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Erratum

Sharp Estimates for Dirichlet Eigenfunctions in Horn-Shaped Regions

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Professors G. Verchota and A. Vogel have pointed out an error in parts (b) and (c) of Theorem 1 in [1]. To correct this, parts (b) and (c) should be replaced by (b′) and (c′) below. These changes do not affect the statement or the proof of part (a), which was the main result of the paper. We adopt the notation of [1] and assume throughout that $\theta(x)>0$, $\theta(x)\downarrow 0$.

(b′) Assume $\theta\in L^1$. Then for all $x>1$,

$$|\varphi_\lambda(x,0)|\leqq C_{\varphi_\lambda} e^{-\pi/2\int_1^x \frac{dt}{\theta(t)}}. \tag{1}$$

(c′) Assume $|\theta'(x)|\leqq m$ for all x. Then for $x>1$,

$$\varphi(x,0)\geqq c_m\varphi(1,0)\exp\left(\frac{-\pi}{2}\int_1^x \frac{dt}{\theta(t)}-\frac{\pi}{24}\int_1^x \frac{\theta'(t)^2}{\theta(t)}\,dt\right). \tag{2}$$

For the proof of (c′) replace Lemma 1.2 of [1] by Theorem 2.1 of Haliste, (reference 7 of [1]), which shows that the harmonic measure $\omega_{(x,0)}^{D_1}(I_1)$ is bounded below by c_m times the exponential appearing in (2), and continue exactly as in the proof of (c). We note that (c′) still proves the sharpness of (a) which was the main reason for (c). Indeed, since θ' is bounded, the Harnack inequality shows that there are constants c_1 and c_2 depending on m such that for $z=(x,y)$, $0<y<c_1\theta(x)$, $\varphi(x,y)\geqq c_2\varphi(x,0)$. This and (c′) with a θ for which the second factor in the exponential is bounded give examples where the area of D_θ is infinite and the left-hand side of (0.3) of [1] is infinite when $\varepsilon=0$, and where the area of D_θ is finite and the left-hand side of (0.3) is infinite for any $\varepsilon<0$.

For (b′), choose x_λ large enough and depending only on λ such that the lowest eigenvalue μ of the domain $D(\lambda)=\{z\in D_\theta: x>x_\lambda\}$ satisfies $\lambda<\mu$. Let $I_\lambda=\{z\in D_\theta: x=x_\lambda\}$. By Theorem 3.2 of reference 7 of [1], it suffices to show that for $x>x_\lambda$, $|\varphi_\lambda(x,0)|\leqq C_{\varphi_\lambda}\omega_{(x,0)}^{D(\lambda)}(I_\lambda)$. If we let B_t be Brownian motion in $D(\lambda)$ and

η its exit time from $D(\lambda)$, it follows from the Feynman–Kac formula that $\varphi_\lambda(x,0)=E_x(e^{\lambda\eta}\varphi_\lambda(B_\eta))$. Since φ_λ is zero on the boundary of $D(\lambda)$ except on I_λ, we have after conditioning, that

$$|\varphi_\lambda(x,0)| \leqq \sup_{\xi\in I_\lambda}|\varphi_\lambda(\xi)| \sup_{\substack{z\in D(\lambda)\\ \xi\in I_\lambda}} E_z^\xi(e^{\lambda\eta})\omega_{(x,0)}^{D(\lambda)}(I_\lambda)\,, \tag{3}$$

where E_x^ξ denotes expectation with respect to the Brownian motion starting at z and conditioned to exit $D(\lambda)$ at ξ. Since $\lambda<\mu$, and the area of $D(\lambda)$ is finite, it follows from Bañuelos and Davis [2], that the second sup in (3) is bounded by a constant depending on λ and this proves (b′).

References

1. Bañuelos, R., Davis, B.: Sharp estimates for Dirichlet Eigenfunctions in horn-shaped regions. Commun. Math. Phys. **150**, 209–215 (1992)
2. Bañuelos, R., Davis, B.: Heat kernel, eigenfunctions and conditioned Brownian motion in planar domains. J. Funct. Anal. **89**, 188–200 (1989)

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