in both cases the cohomology is determined by Laplacians which involve only counting operators and coupled $SU(2)$ angular momentum operators.

A different approach was used by the authors of [2], where a general formula for the creation of Lorentz invariant polynomials in the cohomology space for all compact gauge groups for the restricted BRS operator in Yang–Mills theories and gravity is given. Some aspects of the BRS cohomology of supersymmetric theories in four dimensions, restricted to Lorentz invariant polynomials, are investigated in [3, 4]. The cohomology of local integrated polynomials in field theories was also examined recently in [12, 13]. Spectral sequences have also been used in the BRS cohomology of $2d$ gravity (but without the introduction of a positive Fock space metric) [14]; for a review of this technique applied to CFT and $2d$ gravity see e.g. [1].

3. Action and Supersymmetry Invariance for Wess–Zumino Model

The first correct calculation of the BRS cohomology of the Wess–Zumino theory was made in [7]. That paper used Majorana spinors and real four dimensional Dirac matrices γ_μ. That notation obscures the symmetry of the theory under complex conjugation, and also makes superfield formalism much harder to use. As a consequence, no method was found in [7] to solve Eq. (214) of that paper, except for the simplest case, which occurs when no derivatives are present. We shall show below, in analogy with the work in [10], that Eq. (214) of reference [7], which becomes Eq. (7.12) below, can be solved including the situation when derivatives are present, using the theory of coupled angular momenta. This is most easily seen using the complex notation of [10].

The new result found here is that there are potential anomalies of spins $1/2, 3/2, 5/2 \ldots$, all of which involve derivatives. The results of [7] found only potential anomalies of spin 1/2, none of which involve derivatives.

In complex two-component notation, the (free quadratic) action for the Wess–Zumino chiral model is:

$$S = -\int d^4x[\partial_\mu A \partial^\mu \bar{A} + \psi^\alpha \sigma^\mu_{\alpha\dot{\beta}} \partial_\mu \bar{\psi}^{\dot{\beta}} - F\bar{F}] . \tag{3.1}$$

This action is invariant under the following supersymmetry and translational transformations (which are assumed to imply their complex conjugates):

$$\delta A = c^\alpha \psi_\alpha + \varepsilon^\mu \partial_\mu A , \tag{3.2}$$

$$\delta \psi_\alpha = \partial_\mu A \sigma^\mu_{\alpha\dot{\beta}} \bar{c}^{\dot{\beta}} + F c_\alpha + \varepsilon^\mu \partial_\mu \psi_\alpha , \tag{3.3}$$

$$\delta F = \partial_\mu \psi^\alpha \sigma^\mu_{\alpha\dot{\beta}} \bar{c}^{\dot{\beta}} + \varepsilon^\mu \partial_\mu F . \tag{3.4}$$

Here c^α is a constant (spacetime independent), commuting two component complex chiral spinor and ε_μ is a constant real anticommuting Lorentz vector. Their variations are:

$$\delta \varepsilon^\mu = -c^\gamma \sigma^\mu_{\alpha\dot{\beta}} \bar{c}^{\dot{\beta}} = -c \cdot \sigma^\mu \cdot \bar{c} , \tag{3.5}$$

$$\delta c^\alpha = 0 . \tag{3.6}$$

It is straightforward to show that

$$\delta^2 = 0 \tag{3.7}$$

on any field (including ε_μ and c^γ as constant fields). Note that

$$(c^\alpha \sigma^\mu_{\alpha\dot{\beta}} \bar{c}^{\dot{\beta}})^* = c^\beta \sigma^\mu_{\beta\dot{\alpha}} \bar{c}^{\dot{\alpha}} \tag{3.8}$$

is a real quantity.

Another way to express the generator δ is

$$\delta = \int d^4x \Big\{ [c^\alpha \psi_\alpha + \varepsilon^\mu \partial_\mu A] \frac{\delta}{\delta A} + [\partial_\mu A \sigma^\mu_{\alpha\dot\beta} \bar c^{\dot\beta} + F c_\alpha + \varepsilon^\mu \partial_\mu \psi_\alpha] \frac{\delta}{\delta \psi_\alpha}$$

$$+ [\partial_\mu \psi^\alpha \sigma^\mu_{\alpha\dot\beta} \bar c^{\dot\beta} + \varepsilon^\mu \partial_\mu F] \frac{\delta}{\delta F} + [\bar c^{\dot\alpha} \bar\psi_{\dot\alpha} + \varepsilon^\mu \partial_\mu \bar A] \frac{\delta}{\delta \bar A}$$

$$+ [\partial_\mu \bar A \bar\sigma^\mu_{\dot\alpha\beta} c^\beta + \bar F \bar c_{\dot\alpha} + \varepsilon^\mu \partial_\mu \bar\psi_{\dot\alpha}] \frac{\delta}{\delta \bar\psi_{\dot\alpha}}$$

$$+ [\partial_\mu \bar\psi^{\dot\alpha} \bar\sigma^\mu_{\dot\alpha\beta} c^\beta + \varepsilon^\mu \partial_\mu \bar F] \frac{\delta}{\delta \bar F} \Big\} - c^\alpha \sigma^\mu_{\alpha\dot\beta} \bar c^{\dot\beta} \frac{\partial}{\partial \varepsilon^\mu} . \tag{3.9}$$

4. The Grading of the Spectral Sequence

The final goal is to find the cohomology space

$$H \approx \mathrm{Ker}\,\delta / \mathrm{Im}\,\delta , \tag{4.1}$$

which is isomorphic to the kernel of the Laplacian

$$\Delta = (\delta + \delta^\dagger)^2 . \tag{4.2}$$

Unfortunately, Δ is in general a very complicated operator, and it is not possible to deduce much about kernel Δ from the expression for Δ directly – one just gets a huge number of terms and no insight.

The key idea behind the spectral sequence formalism[1] is to divide δ into parts which are easier to work with. For this purpose, we need to define a suitable counting operator N_{grading}, which assigns to each term of δ a positive (or zero) integral order. We decompose

$$\delta = \sum_{i=0}^{\infty} \delta_i , \tag{4.3}$$

with

$$[N_{\text{grading}}, \delta_i] = i \delta_i . \tag{4.4}$$

The grading is certainly not uniquely defined, and an important and difficult part of the spectral sequence technique is to find a useful grading. The spectral sequence generated by a given grading consists of a sequence of positive semidefinite Laplacian operators

$$\Delta_r = \left(d_r + d_r^\dagger\right)^2 , \quad r \geqq 0 , \tag{4.5}$$

where each successive nilpotent operator d_{r+1} operates in the cohomology space E_{r+1}, defined by $E_{r+1} = \ker \Delta_r$. The spaces satisfy the relation (E_0 is the whole space in which δ acts):

$$E_\infty \subseteq \cdots \subseteq E_{r+1} \subseteq E_r \subseteq \cdots \subseteq E_0 . \tag{4.6}$$

In practice, the sequence collapses (i.e. $d_r = 0$ for $r \geqq r_0$ so that $E_\infty = E_{r_0}$) for some low value r_0 of r ($r_0 = 3$ in the present case).

[1] A complete discussion of the spectral sequence method for finding BRS cohomology can be found in [6].

For the present problem, we will use the counting operator

$$N_{\text{grading}} = N_{\text{grad}} + \bar{N}_{\text{grad}} \tag{4.7}$$

with

$$N_{\text{grad}} = 3N(A) + 2N(\psi) + N(F) + N(c) . \tag{4.8}$$

We easily see that the decomposition

$$\delta = \delta_0 + \delta_2 \tag{4.9}$$

fulfills (4.4), where

$$\delta_0 = c^\alpha \Lambda_\alpha + \bar{c}^{\dot{\alpha}} \bar{\Lambda}_{\dot{\alpha}} + \varepsilon^\mu \partial_\mu \tag{4.10}$$

and

$$\delta_2 = c^\alpha \bar{\nabla}_\alpha + \bar{c}^{\dot{\alpha}} \nabla_{\dot{\alpha}} - c^\alpha \sigma^\mu_{\alpha\dot{\beta}} \bar{c}^{\dot{\beta}} (\varepsilon^\mu)^\dagger . \tag{4.11}$$

Here we define:

$$\Lambda_\alpha = \int d^4x \left\{ \psi_\alpha \frac{\delta}{\delta A} + F \frac{\delta}{\delta \psi^\alpha} \right\} , \tag{4.12}$$

$$\bar{\Lambda}_{\dot{\alpha}} = \int d^4x \left\{ \bar{\psi}_{\dot{\alpha}} \frac{\delta}{\delta \bar{A}} + \bar{F} \frac{\delta}{\delta \bar{\psi}^{\dot{\alpha}}} \right\} , \tag{4.13}$$

$$\bar{\nabla}_\alpha = \int d^4x \left\{ \partial_\mu \bar{A} \sigma^\mu_{\alpha\dot{\alpha}} \frac{\delta}{\delta \bar{\psi}_{\dot{\alpha}}} + \partial_\mu \bar{\psi}^{\dot{\alpha}} \sigma^\mu_{\alpha\dot{\alpha}} \frac{\delta}{\delta \bar{F}} \right\} , \tag{4.14}$$

$$\nabla_{\dot{\alpha}} = \int d^4x \left\{ \partial_\mu A \sigma^\mu_{\alpha\dot{\alpha}} \frac{\delta}{\delta \psi_\alpha} + \partial_\mu \psi^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \frac{\delta}{\delta F} \right\} . \tag{4.15}$$

The grading (4.7) separates δ into parts with no derivatives (Λ) and with derivatives (∇, ∂) and is appropriate for this problem, because the Λ part is particularly easy to deal with. ∂_μ was already fully analyzed in [6].

5. The Operator Δ_0

Now that we have chosen a grading, we go through the steps of the spectral sequence. The starting point is to calculate the kernel of the operator $\Delta_0 = [\delta_0 + \delta_0^\dagger]^2$. From (4.12) and (4.13), it follows that:

$$\{\Lambda_\alpha, \Lambda_\beta\} = 0 , \tag{5.1}$$

$$\{\Lambda_\alpha, (\Lambda_\beta)^\dagger\} = \delta_\alpha^\beta N , \tag{5.2}$$

$$\{\Lambda_\alpha, (\bar{\Lambda}_\beta)^\dagger\} = 0 , \tag{5.3}$$

and, therefore, Δ_0 computed from (4.10) takes the form:

$$\Delta_0 = (\Lambda_\alpha)^\dagger \Lambda_\alpha + (\bar{\Lambda}_{\dot{\alpha}})^\dagger \bar{\Lambda}_{\dot{\alpha}} + \partial_\mu (\partial_\mu)^\dagger + (N + \bar{N}) \varepsilon_\mu^\dagger \varepsilon_\mu + nN + \bar{n}\bar{N} . \tag{5.4}$$

In (5.4) we used the counting operators

$$n = c_\alpha (c_\alpha)^\dagger \,, \tag{5.5}$$

$$\bar{n} = \bar{c}_\alpha (\bar{c}_{\dot\alpha})^\dagger \,, \tag{5.6}$$

and

$$N = \int d^4x \left\{ A \frac{\delta}{\delta A} + \psi_\alpha \frac{\delta}{\delta \psi_\alpha} + F \frac{\delta}{\delta F} \right\} , \tag{5.7}$$

which can also be written as

$$N = \sum_{k=0}^{\infty} \frac{1}{k!} \{ A_{\mu_1 \mu_2 \ldots \mu_k} (A_{\mu_1 \mu_2 \cdots \mu_k})^\dagger + \psi_{\alpha \mu_1 \mu_2 \ldots \mu_k} (\psi_{\alpha \mu_1 \mu_2 \ldots \mu_k})^\dagger + F_{\mu_1 \mu_2 \ldots \mu_k} (F_{\mu_1 \mu_2 \ldots \mu_k})^\dagger \} \,. \tag{5.8}$$

Here the definition

$$A_{\mu_1 \mu_2 \ldots \mu_k} = \partial_{\mu_1} \partial_{\mu_2} \ldots \partial_{\mu_k} A \tag{5.9}$$

is used (same for F and ψ_α). Equation (5.4) also uses the relation:

$$[\partial_\mu^\dagger, \partial_\nu] = \delta_\nu^\mu (N + \bar{N}) \,, \tag{5.10}$$

which was discussed in [6].

6. The Space E_1

Since (5.4) consists of a sum of separately positive semidefinite operators in the form $\sum_i Q_i^\dagger Q_i = \Delta$, the kernel satisfies the equations

$$Q_i \ker \Delta = 0 \,, \tag{6.1}$$

or, more specifically:

$$\Delta_\alpha E_1 = 0 \,, \tag{6.2}$$

$$\bar{\Delta}_\alpha E_1 = 0 \,, \tag{6.3}$$

$$(\partial_\mu)^\dagger E_1 = 0 \,, \tag{6.4}$$

$$n N E_1 = 0 \,, \tag{6.5}$$

$$\bar{n} \bar{N} E_1 = 0 \,, \tag{6.6}$$

$$(N + \bar{N}) \varepsilon_\mu E_1 = 0 \,. \tag{6.7}$$

The solutions of these constraints are much more obvious than they were in the real notation of [7]. If $nN = 0$ then either $n = 0$ or $N = 0$. Hence, any function of the form

$$\mathscr{F} = \mathscr{F}(\partial, A, F, \psi, \bar{c}) \Theta \tag{6.8}$$

satisfies this whole set of equations, if it fulfills (6.2) and (6.4). Functions of this kind and their complex conjugates exhaust the solutions of these equations depending non-trivially on c or $\bar{c}$.

Solutions independent of c and $\bar{c}$ can depend on all variables

$$\mathscr{F} = \mathscr{F}(\partial, A, F, \psi, \bar{A}, \bar{F}, \bar{\psi})\Theta \tag{6.9}$$

and must satisfy all the equations (6.2), (6.3) and (6.4).

The expression Θ is defined as

$$\Theta = \varepsilon^{\mu}\varepsilon^{\nu}\varepsilon^{\lambda}\varepsilon^{\rho}\varepsilon_{\mu\nu\lambda\rho} \tag{6.10}$$

and satisfies

$$\varepsilon_{\mu}^{\dagger}\varepsilon_{\mu}\Theta = (4 - N(\varepsilon))\Theta = 0\,. \tag{6.11}$$

We can think of Θ as being equivalent to $\int d^4x$ (see [6]).

To construct the operator d_1 we will need the explicit form of the operator Π_1, which projects the entire space E_0 onto E_1. A general form is easy to write down:

$$\begin{aligned}\Pi_1 = \Pi_{\varepsilon=0}\,\Pi_{\bar{c}^{\dagger}=0}\{&\Pi_{N\geqq 0}\,\Pi_{\bar{N}\geqq 0}\,\Pi_{\Lambda=0}\,\Pi_{\bar{\Lambda}=0}\,\Pi_{n=0}\,\Pi_{\bar{n}=0}\\ &+\Pi_{N>0}\Pi_{n=0}\,\Pi_{\bar{N}=0}\,\Pi_{\bar{n}>0}\,\Pi_{\Lambda=0}\\ &+\Pi_{\bar{N}>0}\,\Pi_{\bar{n}=0}\,\Pi_{N=0}\,\Pi_{n>0}\,\Pi_{\bar{\Lambda}=0}\} + \Pi_{N=0}\Pi_{\bar{N}=0}\,.\end{aligned} \tag{6.12}$$

Here, the first part projects onto solutions of type (6.9). The second operator projects onto states of type (6.8) and the third projects onto their complex conjugates, i.e.

$$\mathscr{F} = \mathscr{F}(\partial, \bar{A}, \bar{F}, \bar{\psi}, c)\Theta\,. \tag{6.13}$$

The fourth operator projects onto pure ghost states, formed by $\varepsilon_{\mu}, c^{\alpha}$ and $\bar{c}^{\dot{\alpha}}$ only. One can easily verify that these states satisfy (6.2)–(6.7) also, but they do not contain any information of interest for this paper (see [10] for a discussion of these terms).

Now we find explicit forms of the operators contained in (6.12). In addition to the defining equations for an orthogonal projection operator $\Pi^2 = \Pi = \Pi^{\dagger}$, $\Pi_{\Lambda=0}$ is subject to the constraint

$$\Lambda_{\alpha}\Pi_{\Lambda=0} = 0\,. \tag{6.14}$$

It is easy to see that the projection operator

$$\begin{aligned}\Pi_{\Lambda=0} &= 1 - \frac{1}{N}\Lambda_{\alpha}^{\dagger}\Lambda_{\alpha} + \frac{1}{2N^2}\Lambda_{\alpha}^{\dagger}\Lambda_{\beta}^{\dagger}\Lambda_{\beta}\Lambda_{\alpha}\\ &= \frac{1}{4N^2}\Lambda^{\alpha}\Lambda_{\alpha}(\Lambda^{\beta}\Lambda_{\beta})^{\dagger}\end{aligned} \tag{6.15}$$

satisfies (6.14). Note that the Λ_{α} are anticommuting objects, so that (6.15) involves only quadratic terms ($\Lambda^3 = 0$).

Similarly,

$$\Pi_{\bar{\Lambda}=0} = \frac{1}{4\bar{N}^2}\bar{\Lambda}^{\dot{\alpha}}\bar{\Lambda}_{\dot{\alpha}}(\bar{\Lambda}^{\dot{\beta}}\bar{\Lambda}_{\dot{\beta}})^{\dagger} \tag{6.16}$$

fulfills the corresponding condition for $\bar{\Lambda}$. Equation (6.4) requires an operator satisfying

$$\partial_{\mu}^{\dagger}\Pi_{\bar{c}^{\dagger}=0} = 0\,. \tag{6.17}$$

This projection operator has the form

$$\Pi_{\partial^\dagger=0} = \sum_{k=0}^{\infty} \left\{ \frac{-1}{N+\bar{N}} \right\}^k \partial_{\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_k}\partial^\dagger_{\mu_k}\ldots\partial^\dagger_{\mu_2}\partial^\dagger_{\mu_1} \tag{6.18}$$

as can be shown using the commutation relation (5.10). The projection operators $\Pi_{N=0}$ and $\Pi_{N>0}$ for the counting operators are

$$\Pi_{N=0} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} N_k \tag{6.19}$$

with $N_k = \sum_{\{i\}}[\phi_{i_1}\phi_{i_2}\ldots\phi_{i_k}][\phi_{i_1}\phi_{i_2}\ldots\phi_{i_k}]^\dagger$ · ϕ represents any relevant set of fields. Of course, $\Pi_{N>0}$ is given by $\Pi_{N>0} = 1 - \Pi_{N=0}$.

7. The Operator Δ_2

The operator $d_1 = \Pi_1\delta_1\Pi_1$ vanishes since $\delta_1 = 0$, which results in $E_2 = E_1$ and $\Pi_2 = \Pi_1$. Then d_2 reduces to

$$\begin{aligned} d_2 &= \Pi_2\{c^\alpha\bar{\nabla}_\alpha + \bar{c}^{\dot{\alpha}}\nabla_{\dot{\alpha}} - c^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{c}^{\dot{\beta}}(\varepsilon^\mu)^\dagger\}\Pi_2 \\ &= \Pi_2\{c^\alpha\bar{\nabla}_\alpha + \bar{c}^{\dot{\alpha}}\nabla_{\dot{\alpha}}\}\Pi_2\Pi_{\varepsilon=0} - \Pi_{N=\bar{N}=0}(c^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{c}^{\dot{\beta}})(\varepsilon^\mu)^\dagger \,. \end{aligned} \tag{7.1}$$

Now we concentrate on the sector where either $N \neq 0$ or $\bar{N} \neq 0$ or both. Then the operator $c^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{c}^{\dot{\beta}}(\varepsilon^\mu)^\dagger$ can be dropped. Its cohomology can be found in [10]. From the general theory of spectral sequences, it follows that d_2 is nilpotent, as can be verified explicitly. To compute Δ_2, we first evaluate some (anti)commutators. It is a remarkable fact that these (anti)commutators frequently give rise again to the operators we start with (or their adjoints):

$$\{\bar{\nabla}_\alpha, (\Lambda_\beta)^\dagger\} = 0 \,, \tag{7.2}$$

$$\{\bar{\nabla}_\alpha, (\bar{\Lambda}_\beta)^\dagger\} = 0 \,, \tag{7.3}$$

$$[\partial_\mu, \Lambda_\alpha] = 0 \,, \tag{7.4}$$

$$[\partial_\mu, (\Lambda_\alpha)^\dagger] = 0 \,, \tag{7.5}$$

$$[\partial_\mu, \bar{\nabla}_\gamma] = 0 \,, \tag{7.6}$$

$$[(\partial_\mu)^\dagger, \bar{\nabla}_\alpha] = \sigma^\mu_{\alpha\dot{\beta}}(\bar{\Lambda}_{\dot{\beta}})^\dagger \,, \tag{7.7}$$

$$\{\Lambda_\alpha, \nabla_{\dot{\beta}}\} = \sigma^\mu_{\alpha\dot{\beta}}\partial_\mu \,, \tag{7.8}$$

$$\{\Lambda_\alpha, \bar{\nabla}_\beta\} = 0 \,, \tag{7.9}$$

$$\{\bar{\nabla}_\alpha, (\bar{\nabla}^\beta)^\dagger\} = \varepsilon_{\alpha\beta}\bar{M} + \sigma^{\mu\nu}_{\alpha\beta}\bar{L}_{\mu\nu} \,. \tag{7.10}$$

Using the above relations, we find

$$\Pi_2\bar{\nabla}_\alpha\Pi_2 = \Pi_2\bar{\nabla}_\alpha \,, \tag{7.11}$$

and hence,

$$\begin{aligned}\Delta_2 &= \Pi_2\{c^\alpha(c^\beta)^\dagger\{\bar\nabla_\gamma,(\bar\nabla_\beta)^\dagger\} + \bar c^\alpha(\bar c^\beta)^\dagger\{\nabla_{\dot\gamma},(\nabla_\beta)^\dagger\} \\ &\quad + c^\alpha(c^\beta)^\dagger(\bar\nabla_\beta)^\dagger(\Pi_2-1)\bar\nabla_\gamma + \bar c^\alpha(\bar c^\beta)^\dagger(\nabla_\beta)^\dagger(\Pi_2-1)\nabla_\gamma \\ &\quad + (\bar\nabla_\gamma)^\dagger\Pi_2\bar\nabla_\gamma + (\nabla_\alpha)^\dagger\Pi_2\nabla_\gamma\}\Pi_2 \\ &= \Pi_2\{(\nabla_\gamma)^\dagger\Pi_2\nabla_\beta + \bar n[M-4] - 4\bar J_i L_i \\ &\quad + (\bar\nabla_\gamma)^\dagger\Pi_2\bar\nabla_\gamma + n[\bar M - 4] - 4J_i\bar L_i\}\Pi_2\,. \end{aligned} \tag{7.12}$$

We use the following abbreviations

$$M = N(\partial) + 4N(F) + 2N(\psi)\,, \tag{7.13}$$

$$\bar M = \bar N(\partial) + 4N(\bar F) + 2N(\bar\psi)\,, \tag{7.14}$$

$$N(\partial) = \sum_{k=0}^{\infty}\frac{1}{k!}[\phi_{\mu_1\ldots\mu_{k+1}}(\phi_{\mu_1\ldots\mu_{k+1}})^\dagger]\,; \tag{7.15}$$

$$\bar N(\partial) = \sum_{k=0}^{\infty}\frac{1}{k!}[\bar\phi_{\mu_1\ldots\mu_{k+1}}(\bar\phi_{\mu_1\ldots\mu_{k+1}})^\dagger]\,. \tag{7.16}$$

$N(\partial)$ and its complex conjugate count the number of derivatives in each expression.

The coupled angular momenta $\bar J_i L_i$ arise as follows. Computation yields the term

$$\frac{1}{2}\bar c^\gamma(\bar c^{\dot\beta})^\dagger(\bar\sigma^{\mu\nu})_{\alpha\beta}L_{\mu\nu}\,, \tag{7.17}$$

where

$$L_{\mu\nu} = \sum_{k=0}^{\infty}\frac{1}{k!}[\phi_{\mu_1\ldots\mu_k\mu}(\phi^\nu_{\mu_1\ldots\mu_k})^\dagger - \phi_{\mu_1\ldots\mu_k\nu}(\phi^\mu_{\mu_1\ldots\mu_k})^\dagger]; \quad (\phi = A,\psi_\gamma,F)\,. \tag{7.18}$$

Now using the identities

$$(\bar\sigma^{0i})_{\dot\gamma\dot\beta} = (\sigma^i)_{\dot\gamma\beta} = i\varepsilon^{ijk}(\sigma^k)_{\alpha\dot\beta}\,, \tag{7.19}$$

(7.17) can be written as $\bar J_i L_i$, where

$$\bar J_i = \frac{1}{2}\bar c^{\dot\gamma}(\bar\sigma_i)_\alpha^\beta(\bar c^{\dot\beta})^\dagger\,, \tag{7.20}$$

$$L_i = -\frac{1}{2}L_{0i} - \frac{i}{4}\varepsilon_{ijk}L_{jk}\,. \tag{7.21}$$

Equivalently, one could write

$$L_i(\sigma^i)_{\alpha\beta} = L_{\mu\nu}(\sigma^{\mu\nu})_{\dot\alpha\dot\beta}\,. \tag{7.22}$$

It is easy to verify that L_i and J_i obey the commutation rules of the $SU(2)$ Lie algebra:

$$[J_i,J_j] = i\varepsilon^{ijk}J_k\,, \tag{7.23}$$

$$[L_i,L_j] = i\varepsilon^{ijk}L_k\,. \tag{7.24}$$

The complex conjugate equations are:

$$[-\bar{J}_i, -\bar{J}_j] = i\varepsilon^{ijk}(-\bar{J}_k)\,, \tag{7.25}$$

$$[-\bar{L}_i, -\bar{L}_j] = i\varepsilon^{ijk}(-\bar{L}_k)\,. \tag{7.26}$$

and we note that

$$[J_i, L_j] = 0\,. \tag{7.27}$$

From this we see that the effect of the L_i is to "rotate" the dotted indices that arise in derivatives $\partial_{\dot{\alpha}\beta}$. It follows that the Laplacian contains only counting operators, coupled angular momenta and the operator $\Pi_2\nabla_{\dot{\alpha}}\Pi_2$.

8. The space E_3

Given the manifest $SU(2)$ structure of the Laplacian

$$\begin{aligned}\nabla_2 = \Pi_2\{(\nabla_\alpha)^\dagger\Pi_2\nabla_{\dot{\alpha}} + \bar{n}[M-4] - 2[(\bar{J}_i + L_i)(\bar{J}_i + L_i) - \bar{J}_i\bar{J}_i - L_iL_i] \\ +(\bar{\nabla}_\alpha)^\dagger\Pi_2\bar{\nabla}_\alpha + n[\bar{M}-4] - 2[(J_i + \bar{L}_i)(J_i + \bar{L}_i) - J_iJ_i - \bar{L}_i\bar{L}_i]\}\Pi_2\,,\end{aligned} \tag{8.1}$$

the problem of finding the kernel reduces to the determination of the eigenvalues and eigenstates of the operators. We assume $n = 0$, $\bar{n} \neq 0$ here (if $n = \bar{n} = 0$, we just get (8.6) below.)

Now the eigenvalue of the $SU(2)$ operator $\bar{J}^2 = \bar{J}_i\bar{J}_i$ is given by (see [10])

$$\bar{J}^2 = \bar{j}(\bar{j}+1) \quad \text{with} \quad \bar{j} = \frac{\bar{n}}{2}\,, \tag{8.2}$$

since the operator $\bar{J}^2$ is acting on polynomials in $\bar{c}_{\dot{\alpha}}$ which are totally symmetric.

As is clear from (7.22), the operator L_i rotates the dotted indices occurring in derivatives. Hence, the maximum possible eigenvalue of $L^2 = L_iL_i$ is $\frac{N(\partial)}{2}$. Therefore, we find

$$L^2 = l(l+1) \text{ with } l = \frac{N(\partial) - 2s}{2}, \quad s = 0,1,2\ldots\left[\frac{N(\partial)}{2}\right]\,, \tag{8.3}$$

where $[x]$ is the greatest integer in x.

The coupling operator of the two angular momenta leads to the term

$$(\bar{J} + L)^2 = k(k+1), \text{ where } k = \bar{j} + l - r\,, \tag{8.4}$$

and $r = 0,1,2,\ldots,\min(\bar{n}, N(\partial) - 2s)$.

Now it follows from (8.1) that

$$\begin{aligned}\Delta_2 = \Pi_2\{(\nabla_{\dot{\alpha}})^\dagger\Pi_2\nabla_\alpha + \bar{n}[4N(F) + 2N(\psi) - 4 + 2r + 2s] + 2r[N(\partial) + 1 - r - 2s]\} \\ \times\, \Pi_2 + \text{complex conjugate}\,.\end{aligned} \tag{8.5}$$

Equation (6.15) implies that $N(F) \geqq 1$ or $N(\psi) \geqq 2$ for each term in every polynomial in $\ker \Lambda_\alpha$ and since $r \leqq N(\partial) - 2s$, the following terms in (8.5) are positive semidefinite:

$$\Pi_2 \nabla_{\dot\beta} E_3 = 0 \,, \tag{8.6}$$

$$\bar n [4N(F) + 2N(\psi) - 4 + 2r + 2s] E_3 = 0 \,, \tag{8.7}$$

$$r[N(\partial) + 1 - r - 2s] E_3 = 0 \,. \tag{8.8}$$

The complex conjugates of these equations are also true of course, for the case when $\bar n = 0$ and $n \neq 0$.

The only possible solutions for (8.8) occur when $r = 0$, hence, the only possible solutions of (8.7) occur for $s = 0$. This result implies $k = \bar j + l$. It follows from this and $l = \frac{N(\partial)}{2}$, that the polynomials in E_∞ are totally symmetric in the dotted indices.

With $r = s = 0$, (8.7) reduces to

$$\bar n [4N(F) + 2N(\psi) - 4] E_3 = 0 \,. \tag{8.9}$$

9. The Cohomology Spaces E_∞ and H

The higher operators d_r for $r = 3, 4, 5, \ldots$ vanish due to (8.6),(6.17) and (6.2) (these operators are written down explicitly in terms of δ_0 and δ_2 in [6]). Hence, the spectral sequence collapses at this point, resulting in $E_\infty = E_3$, and the defining equations of E_∞ are:

$$\Lambda_\alpha E_\infty = 0 \,, \tag{9.1}$$

$$(\partial_\mu)^\dagger E_\infty = 0 \,, \tag{9.2}$$

$$n N E_\infty = 0 \,, \tag{9.3}$$

$$(N + \bar N) \varepsilon_\mu E_\infty = 0 \,, \tag{9.4}$$

$$\Pi_2 \nabla_{\dot\beta} E_\infty = 0 \,, \tag{9.5}$$

$$\bar n [4N(F) + 2N(\psi) - 4] E_\infty = K E_\infty = 0 \,, \tag{9.6}$$

and their complex conjugates. In addition, for $\bar n \neq 0$ ($n \neq 0$) the objects in E_∞ have to be symmetric in their dotted (undotted) indices, as discussed above.

We can construct solutions of these equations using superfields in the following way. The basic chiral superfield is:

$$S(x, \theta, \bar\theta) = A(y) + \theta\psi(y) + \frac{1}{2}\theta^2 F(y) \,, \tag{9.7}$$

where

$$y^\mu = x^\mu + \frac{1}{2}\theta^\alpha \sigma^\mu_{\alpha\dot\beta} \bar\theta^{\dot\beta} \,. \tag{9.8}$$

To find a solution of the above equations for E_∞, we simply take any polynomial $P(S, \partial, \bar c)$ of the superfields, the derivative operator and the antighost $\bar c$. Then we choose its "F" component, i.e. the coefficient of θ^2 in the polynomial. Let us denote this $f = [P(S, \partial, \bar{c})]_F$. Then we claim that the following is a solution of the above equations plus the symmetry requirements discussed in the previous section:

$$e = \mathrm{Sym}_{\text{dotted indices}}\, \Pi_{\bar{c}^\dagger=0} f \Theta \in E_\infty\,. \tag{9.9}$$

We believe that all solutions of these equations are obtained in this manner, but will not attempt to prove that here. We have shown that there are an infinite set of objects in the cohomology space, and the missing proof would only establish that there are no more. The more interesting question now is whether any of the known ones correspond to anomalies.

Now we show that, in fact, (9.9) is in the cohomology space E_∞. To see this, we notice that the grading operator

$$N'_{\text{grad}} = N_{\text{grad}} + N(\theta)\,, \tag{9.10}$$

where N_{grad} is given by (4.8), satisfies

$$N'_{\text{grad}} S = 3S\,, \tag{9.11}$$

so each term in (9.7) has eigenvalue $N'_{\text{grad}} = 3$. Obviously, for a product of kS fields (including arbitrary number of space-time derivatives), $N'_{\text{grad}} S^k = 3kS^k$ holds. In terms of this new grading, operator K in (9.6) becomes

$$K = \bar{n}[2N(\theta) + 2N(c) + 2(3N - N'_{\text{grad}}) - 4]\,. \tag{9.12}$$

Recalling $NS^k = kS^k$, we note that

$$(3N - N'_{\text{grad}})P = 0\,. \tag{9.13}$$

Note that this would not be true in general if P contained any covariant derivatives D_α. Using this result we see that

$$KP = \bar{n}[2N(\theta) - 4]P\,. \tag{9.14}$$

Now P is not homogeneous under the action of K. This equation shows that the terms of P which are homogeneous in θ are also homogeneous under the action of K. In fact, the θ^2 terms of P is homogeneous of degree zero when acted upon by the operator K. So we find that the operator K acting on the θ^2 term of any product of chiral superfields involving only partial derivatives vanishes assuring that the F component (with θ^2) of this product is a solution of (9.6).

Let us generalize our analysis to the case where there are several superfields S_a. Then a more explicit form of our solution is:

$$e = \mathrm{Sym}_{\text{dotted indices}} \int d^2\theta\, \Pi_{\bar{c}^\dagger=0} P(S_{a_1}, \partial_{\alpha\dot{\alpha}} S_{a_2}, \partial_{\beta\dot{\beta}} \partial_{\gamma\dot{\gamma}} S_{a_3}, \ldots) \bar{c}_{\dot{\kappa}_1} \bar{c}_{\dot{\kappa}_2} \cdots\,, \tag{9.15}$$

and the undotted indices are "free." In the above expression, $\partial_{\alpha\dot{\beta}} = \partial_\mu \sigma^\mu_{\alpha\dot{\beta}}$ and P represents any polynomial of its argument fields. Acting with $\Pi_{\bar{c}^\dagger=0}$ on any expression simply corresponds to a subtraction of the total derivative part. e, as defined above, satisfies (9.2), (9.3), (9.4) and (9.6) trivially; (9.1) and (9.5) need to be checked explicitly. As stated above, any product of S's is homogenous in N'_{grad}, therefore, its F component is also homogeneous in N'_{grading} and N_{grading}, leading to $f = f_k$ with $N_{\text{grading}} f_k = k f_k$. As a consequence, the supersymmetry transformation

$\delta' = \delta - \varepsilon^\mu \partial_\mu + c^\alpha \sigma^\mu_{\alpha\dot\beta} \bar{c}^{\dot\beta} (\varepsilon^\mu)^\dagger = c\Lambda + \bar{c}\bar{\Lambda} + c\bar{\nabla} + \bar{c}\nabla$, acting on f_k splits into two independent parts:

$$\delta' f_k = (\delta'_0 + \delta'_2) f_k = \partial_\mu X^\mu_k + \partial_\mu X^\mu_{k+2}\,, \tag{9.16}$$

where $\delta'_0 = c\Lambda + \bar{c}\bar{\Lambda}$ and $\delta'_2 = c\bar{\nabla} + \bar{c}\nabla$; X^μ_k and X^μ_{k+2} don't need to be further specified. Equation (9.16) reiterates the well-known fact that the F component of any superfield transforms into a total derivative. Using (7.4) and (7.5), we find that $[\Lambda, \Pi_{\bar{c}^\dagger=0}] = 0$ and, therefore,

$$\Lambda_\alpha e = \Lambda_\alpha \Pi_{\bar{c}^\dagger=0} f_k \Theta = \Pi_{\bar{c}^\dagger=0} \Lambda_\alpha f_k \Theta = \Pi_{\bar{c}^\dagger=0} \partial_\mu X^\mu_{\alpha,k-1} \Theta = 0\,, \tag{9.17}$$

since $\Pi_{\bar{c}^\dagger=0}\partial = 0$. Hence, (9.1) is fulfilled. Using (7.11), we see (9.5) is obeyed as well:

$$\begin{aligned}\Pi_2 \nabla_{\dot\beta} e &= \Pi_2 \nabla_{\dot\beta} \Pi_2 \Pi_{\bar{c}^\dagger=0} f_k \Theta \\ &= \Pi_2 \nabla_{\dot\beta} \Pi_2 f_k \Theta = \Pi_2 \nabla_{\dot\beta} f_k \Theta = \Pi_2 \partial_\mu X^\mu_{\dot\beta,k+1} \Theta = 0\,.\end{aligned} \tag{9.18}$$

This proves that the objects of the form (9.9) (or their complex conjugates) are indeed in E_∞.

The states with $n = \bar{n} = 0$ can be built with chiral and antichiral superfields, supercovariant derivatives D_a and $\bar{D}_{\dot\alpha}$, and their anticommutator $\partial_{\alpha\dot\beta}$. This result reflects the fact that the usual supersymmetric actions with ghost charge zero are BRS-invariant.

The one-to-one relation between E_∞ and H [6] leads to the following objects $\mathcal{X}$ in the cohomology space:

$$\begin{aligned}\mathcal{X} &= \int d^4x f \\ &= \mathrm{Sym}_{\text{dot. ind.}} \int d^4x \int d^2\theta \mathrm{P}(S_{a_1}, \partial_{\alpha\dot\alpha} S_{a_2}, \partial_{\beta\dot\beta}\partial_{\gamma\dot\gamma} S_{a_3}, \ldots) \bar{c}_{\dot\kappa_1} \ldots\,.\end{aligned} \tag{9.19}$$

(The operator $\Pi_{\bar{c}^\dagger=0}\Theta$ has been mapped into $\int d^4x$.)

10. Conclusion

We have shown that there are no polynomials in H for the Wess–Zumino theory that contain both c and $\bar{c}$. The complex conjugate of every solution of the defining equations for E_∞ is also a solution. Hence, we can restrict the discussion of the complete cohomology space to objects containing only the antighosts $\bar{c}$. Revealing the index structure explicitly, we have shown that there is an infinite set of states of the form

$$\begin{aligned}\mathcal{X}_{\alpha_1 \ldots \alpha_{k_n} \dot\beta_1 \ldots \dot\beta_{k_n+g}} &= \mathrm{Sym}_{\dot\beta_1 \ldots \dot\beta_{k_n+g}} \int d^4x\, d^2\theta \\ \times \{\partial_{\alpha_1\dot\beta_1} \ldots \partial_{\alpha_{k_1}\dot\beta_{k_1}} S_{a_1} \ldots \partial_{\alpha_{k_{(n-1)}+1}\dot\beta_{k_{(n-1)}+1}} &\ldots \partial_{\alpha_{k_n}\dot\beta_{k_n}} S_{a_n} \bar{c}_{\dot\beta_{k_n+1}} \bar{c}_{\dot\beta_{k_n+2}} \ldots \bar{c}_{\dot\beta_{k_n+g}}\}\end{aligned} \tag{10.1}$$

in the cohomology space of the Wess–Zumino model. The states found in [7] are those states of this form that contain no derivatives – so that for ghost charge one they are necessarily of spin $\frac{1}{2}$. To get spin $\frac{3}{2}$, one needs at least one derivative, for spin $\frac{5}{2}$, one needs at least two derivatives, etc. The corresponding complex conjugate expressions are obtained by converting dotted into undotted indices and vice versa, and $S \to \bar{S}$, $\bar{c} \to c$. By contraction and symmetrization of the undotted indices, we can decompose $\mathscr{X}$ into operators of the form

$$\mathscr{A}_{(a,b)} = \mathscr{A}_{(\alpha_1\alpha_2\ldots\alpha_a)(\dot\beta_1\dot\beta_2\ldots\dot\beta_b)}\,, \tag{10.2}$$

where $b = k_n + g$ and $k_n - a$ is even and greater than or equal to 0, since contractions always involve pairs of undotted indices. In particular, we are interested in polynomials with ghost charge $g = 1$, which correspond to anomalies. For these objects, we find $b - a$ is odd and positive. Therefore, the operators $\mathscr{A}$ are spinors.

These objects could appear as anomalies in the renormalization of composite operators with the same spin structure as the anomaly. To compute the anomalies of a given such composite operator, a term of the form

$$S_\Psi = \int d^4x\, d^4\theta [\Psi_{\alpha_1\ldots\alpha_a\dot\beta_1\ldots\dot\beta_b}\Phi^{\alpha_1\ldots\alpha_a\dot\beta_1\ldots\dot\beta_b}] \tag{10.3}$$

would be introduced into the action. Here Ψ is a composite operator with ghost charge zero and $\Phi^{\alpha_1\ldots\alpha_a\dot\beta_1\ldots\dot\beta_b}$ is a chiral source superfield. There is a matching between the indices of the anomaly and those of the anomalous operator, because both must couple to the source Φ. Now to compute the anomaly, some specific form for the composite operator Ψ would be chosen and then the one particle irreducible generating functional Γ including one vertex (10.3) would be calculated. If there is an anomaly, one would find that the supersymmetric variation of this part of Γ would be of the form

$$\delta\Gamma = \kappa \int d^4x\, d^2\theta [\mathscr{A}_{\alpha_1\ldots\alpha_a\dot\beta_1\ldots\dot\beta_b}\Phi^{\alpha_1\ldots\alpha_a\dot\beta_1\ldots\dot\beta_b}]\,, \tag{10.4}$$

where κ is a calculable coefficient.

Note that (10.4) is still in the cohomology space because Φ transforms simply as a chiral superfield as does S. Hence, our derivation of the cohomology applies to Φ just as it did to S. The indices of Φ play no role in the cohomology discussion.

Because all possible anomalies have half-integer spin (see discussion of (10.2)), it follows that all operators which can be anomalous also have half-integer spin. Generally, the entire class of spinor operators in supersymmetric theories containing chiral matter can be anomalous.

It would be interesting to extend the results of the present paper to the BRS cohomology of the supersymmetric Yang–Mills theory. This appears to be rather difficult, because the gauge symmetry mixes with supersymmetry in a tricky way. However, it is important. In fact, we conjecture that if there are one-loop supersymmetry anomalies in rigid supersymmetric theories, they will require at least one gauge propagator in the diagram – i.e. they will occur only in supersymmetric theories where chiral matter is coupled to gauge theory.

References

1. Bouwknegt, P., McCarty, J., Pilch, K.: Semi-infinite cohomology in conformal field theory and 2*d* gravity. Preprint CERN-TH.6646/92 (hepth@xxx9209034)
2. Brandt, F., Dragon, N., Kreuzer, M.: Phys. Lett. **B231**, 263–270 (1989), Nucl. Phys. **B332**, 224–249 (1990), Nucl. Phys. **B332**, 250–260 (1990), Nucl. Phys. **B340**, 187–224 (1990)
3. Brandt, F.: Lagrangedichten und Anomalien in vierdimensionalen supersymmetrischen Theorien. PhD-Thesis, Universität Hannover (1991)

4. Brandt, F.: Lagrangians and anomaly candidates of $D = 1$, $N = 4$ rigid supersymmetry. Nucl. Phys. **B392**, 928 (1993)
5. Dixon, J.A.: A natural mechanism for supersymmetry breaking with zero cosmological constant. CTP-TAMU-69/92. To be published in: Duff, M.J., Khuri, R.R., Ferrara, S. (eds.): From superstrings to supergravity. Workshop Proceedings, Erice 1992. Singapore, New Jersey, Hong Kong: World Scientific
6. Dixon, J.A.: Calculation of BRS cohomology with spectral sequences. Commun. Math. Phys. **139**, 495–526 (1991)
7. Dixon, J.A.: BRS cohomology of the chiral superfield. Commun. Math. Phys. **140**, 169–201 (1991)
8. Dixon, J.A.: Supersymmetry is full of holes. Class. Quant. Grav. **7**, 1511–1521 (1990)
9. Dixon, J.A.: Anomalies, Becchi–Rouet–Stora cohomology, and effective theories. Phys. Rev. Lett. **67**, 797–800 (1991)
10. Dixon, J.A., Minasian, R.: BRS cohomology of the supertranslation in $D = 4$. Preprint CPT-TAMU-13/93 (hep-th@xxx/9304035). Commun. Math. Phys.
11. Dixon, John: Supersymmetry Anomalies, in Witten Index and the Standard Model. Preprint CPT-TAMU-47/93 (hep-th@xxx/9311305)
12. Dubois-Violette, M., Henneaux, M., Tallon, M., Viallet, C.M.: Some results on local cohomologies in field theory. Phys. Lett. **B267**, 81–87 (1991)
13. Dubois-Violette, M., Henneaux, M., Tallon, M., Viallet, C.M.: General solution of the consistency equation. Phys. Lett. **B289**, 361–367 (1992)
14. Lian, B.H.: Semi-infinite homology and 2*D* Quantum Gravity. PhD-Thesis, Yale University, May 1991
15. Weinberg, S.: Dreams of a final theory. New York: Pantheon Books, 1992

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