Absence of Symmetry Breaking for N-Vector Spin Glass Models in Two Dimensions

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Abstract. We prove the absence of continuous symmetry breaking at arbitrary temperatures for two-dimensional N-vector spin glass models with Hamilton function

$$H = - \sum_{i,j} J(i,j)|i-j|^{-2-\varepsilon} \mathbf{S}_i \cdot \mathbf{S}_j, \quad \varepsilon > 0,$$

where $J(i,j)$ has mean 0 and variance 1, for all i,j. We comment on the role of boundary conditions in spin glasses and on their critical behaviour in high dimensions.

1. Introduction

Introducing random signs into long-range interactions, such as exchange couplings between magnetic ions, tends to shorten their effective range (competing interactions may cancel each other statistically). Examples for this phenomenon are the existence of the free energy in magnetic systems with interactions of mean zero which are square summable, but not absolutely summable [1–3] and the absence of phase transitions in low-dimensional systems with interactions of moderately long range [4–8].

Consider, for example, a Hamilton function of the form

$$H = - \sum_{i,j} J(i,j)|i-j|^{-\alpha d} \mathbf{S}_i \cdot \mathbf{S}_j, \tag{1}$$

where the couplings $J(i,j)$ are independent, identically distributed random variables with mean 0 and variance one, and the $\mathbf{S}_j$'s may be Ising- or N-vector spins. Then the thermodynamic limit of the free energy of this system exists if $\alpha > \frac{1}{2}$. This is in contrast to the condition $\alpha > 1$ required in magnets with deterministic, e.g. purely ferromagnetic, exchange couplings. Furthermore, in one dimension there is no phase transition if $\alpha > 1$, in the sense that the spin flip symmetry of the Hamiltonian remains unbroken at all temperatures [8]. We comment on this result below (Sect. 3). For a stronger notion of absence of transitions which

requires more stringent hypotheses see [4, 5]. We recall that the one-dimensional Ising ferromagnet exhibits a transition for $1 < \alpha \leq 2$ [9,10], but has no transitions, in the strongest possible sense (analyticity of the free energy at all temperatures) for $\alpha > 2$ [11].

For models of N-vector spins in two dimensions (with deterministic exchange couplings), it is known that the continuous ($O(N)$-)symmetry is unbroken at all temperatures if $\alpha \geq 2$, but the models with ferromagnetic couplings exhibit a transition accompanied by spontaneous symmetry breaking if $1 < \alpha < 2$, [13]. For two-dimensional models with continuous symmetry and random exchange couplings of mean 0, Picco [6, 7] has applied the probabilistic energy estimates of [1, 4], combined with the use of relative entropy as in [12], to prove the absence of symmetry breaking at all temperatures, for $\alpha \geq \frac{3}{2}$.

As in the one-dimensional models, one may actually improve this condition to $\alpha \geq 1$ by estimating the relative entropy directly. This is proven in the following section. Our paper is basically just a comment on references [6] and [8] and is based on ideas already appearing in [8] and [12]. The main point we wish to make is that it is possible to provide a simple proof for the absence of continuous symmetry breaking in two dimensions, even if the couplings are random and of very long range, by a straightforward, but careful use of sign cancellations in the exchange interactions.

In Sect. 3 we comment on the notions of "absence of phase transitions" and "boundary conditions" in random magnets and speculate on possible transitions in spin glass models with interactions of very long range.

2. Proof of the Main Result

For simplicity we discuss the XY spin glass, i.e. we consider two-component rotator spins. The extension to general N-vector models will turn out to be obvious. In our proof we shall rely heavily on [12, 14] for many of the detailed estimates.

The Hamilton function of the XY spin glass is, formally, given by

$$H = - \sum_{i,j} \tilde{J}(i,j) \cos(\theta_i - \theta_j), \tag{2}$$

where

$$\tilde{J}(i,j) = J(i,j)|i-j|^{-2\alpha}. \tag{3}$$

Theorem. *For $\alpha \geq 1$, the $O(2)$-symmetry of the XY spin glass introduced in (2) and (3) remains unbroken at all temperatures, provided the expectation of $J(i,j)$ vanishes and the support of the distribution of $J(i,j)$ is bounded.*

Remark. At the end of this section we show how to include couplings, $J(i,j)$, with Gaussian distribution, or other distributions of unbounded support.

Proof. Choose an arbitrary extremal Gibbs state ω of the XY spin glass, and consider the relative entropy, $S(\omega|\omega_l)$, of a state ω_l obtained from ω by rotating all the spins inside a disk, D_l, of radius l through an angle $\hat{\theta}$ and rotating the spins in a large annulus, A_L, of inner radius l and outer radius L by an angle that interpolates linearly between $\hat{\theta}$ and 0 as the site varies from the inner to the outer boundary of A_L. For details concerning this construction see [14, 12].

We denote by R_i the rotation of the spin at site i and set $R_{ij} = R_i R_j^{-1}$. Let $\gamma_{ij} = \frac{1}{2} \operatorname{tr} R_{ij}$, i.e. γ_{ij} is the cosine of the difference of the rotation angles at i and at j. We note that $\gamma_{ij} = 1$ if i and j are contained in D_l, or if i and j are contained in the complement of $D_l \cup A_L$. Moreover,

$$0 \leq 1 - \gamma_{ij} \leq \min(2, \operatorname{const} |i-j|^2 (L-l)^{-2})$$

if i or j are in A_L. For more details see [14].

Using the duplicate system trick of [12] we see that the relative entropy is bounded by

$$0 \leq S(\omega|\omega_l) \leq \bar{S}(\omega|\omega_l),$$

where

$$\bar{S}(\omega|\omega_l) = 2\beta \cdot \omega \left(\sum_{i,j} (1-\gamma_{ij}) \tilde{J}(i,j) \cos(\theta_i - \theta_j) \right). \tag{5}$$

From this we conclude, using standard measure-theoretic arguments, that it is enough to prove estimates on the expectation value of $\bar{S}(\omega|\omega_l)$ which are *uniform* in l, provided $L \gg l$ is chosen appropriately. For, it clearly follows from such estimates that for almost every configuration $\{J(i,j)\}$ there exists a sequence $\{l_n\}_{n=1}^\infty$ diverging to $+\infty$ such that

$$\bar{S}(\omega|\omega_{l_n}) \leq C, \tag{6}$$

for some finite constant C depending on $\{J(i,j)\}$. Hence, for almost all $\{J(i,j)\}$, $S(\omega|\omega_{l_n})$ is bounded uniformly in n, and this implies that the state ω_∞ obtained from ω by rotating *all* spins through an angle $\hat{\theta}$ is absolutely continuous with respect to ω (see [12, 15]).

Let $\mathbb{E}$ denote the expectation over $\{J(i,j)\}$. In order to prove a uniform bound on

$$\mathbb{E} \bar{S}(\omega|\omega_l),$$

it is enough to estimate the contribution to $\bar{S}(\omega|\omega_l)$ of the tail interactions with $|i-j| > R$, for some conveniently chosen, sufficiently large constant R. The contribution of the short range interactions, corresponding to $|i-j| \leq R$, can be estimated as in [12, 14] [since the distribution of $J(i, j)$ has bounded support]. Put differently, for every value of β and every $\varepsilon > 0$, we may choose R so large that

$$\sup_{|i-j|>R} \beta |\tilde{J}(i,j)| < \varepsilon. \tag{7}$$

Without loss of generality we may require (7) for arbitrary i, j, assuming we have already dealt with the short range interactions; ($|i-j| \leq R$). We now estimate

$$\mathbb{E}\bar{S}(\omega|\omega_l) = 2\beta \mathbb{E} \sum_{i,j} (1-\gamma_{ij}) \omega(\tilde{J}(i,j) \cos(\theta_i - \theta_j))$$

$$= 2\beta \sum_{i,j} (1-\gamma_{ij}) \mathbb{E}[\tilde{J}(i,j) \omega(\cos(\theta_i - \theta_j))]. \tag{8}$$

We denote by ω_{ij} the state obtained from ω by deleting the term $\tilde{J}(i,j) \cos(\theta_i - \theta_j)$ from the Hamiltonian. Then

$$\mathbb{E}[\tilde{J}(i,j)\omega(\cos(\theta_i - \theta_j))] = \mathbb{E}\left[\tilde{J}(i,j) \frac{\omega_{ij}(\cos(\theta_i - \theta_j) \exp\{\beta \tilde{J}(i,j) \cos(\theta_i - \theta_j)\})}{\omega_{ij}(\exp\{\beta \tilde{J}(i,j) \cos(\theta_i - \theta_j)\})} \right]. \tag{9}$$

(For comparison, note that in [8] infinitely many terms $\tilde{J}(i,j)\sigma_i\sigma_j$ were removed from the Hamiltonian H at once, a procedure which does not appear to work in the present situation.)

Next, we expand the exponentials on the right side of (9) in $\tilde{J}(i,j)$, making use of (7). This yields

$$\mathbb{E}[\tilde{J}(i,j)\omega(\cos(\theta_i-\theta_j))] = \mathbb{E}\left[\tilde{J}(i,j)\omega_{ij}\left(\cos(\theta_i-\theta_j)\sum_{n=0}^{\infty}\frac{(\beta\tilde{J}(i,j))^n\cos(\theta_i-\theta_j)^n}{n!}\right)\right.$$
$$\left.\cdot\sum_{m=0}^{\infty}(-1)^m\left\{\omega_{ij}\left(\sum_{k=1}^{\infty}\frac{(\beta\tilde{J}(i,j))^k\cos(\theta_i-\theta_j)^k}{k!}\right)\right\}^m\right]. \quad (10)$$

Choosing ε so small that $e^\varepsilon-1<1$ we observe that all series on the right side of (10) are absolutely convergent, and that the right side of (10) can be estimated by

$$|\mathbb{E}[c_1\tilde{J}(i,j)+c_2\beta\tilde{J}(i,j)^2+O(\beta^2|\tilde{J}(i,j)|^3)]|,$$

with $c_1\leq 1$, $|c_2|\leq 2$. Hence, using (8)–(10) and the fact that $\mathbb{E}\tilde{J}(i,j)=0$, we get

$$\mathbb{E}\bar{S}(\omega|\omega_l)\leq\beta^2\sum_{i,j}(1-\gamma_{ij})\,\text{const}\,\mathbb{E}\tilde{J}(i,j)^2\leq\text{const}\,\beta^2\sum_{i,j}(1-\gamma_{ij})|i-j|^{-4\alpha}. \quad (11)$$

At this point we can use the estimates in [14] to prove convergence of the right side of (11) for $\alpha\geq 1$. $\square$

Next, we sketch how to extend the result just proven to a class of models with random exchange couplings $\tilde{J}(i,j)$ that are not necessarily bounded. We suppose that

$$\tilde{J}(i,j)=J(i,j)|i-j|^{-(2+\varepsilon)}, \quad (12)$$

for some $\varepsilon>0$, where the $J(i,j)$'s are i.i.d. random variables with distribution $d\varrho(J(i,j))$ which we require to have the following properties.

1) $d\varrho$ is even in $J(i,j)$.
2) $\mathbb{E}J(i,j)^2\equiv\int J(i,j)^2d\varrho(J(i,j))=1$.
3) Let B_{ij}^δ denote the characteristic function of the set $\{J(i,j):|J(i,j)|\geq|i-j|^\delta\}$, and let G_{ij}^δ be the characteristic function of the complement. We require that

$$\text{Prob}\,B_{ij}^\delta\equiv\mathbb{E}B_{ij}^\delta\leq\text{const}|i-j|^{-4}, \quad (13)$$

for some $\delta\in(0,\varepsilon)$.

These conditions obviously hold if $d\varrho$ is the Gaussian with mean 0 and variance 1. In this case

$$\text{Prob}\,B_{ij}^\delta\sim\exp(-\tfrac{1}{2}|i-j|^{2\delta}).$$

We now estimate the term in $\mathbb{E}\bar{S}(\omega|\omega_l)$ indexed by the pair i,j [see (8)].

$$\mathbb{E}[\tilde{J}(i,j)\omega(\cos(\theta_i-\theta_j))] = \mathbb{E}[G_{ij}^\delta\tilde{J}(i,j)\omega(\cos(\theta_i-\theta_j))]$$
$$+\mathbb{E}[B_{ij}^\delta\tilde{J}(i,j)\omega(\cos(\theta_i-\theta_j))]. \quad (14)$$

Since $\delta<\varepsilon$, the first term on the right side of (14) can be analyzed as above. In order to bound the second term we first note that $|\omega(\cos(\theta_i-\theta_j))|\leq 1$, and then use the Cauchy-Schwarz inequality which yields

$$|\mathbb{E}[B_{ij}^\delta\tilde{J}(i,j)\omega(\cos(\theta_i-\theta_j))]|\leq(\mathbb{E}\tilde{J}(i,j)^2)^{1/2}(\mathbb{E}B_{ij}^\delta)^{1/2}\leq\text{const}|i-j|^{-(4+\varepsilon)},$$

by (12) and (13). Hence we may again refer to the estimates in [14] which imply that

$$\sum_{i,j}(1-\gamma_{ij})|i-j|^{-(4+\varepsilon)}\leqq\text{const},$$

uniformly in l, if L is chosen sufficiently large, depending on l.

Remarks. 1) For general N-vector spin glasses or models with a more general, nonabelian, connected symmetry group, G, the same results hold. To see this it suffices to choose one-parameter subgroups of G to which one can apply the above arguments verbatim.

    2) The methods and results of [8] and of the present paper extend to analogous spin glass models with quantum mechanical spins in a straightforward manner. The basic ideas of the proofs remain unchanged, but one must use the quantum mechanical notion of relative entropy and, in (9), Araki's Gibbs condition to express ω in terms of ω_{ij}. For details see [15], and for similar uses of Araki's basic results see also [12].

3. Transitions and Absence of Transitions in Spin Glasses

We wish to start this section with a comment on one-dimensional spin glasses, more specifically on the uniqueness of the Gibbs state of such systems. The Hamiltonian of the system we propose to consider is given by

$$H=-\sum_{i,j}\tilde{J}(i,j)\sigma_j\sigma_j,\tag{15}$$

where

$$\tilde{J}(i,j)=J(i,j)|i-j|^{-\alpha},\quad\alpha>1,\tag{16}$$

and $\mathbb{E}J(i,j)=0$, $\mathbb{E}J(i,j)^2=1$, for all i,j. For definiteness we suppose that the spins, σ_i, are bounded random variables, but our arguments can be extended to more general models, in particular to quantum mechanical spin glasses. Furthermore, we assume that $J(i,j)$ is bounded, but the general case may be disposed of as explained in Sect. 2.

    We now consider a system, with the above features, confined to the region

$$\Lambda=\{j\in\mathbb{Z}:-l\leqq j\leqq l\},$$

for some $l=1,2,3,\ldots$. The spins outside Λ are distributed by some boundary conditions (b.c.), i.e. according to some probability measure $db(\sigma|_{\Lambda^c})$. For a fixed configuration $\sigma|_{\Lambda^c}$ the interaction, $W(\sigma|_{\Lambda^c})$, of the spins in Λ with $\sigma|_{\Lambda^c}$ is given by

$$W(\sigma|_{\Lambda^c},\sigma|_{\Lambda})=-\sum_{\substack{i\in\Lambda\\j\in\Lambda^c}}\tilde{J}(i,j)\sigma_i\sigma_j.\tag{17}$$

The Gibbs state, $\omega_{l,\varrho}$, of the system in Λ with b.c. b at inverse temperature β is given by the measure

$$Z_{\beta,b}^{-1}\exp[-\beta(H(\sigma|_{\Lambda})+W(\sigma|_{\Lambda^c},\sigma|_{\Lambda}))]\prod_{j\in\Lambda}d\lambda(\sigma_j)db(\sigma|_{\Lambda^c}),\tag{18}$$

where $d\lambda$ is the a priori distribution of the spin.

    We now propose to show that the expectation value (over $\{\tilde{J}(i,j)\}$) of the relative entropy of two states $\omega_{l,b}$ and $\omega_{l,b'}$ remains *bounded, uniformly in Λ*, if db

and db' are measures which do *not* depend on the exchange couplings $\{\tilde{J}(i,j)\}$. Clearly

$$0 \leq \mathbb{E}S(\omega_{l,b}|\omega_{l,b'})$$
$$= \mathbb{E}\omega_{l,b}(\log\{Z_{\beta,b}^{-1}\int e^{-\beta W}db/Z_{\beta,b'}^{-1}\int e^{-\beta W}db'\}). \tag{19}$$

Let $\omega_{l,0}$ be the state for which $W \equiv W(\sigma|_{A^c})$ is set equal to zero. Then (19) yields

$$\mathbb{E}\dot{S}(\omega_{l,b}|\omega_{l,b'}) = \mathbb{E}(I+II)\{\int \omega_{l,0}(e^{-\beta W})db\}^{-1} \leq \mathbb{E}(I+II)\exp\{\beta\int\omega_{l,0}(W)db\},$$

by Jensen's inequality. Here
$$\tag{20}$$

$$I \equiv \int \omega_{l,0}(e^{-\beta W}\log\{\int e^{-\beta W}db/\int e^{-\beta W}db'\})db,$$

and

$$II \equiv \int \omega_{l,0}(e^{-\beta W})db \log\{Z_{\beta,b'}/Z_{\beta,b}\},$$

We estimate I by using

$$\log\{e^{-\beta W}db/\int e^{-\beta W}db'\} \leq \int(e^{-\beta W}-1)db + \beta\int Wdb'. \tag{21}$$

(Estimates on II may be obtained in a very similar way.) Let db'' denote either db or db'. If (21) is inserted into (20) and, subsequently, all exponentials are expanded into Taylor series, one obtains an upper bound on $\mathbb{E}S(\omega_{l,b}|\omega_{l,b'})$ in terms of a series over terms of the form

$$\left|\mathbb{E}\left[\omega_{l,0}\left(\frac{1}{n!}\int(-\beta W)^n db \cdot \frac{1}{m!}\int(-\beta W)^m db''\right)\frac{1}{k!}(\beta\omega_{l,0}(\int Wdb))^k\right]\right| \tag{22}$$

with $m \geq 1$. Under our hypotheses on $\tilde{J}(i,j)$, [(16) and boundedness of $J(i,j)$], one verifies easily that the sum over n, m and k of (22) converges for arbitrary finite β. This proves that the expectation of the relative entropy is bounded uniformly in l and in db, db'. *As a consequence the thermodynamic limits of all equilibrium states constructed with $\{J(i,j)\}$-independent b.c. are absolutely continuous with respect to each other.*[1]

While it is possible to have db, db' depend on $\{J(i,j)\}$ in suitable ways without invalidating the above conclusion, *some* restrictions on that dependence are necessary for our arguments to go through, since

$$\sup_{\sigma|_{A^c}} W(\sigma|_{A^c}, \sigma|_A) \to \infty, \quad \text{as} \quad l \to \infty,$$

$\{J(i,j)\}$-almost surely, for $\alpha < 3/2$ [8].

One may then ask whether the above result is satisfactory, since we have not ruled out that one may find further equilibrium states by choosing *sample-dependent*, (i.e. $\{J(i,j)\}$-dependent) b.c. As an answer to this question we argue that b.c. in statistical mechanics represent an idealized description of part of the experimental set-up that serves to measure properties of a statistical system in thermal equilibrium. But the experimental set-up is usually independent, statistically, of the sample on which the experiment is done. Hence, in our example, one would find the same Gibbs state, as the thermodynamic limit is approached, in almost every experiment, i.e. there are almost surely no transitions in those one-

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1   This general method of proving uniqueness was introduced by Araki [15]

dimensional spin glasses that can be observed experimentally. The relevant question then is whether the state constructed by means of sample-independent b.c. is *extremal*, or whether, for large β, it may exhibit some kind of *long range order* or divergent relaxation times. The results of ref. [8] show that if the state were not extremal that would not have anything to do with spontaneous breaking of the $\sigma \to -\sigma$ symmetry. Moreover, for $\alpha > \frac{3}{2}$, the unique state is extremal [4, 5] (as can be shown by improved relative entropy arguments), but that remains open for $1 < \alpha < \frac{3}{2}$.

As argued by Kotliar et al. [16] one *does* expect transitions if the interactions are of very long range, in the sense that $1/2 < \alpha < 1$.

More challenging problems concerning the absence of transitions in spin glass models are encountered in two or more dimensions. It is a reasonable conjecture that the Ising spin glass with nearest-neighbor interactions, $J(i,j)$, with $\mathbb{E}J(i,j) = 0$, $\mathbb{E}J(i,j)^2 = 1$, does not exhibit any equilibrium transitions at arbitrary temperatures and zero magnetic field, in dimension 2. (In dimension 3, recent numerical experiments suggest a transition [20].) An easier problem would be to prove that a spin glass with Hamiltonian

$$H = -\sum_{i,j} \tilde{J}(i,j)\sigma_i\sigma_j, \qquad \tilde{J}(i,j) = J(i,j)|i-j|^{-\alpha d},$$

$$\mathbb{E}J(i,j) = 0, \qquad \mathbb{E}J(i,j)^2 = 1, \qquad J(i,j) \text{ bounded i.i.d.}$$

random variables, has a high-temperature phase with unique Gibbs state and correlations which have cluster properties, provided $\alpha > \frac{1}{2}$. This might follow from improved high-temperature expansions or Dobrushin-type uniqueness theorems, but the details have not been worked out, yet.

There do not appear to exist any mathematically rigorous results on the question of whether there are equilibrium phase transitions in spin glass models and what the main features of such transitions would be. We wish to suggest that one might try to extend Israel's general theory [17] to duplicate systems of spin glasses with random interactions which could permit one to show that, for $\alpha = \frac{1}{2}$, there are distributions of exchange interactions such that the corresponding spin glass model has long range order at low temperatures. (We have, however, no precise results in this direction.) Another, more concrete line of attack would consist in analyzing the behaviour of spin glasses in high dimensions. In the formal $d \to \infty$ limit, the spin-spin correlation is given by

$$\omega(\sigma_x\sigma_y) = a(-\beta J + b)^{-1}_{xy}, \quad \text{for small} \quad \beta, \tag{23}$$

where J is the operator on $l_2(\mathbb{Z}^d)$ with matrix elements equal to the exchange couplings, $\tilde{J}(i,j)$, and a and b are constants depending on the a priori distribution of σ_j; $\beta b^{-1} \sim d^{-1}$. Thus, as $d \to \infty$ and for small β, $\omega(\sigma_x\sigma_y)$ formally approaches a limit proportional to the Green function of the tight binding Hamiltonian, $h = -\beta J$, with off-diagonal disorder. One may expect, therefore, that, in high dimensions and for small β, the spin glass- and the localization problem are related. We argue that the Griffiths singularities [18] in β of the spin glass models are related to the low-energy regime near the lower band edge of the tight binding model, where the spectrum of βJ is pure point, and $(-\beta J + b + i0)^{-1}_{xy}$ still exhibits exponential decay in $|x-y|$ with probability 1, [19]. Furthermore, the lower mobility edge of the tight binding Hamiltonian might be related to some transition encountered in a high-dimensional spin glass, as β is increased. Since the

localization length in the tight binding model appears to tend to ∞ continuously at the mobility edge (with an exponent $v = \frac{1}{2}$, for large d), some transitions in a high-dimensional spin glass, as β is increased, may be expected to be continuous, as well. However, Eq. (23) is actually valid at best for small β, (where one has real analyticity in β), so the above conjectures must be taken with some caution.

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