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Finiteness of Ricci Flat $N=2$ Supersymmetric σ-Models

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Abstract. We study the ultraviolet behavior of two dimensional supersymmetric non-linear σ-models with target space an arbitrary Kähler manifold M, so that the models are $N=2$ supersymmetric. We point out that these models have an additional fermionic axial symmetry if and only if the metric on M is Ricci flat. We show that the preservation of this symmetry in perturbation theory implies that both bare and renormalized metrics on M are Ricci flat. Combining this result with the constraint of $N=2$ supersymmetry requiring that all counterterms to the metric beyond one-loop order be cohomologically trivial, we argue that $N=2$ models defined on Ricci flat Kähler manifolds are on-shell ultraviolet finite to all orders of perturbation theory.

This is the second of two papers on the renormalization of supersymmetric non-linear σ-models in two spacetime dimensions. The first paper [1] treated $N=4$ supersymmetric models, and showed that they are on-shell ultraviolet finite to all orders in perturbation theory. (This means that to all orders in perturbation theory, all ultraviolet divergences can be removed from all Greens functions by using only renormalization counterterms that arise from a (possibly non-linear) redefinition of the fields.) In this paper we extend this result to $N=2$ supersymmetric models, provided that the target manifold M on which the model is defined is Ricci flat. Since all $N=4$ supersymmetric non-linear σ-models necessarily have Ricci flat target manifolds [1] (and, of course, are $N=2$ supersymmetric), the result of this paper implies that of its predecessor.

We will make use here of results presented in [1] and in earlier literature without reiterating their proofs. The main technical details necessary involve the topological triviality of the counterterms on Kähler manifolds beyond one-loop, and the detailed structure of the tensor counterterms generated by performing perturbation theory using the background field expansion. The interested reader is referred to [1] for detailed explanations of these results and for references to the earlier literature.

The remainder of this paper gives the proof of our result. We first define the models under study, and show that renormalization preserves Ricci flatness. In other words, if one starts with a Ricci flat M, then, to all orders in perturbation theory, all ultraviolet divergences can be removed from all Greens functions without introducing counterterms that spoil Ricci flatness. We then use this property to demonstrate on-shell ultraviolet finiteness.

We begin with an m-dimensional riemannian manifold M with metric g_{ij}. For two dimensional spacetimes, an $N=1$ supersymmetric σ-model with scalar fields taking values on M is given by the lagrangian [2]

$$\mathscr{L} = \tfrac{1}{2} g_{ij}(\phi)\partial_\mu\phi^j\partial^\mu\phi^j + \frac{i}{2} g_{ij}\bar{\psi}^i\gamma^\mu D_\mu\psi^j + \tfrac{1}{12} R_{ijkl}\bar{\psi}^i\psi^k\bar{\psi}^j\psi^l,$$

$$D_\mu\psi^i = \partial_\mu\psi^i + \Gamma^i_{jk}\partial_\mu\phi^j\psi^k, \quad \gamma^0 = \sigma_2, \quad \gamma^1 = i\sigma_1, \tag{1}$$

where Γ^i_{jk} is the Christoffel connection and R_{ijkl} is the Riemann curvature tensor. The ϕ's are bosonic fields which represent geometrically a coordinate system on M, and the fermions ψ^i are two-component Majorana fermions which behave like vectors under coordinate reparametrizations. The supersymmetry transformation rules for (1) are given by

$$\delta\phi^i = \bar{\varepsilon}\psi^i, \quad \delta\psi^i = -i\partial\!\!\!/\phi^i\varepsilon - \Gamma^i_{jk}(\bar{\varepsilon}\psi^j)\psi^k \tag{2}$$

$N=2$ supersymmetry implies that M must be Kähler [3, 4]. This means that there exists a tensor $f^i{}_j$ (the complex structure) on M satisfying

$$f^i{}_k f^k{}_j = -\delta^i{}_j, \tag{3a}$$

$$g_{ij} f^i{}_k f^j{}_l = g_{kl}, \tag{3b}$$

$$\nabla_i f^j{}_k = 0. \tag{3c}$$

These equations imply that there exist complex coordinates ϕ^α and $\bar{\phi}^\alpha \equiv \phi^{\bar{\alpha}}$, where α and $\bar{\alpha}$ run from 1 to $m/2$, such that the only non-vanishing components of the metric are the mixed components $g_{\alpha\bar{\beta}}$. Furthermore, the metric must obey

$$\partial_\gamma g_{\alpha\bar{\beta}} = \partial_\alpha g_{\gamma\bar{\beta}}, \quad \partial_{\bar{\gamma}} g_{\alpha\bar{\beta}} = \partial_{\bar{\beta}} g_{\alpha\bar{\gamma}}. \tag{4}$$

The existence of these coordinates and these conditions on the metric can be used as an alternative definition of a Kähler manifold.

In any fixed coordinate patch on M, (4) implies that there exists a scalar function $K(\phi, \bar{\phi})$, called the Kähler potential, such that $g_{\alpha\bar{\beta}} = \partial_\alpha\partial_{\bar{\beta}}K$. In general, K is only defined patch by patch; it is impossible to choose the definitions in all the patches to fit together to make a single valued scalar function on all of M. We shall not need the Kähler potential for most of our analysis, but it will become important to us at the very end. Another useful property of Kähler metrics which we shall later need is that the Ricci tensor can be written in terms of the metric $g_{\mu\bar{\lambda}}$ as

$$R_{\alpha\bar{\beta}} = -\partial_\alpha\partial_{\bar{\beta}} \ln\det g. \tag{5}$$

In terms of the complex coordinates, (1) becomes

$$\mathscr{L} = g_{\alpha\bar{\beta}}\partial_\mu\phi^\alpha\partial^\mu\bar{\phi}^\beta + i g_{\alpha\bar{\beta}}\bar{\psi}^\alpha D\!\!\!\!/\,\psi^{\bar{\beta}} + \tfrac{1}{4} R_{\alpha\bar{\beta}\mu\bar{\lambda}}\bar{\psi}^\alpha\psi^\mu\bar{\psi}^{\bar{\beta}}\psi^{\bar{\lambda}}. \tag{6}$$

This has certain symmetries which become manifest if we write the fermions in Weyl form, as $\psi^\alpha_\pm$ and $\psi^{\bar{\alpha}}_\pm$, where $\pm$ denotes the eigenvalue of γ_5. The terms bilinear in the fermions take the form

$$\frac{i}{2} g_{\alpha\bar{\beta}}\psi^\alpha_+ \overleftrightarrow{D}_+ \psi^{\bar{\beta}}_+ + \frac{i}{2} g_{\alpha\bar{\beta}}\psi^\alpha_- \overleftrightarrow{D}_- \psi^{\bar{\beta}}{}_-, \tag{7}$$

where

$$D_{\pm}\psi^{\alpha}_{\pm}=\partial_{\pm}\psi^{\alpha}_{\pm}+\Gamma^{\alpha}_{\gamma\beta}\partial_{\pm}\phi^{\gamma}\psi^{\beta}_{\pm},$$
$$\partial_{\pm}=\frac{\partial}{\partial x^{\pm}},\quad x^{\pm}=\frac{1}{\sqrt{2}}(x^0\pm x^1),$$

and the quartic fermion term is proportional to

$$R_{\alpha\bar{\beta}\gamma\bar{\delta}}\psi^{\alpha}_{+}\psi^{\bar{\beta}}_{+}\psi^{\gamma}_{-}\psi^{\bar{\delta}}_{-}. \tag{8}$$

All this is invariant under the $U(1)_{+}\times U(1)_{-}$ transformations

$$\begin{aligned}\psi^{\alpha}_{+}&\to e^{i\eta}\psi^{\alpha}_{+}, & \psi^{\alpha}_{-}&\to e^{i\eta'}\psi^{\alpha}_{-},\\ \psi^{\bar{\alpha}}_{+}&\to e^{-i\eta}\psi^{\bar{\alpha}}_{+}, & \psi^{\bar{\alpha}}_{-}&\to e^{-i\eta'}\psi^{\bar{\alpha}}_{-},\end{aligned} \tag{9}$$

(with $\phi^{\alpha}\to\phi^{\alpha}$, $\bar{\phi}^{\alpha}\to\bar{\phi}^{\alpha}$). This symmetry group includes both vector transformations, $\eta=\eta'$, and axial transformations, $\eta=-\eta'$. These are on much the same footing in classical field theory. They play strikingly different roles in quantum field theory, however, and therefore we will consider them separately.

The vector transformations, together with $N=1$ supersymmetry, generate $N=2$ supersymmetry [3, 4]. Equivalently, they generate the full Kähler structure discussed above. In the quantum theory, there are supersymmetric regularizations of perturbation theory which do not break the vector symmetry. (There is, for example, superspace regularization by dimensional reduction, reviewed in [5], and regularization by higher covariant derivative regulators in superspace, recently reviewed in [6].)[1] Thus the Kähler structure of the theory is preserved by renormalization [7]. To be more explicit, let us consider constructing renormalized perturbation theory in the usual way: we begin with some initial lagrangian, and, as we encounter divergences in each order of perturbation theory, we change the lagrangian, adding counterterms to it to cancel the divergences. In the case at hand, standard power-counting shows that the only counterterms needed are of the same general form as the terms already present in the lagrangian: any number of scalar fields with two derivatives, any number of scalar fields with two spinor fields and one derivative, and any number of scalar fields with four spinor fields and no derivatives. Because our regularization procedure is assumed to preserve $N=2$ supersymmetry, the divergences in each order must be connected by the supersymmetry, and our counterterms can be chosen to preserve it. The net effect of renormalization is merely to replace the initial metric defining the theory (the "renormalized metric") by some other Kähler metric (the "bare metric"). There is nothing of course in our reasoning thus far that prevents the counterterms from being cutoff dependent, and the bare metric from being consequently divergent.

The situation is very different for the axial transformations, for here the symmetry can be spoiled by an anomaly. Computation of the anomaly is

1 Although we are concerned here only with the ultraviolet behavior of these theories, we point out that any infrared problems may be resolved by, for example, compactifying the space dimension and imposing antiperiodic boundary conditions, or by adding a holomorphic potential term to (6) to provide a small regulator mass to the particle content

straightforward, because we can consider $\Gamma^{\alpha}_{\gamma\beta}\partial_{\mu}\phi^{\gamma}$ as a (composite) gauge field, and, in two dimensions, the anomaly is proportional to the trace, i.e. the $U(1)$ part of the field strength tensor ($\partial_{\mu}j^{\mu}_{5} \sim \operatorname{tr} F$ for a two dimensional gauge theory). The relevant field strength here is related to the curvature tensor, which, on a Kähler manifold, is just $R^{\alpha}{}_{\beta\bar{\rho}\gamma} = \partial_{\bar{\rho}}\Gamma^{\alpha}_{\beta\gamma}$. Its trace, picking out the U(1) part of the holonomy group, turns out by the Kähler cyclic and Bianchi identities to be simply the Ricci tensor, $R^{\gamma}{}_{\gamma\alpha\bar{\beta}} = R_{\alpha\bar{\beta}}$. A one-loop calculation thus results in

$$\partial_{\mu}j^{\mu}_{5} = \frac{1}{\pi}\varepsilon^{\mu\nu}R_{\alpha\bar{\beta}}\partial_{\mu}\phi^{\alpha}\partial_{\nu}\bar{\phi}^{\beta}, \tag{10}$$

where j^{μ}_{5} is the current associated with the axial symmetry. Notice that in two dimensions the anomaly equation can be non-polynomial in the scalar fields because they have zero canonical dimension.

Thus the statement that the manifold is Ricci flat is the same as the statement that the axial current is conserved to one-loop. This observation is the key to our argument, but to exploit it to all orders of perturbation theory, we need an analog of the Adler–Bardeen theorem [8], the statement that (10) receives no corrections from higher orders of perturbation theory.

To prove the Adler–Bardeen theorem requires a regulator which maintains the axial symmetry while regulating all diagrams beyond one-loop. The most convenient regularization for supersymmetric calculations is superspace regularization by dimensional reduction [5]. This scheme is defined by keeping spacetime indices in two dimensions whereas momentum integrals are done in $(2-\varepsilon)$-dimensions. It thus respects axial invariance at the classical level, since spinor algebra is done in two dimensions, but allows the existence of ε-dimensional operators that mix with the classical current. At the quantum level it is this mixing, which occurs only at the one-loop level, that breaks the conservation of the γ_5 current [9]. We do not wish to enter in detail into possible ambiguities in this procedure which may appear in high loop orders. As emphasized in [1], all of our results are dependent in any event on the assumption that there exists some consistent supersymmetric regulator. We point out, though, that any possible difficulties in the two dimensional non-gauge theories we consider here are much less severe than those which may appear in four dimensional gauge theories. In addition, many of the subtleties addressed in [9] concerning the difference between the supersymmetric axial current (the one in a supersymmetry multiplet with the energy-momentum tensor) and the Adler–Bardeen axial current disappear here, precisely because we are interested in the case when the anomaly vanishes. Indeed, although we will not use it explicitly here, we expect that the vanishing of the β-function which we will prove for these theories when the Ricci tensor vanishes and the vanishing of the axial anomaly are tied together neatly in the supermultiplet containing the trace and conformal anomalies.

We mention that we could also prove the Adler–Bardeen theorem by using a higher covariant derivative regulator [6]. Such a regulator regulates all diagrams beyond one-loop because of their negative superficial degree of divergence. Moreover on a Ricci flat manifold, the one-loop divergence vanishes and thus cannot enter as a subdivergence in higher order diagrams. We recall [2] that the superspace form of the action (1) is

$$\mathscr{L}=\frac{1}{4i}\int d^2\theta g_{ij}(\Phi)\bar{D}\Phi^i D\Phi^j,$$
$$D_\alpha \Phi^i=\left(\frac{\partial}{\partial\bar{\theta}^\alpha}-i(\gamma^\mu\theta)_\alpha\frac{\partial}{\partial x^\mu}\right)\Phi^i, \tag{11}$$

where $\Phi^i=\phi^i+\bar{\theta}\psi^i+\frac{1}{2}\bar{\theta}\theta F^i$ is a 2-dimensional real superfield and θ is a real two-component constant Majorana spinor. A higher covariant derivative regulator can be introduced into (11) via the substitution $g_{ij}(\Phi)\bar{D}\Phi^i D\Phi^j \to g_{ij}(\Phi)\bar{D}\Phi^i_\Lambda D\Phi^j_\Lambda$. Here we have defined the bare superfield $\Phi^i_\Lambda=f(\hat{\bar{D}}\hat{D}/\Lambda)\Phi^i$, with f some regulator function satisfying $f(x/\Lambda)\to 1$ as $\Lambda\to\infty$. $\hat{D}$ is a supercovariant derivative defined to act on the superspace coordinate Φ^i by $\hat{D}\Phi^i=D\Phi^i$, and to act on a vector W^i by $\hat{D}W^i=DW^i+\Gamma^i_{jk}D\Phi^j W^k$. This definition ensures the manifest coordinate reparametrization invariance of the regulator. Although this regularization prescription is not practical for doing computations, it does show that if the anomaly does not appear at the one-loop level, it will not appear in any order of perturbation theory. Our ultimate results, of course, are independent of the choice of regulator.

After these preliminaries, we are now in a position to give our proof. Because our regulator preserves the vector symmetry, we know by standard arguments that the canonical vector current

$$j^\mu=g_{\alpha\beta}\bar{\psi}^\alpha\gamma^\mu\psi^\beta \tag{12}$$

will have finite matrix elements which satisfy the vector Ward identities. The current is to be understood as built up as a sum in terms of the renormalized metric plus its counterterms. We shall denote by $g^{(n)}_{\alpha\beta}$ and $j^{(n)}_\mu$ the metric and current up to and including the n^{th} order counterterms. Now in two dimensions the canonical axial-vector current is given naively in terms of the canonical vector current by

$$j^\mu_5=\varepsilon^{\mu\nu}j_\nu. \tag{13}$$

This remains true in the presence of the supersymmetric dimensional regulator, so (13) allows us to determine the canonical axial-vector current from the vector current. Since (13) is a purely algebraic relation, it is trivial that the axial-current matrix elements are also finite. (This step would have been much more difficult if we had used higher covariant derivative regularization, for which (13) holds only up to terms which vanish in the limit as the regulator is removed.)

The Adler–Bardeen theorem states that

$$\partial_\mu j^{\mu(n)}_5=\frac{1}{\pi}\varepsilon^{\mu\nu}R^{(n-1)}_{\alpha\bar{\beta}}\partial_\mu\phi^\alpha\partial_\nu\bar{\phi}^\beta. \tag{14}$$

(This is of course a shorthand notation for expressing the effect of an operator insertion in arbitrary Green's functions.) The n^{th} order current $j^{\mu(n)}_5$ is related to the $(n-1)^{st}$ order Ricci tensor $R^{(n-1)}_{\alpha\bar{\beta}}$ because of an implicit factor of the loop counting parameter $\hbar$ on the right-hand side. We now wish to show that when the renormalized metric is Ricci flat, no counterterms to the metric need be added which induce a non-vanishing Ricci tensor. We shall proceed inductively, by assuming that to $(n-1)^{st}$ order the theory has been renormalized with the $(n-1)^{st}$ order metric, $g^{(n-1)}_{\alpha\bar{\beta}}$, Ricci flat, and show that this implies the same can be made true to n^{th} order. We know by renormalizability that the theory in n^{th} order can be made to have finite Greens functions by addition of a suitable counterterm $\delta g^{(n)}_{\alpha\bar{\beta}}$ to the metric. The question is whether the induced n^{th} order Ricci tensor will then continue to vanish. To assess this requires an additional step forward to $(n+1)^{st}$ order. Renormalizability of the theory insures that it can to this order also be made to have only finite matrix elements (with the $(n+1)^{st}$ order counterterms depending on the n^{th} order ones). But it now follows from (14) that the divergence of the $(n+1)^{st}$ order axial current is given by

$$\partial_\mu j_5^{\mu(n+1)} = \frac{1}{\pi}\varepsilon^{\mu\nu} R^{(n)}_{\alpha\bar{\beta}} \partial_\mu \phi^\alpha \partial_\nu \bar{\phi}^\beta, \tag{15}$$

where $R^{(n)}_{\alpha\bar{\beta}}$ is the Ricci tensor calculated from the n^{th} order metric. But since the axial current is finite to $(n+1)^{st}$ order, the right-hand side of (15) must be finite. (Any divergences due to the compositeness of the operator on the right-hand side involve additional bosonic loops and so only appear in higher order.) This shows that any non-Ricci flat part of the metric is at worst a finite piece which can be removed from n^{th} order by a finite metric renormalization. (This is easily seen in perturbation theory, where the Ricci tensor $R^{(n)}_{\alpha\bar{\beta}} = -\partial_\alpha \partial_{\bar{\beta}} \ln \det g^{(n)}$ induced by the n^{th} order metric $g^{(n)} = g^{(n-1)} + \delta g^{(n)}$ reduces, to linear order in $\delta g^{(n)}$, to $R^{(n)}_{\alpha\bar{\beta}} = -\partial_\alpha \partial_{\bar{\beta}} (g^{\mu\bar{\lambda}}_{(n-1)} \delta g^{(n)}_{\mu\bar{\lambda}})$ by assumption of Ricci flatness of $g^{(n-1)}$. $\delta g^{(n)}$ can then be decomposed into a part which maintains the vanishing of the Ricci tensor and a part which does not, but which is necessarily finite (as the regulator is removed) and so can be eliminated by a finite renormalization of the metric. This part of the argument is especially simple if we adopt minimal subtraction, fixing our counterterms to be sums only of negative powers of ε. In this case, the finiteness of the axial divergence to $(n+1)^{st}$ order directly implies that there are no metric counterterms of the undesired form in n^{th} order, and there is no need to go back to make a further finite renormalization.) This completes our inductive step; we have shown that the theory to n^{th} order can be renormalized, possibly of course by divergent counterterms, while still retaining the Ricci flatness of the n^{th} order metric.

To our knowledge the peculiar structure of this induction (two steps forward, one step back), is unique in renormalization theory; we have used a Ward identity in $(n+1)^{st}$ order to prove that a counterterm can be properly chosen in n^{th} order. This stutter-step induction is a natural consequence of the fact that the breaking of the axial symmetry is itself a quantum loop effect.

The remainder of the proof of on-shell ultraviolet finiteness for $N=2$ supersymmetric σ-models defined on Ricci flat Kähler manifolds follows the discussion of $N=4$ theories in [1]. Rather than repeating directly the argument there, which made use of a uniqueness result for the Ricci flat metric on a Kähler manifold due to Calabi and Yau [10], we give here instead for completeness a straightforward perturbative argument. This argument suffices for our purposes since we do not need the far more difficulty proven existence result of [10]. We are instead here given a manifold admitting a Ricci flat metric (and hence a theory which is one-loop finite), and need only the result that for a given complex structure on the manifold, the Ricci flat metric is unique within a given cohomology class of the Kähler form. (Recall that preservation of supersymmetry means that perturbation theory acts only to renormalize the metric on the manifold, not the intrinsic complex structure [4].)

We organize our counterterms by computing the effective action (evaluated to n^{th} order) at a solution of the classical equations of motion (computed from the $(n-1)^{st}$ order action). There are two advantages to this procedure: (1) Because we are at a stationary point of the $(n-1)^{st}$ order action, counterterms arising from the change in the action induced by a change in coordinates (and only such counterterms) will disappear. (2) If we compute the effective action by the background field method, every order of perturbation theory is manifestly invariant under general coordinate transformations, and therefore all counterterms needed to cancel divergences in n^{th} order must be polynomials in the curvature tensor and its covariant derivatives. It can be shown [1] that this implies that the n^{th} order addition to the Kähler potential is a globally well-defined scalar function on M. (This result also follows immediately [11] from a manifest $N = 2$ formulation of the theory, in which the corrections to the metric are calculated directly in terms of their corrections to the Kähler potential. The only cohomologically non-trivial counterterm generated in any order of perturbation theory is the Ricci tensor $R_{\alpha\bar{\beta}}$ generated in one-loop, and this vanishes here by assumption.) Thus

$$\delta g^{(n)}_{\alpha\bar{\beta}} = \partial_\alpha \partial_{\bar{\beta}} S^{(n)}, \tag{16}$$

for some scalar field $S^{(n)}$, modulo terms arising from coordinate redefinitions. Because Ricci flatness is preserved, the n^{th} order correction to the Ricci tensor (5) must vanish. For our purposes, it suffices to compute only the change in the scalar curvature.

$$0 = \delta R^{(n)} = -\tfrac{1}{2} \square \square S^{(n)}, \tag{17}$$

where $\square$ is the usual Laplace operator $g^{ij}\nabla_i \nabla_j$ acting on scalar functions.

If M is compact, this implies that $S^{(n)}$ is constant and thus that $\delta g^{(n)}_{\alpha\bar{\beta}}$ vanishes. (*Proof.* Multiply by $S^{(n)}$ and integrate by parts over the whole manifold. This shows that $\square S^{(n)}$ vanishes. Now multiply again by $S^{(n)}$ and integrate once more, showing that $dS^{(n)}$ vanishes.) Thus, for compact M, there can be no divergent counterterms to any order of perturbation theory. Although our argument has been formulated for compact Ricci flat Kähler manifolds, we may now appeal to a certain universality property [7] to extend the result to arbitrary Ricci flat Kähler manifolds. Because all the counterterms calculated in the background field normal coordinate expansion are polynomials in the curvature and its derivatives, they make no reference to the global structure of the target manifold; a divergent counterterm which would spoil on-shell finiteness for a noncompact manifold would spoil it as well for a compact one, where we know it does not. This completes the argument.

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