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The Obsessions of Time

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To Rudolf Haag

When I was in high school, a favorite pastime during lectures was to detect obscene meanings in the phrases of the teacher. What we lost of latin grammar or spherical trigonometry, we gained in psychological insight. During the seminars which I hear today I again sometimes find that my mind wanders away from the manifest subject of the talk; I listen instead to the words which are stressed, to those which are mispronounced, and the obsessions of the speaker come out with compelling evidence[1]. In some cases of course, the speaker is aware of the second meaning of his words. There is also the very serious danger, in trying to guess someone else's unconscious thoughts, that one simply reads off one's own obsessions. Altogether I am convinced that scientific words and concepts are loaded, for many or all scientists, with subconscious meanings, which may be revealed in the "scientific dreams" which many or all of us have.

This association of parasitic notions with scientific concepts is usually rejected more or less indignantly by the conscious mind. A very different situation occurs when a scientist deliberately chooses to study a subject, like form or time, which is rather explicitly loaded with emotional meaning. The rest of this paper is dedicated to a short and modest excursion in this difficult subject.

Theophile De Donder was the father of mathematical physics at the Free University of Brussels and, while active, he was a scientist of some distinction. I saw him however only at the end of his life, a very old man, shrivelled up, dessicated, and senile. He walked with a cane, and his tiny steps made a queer sound, tip-tip-tap, tip-tip-tap, as he progressed through the corridors of the "Faculté des Sciences." When they heard this tip-tip-tap, his younger colleagues, professors and assistants, rushed to an office, locked themselves in, and stayed very quiet. If by misfortune they were caught by the little grand old man, they would have to listen to a long talk on his theory of the shape of the liver, or his mathematical theory of music. I don't know why De Donder was interested in the

1 I had thought of giving examples, but they are somewhat unpleasant and could be traced all too easily to their authors

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shape of the liver. As for music, he tried to make mathematical sense of his intuitive perception – a rather natural idea for a scientist.

It should be pointed out here that the intuitive perception we have of time, space, shapes, music, or the smell of a flower, is rather different from what we call physical intuition. Physical intuition and mathematical intuition are based on familiarity with technical tools which are at first rather unintuitive. History shows how hard it was to come to the idea that motion is due to forces which change the momentum of objects. An important motor of scientific activity has been, and remains, to reconcile primitive intuition with more technical knowledge. The examples of special relativity and quantum mechanics have shown how painful the reconciliation could sometimes be.

Time is a very primitive psychological notion, and its mathematical idealization by the real line is a resonably simple technical object. At least as early as Zeno the intuitive grasp which we have of time has led to difficulties (in the Gedankenexperiment of Achilles and the tortoise). The fact is that in our intuitive perception of time the order relation is clear and strong, while our appreciation of the metric is uncertain. It has thus been a recurrent temptation to construct a mental image of time preserving the order relation but with flexible metric. Literary use of the possibility of distorting time occurs for instance in the short story "El milagro secreto" by Borges [1].

Time occurs in physics in the study of time evolutions or, if one prefers, of *dynamical systems*. Macroscopic dynamical systems are conservative or dissipative, and dissipative systems may have some external force to keep them from going to rest. These are however technical subtleties. More important from a primitive psychological viewpoint is the ultimate fate of the system. For instance the ultimate triumph of the true religion or of the right ideology. It is assumed that a stationary state is eventually reached, and no thought is given to the possibility of a more complicated time behavior. Let us give a more scientific example. After a disturbance (like a fire) there is, in a given environment, a succession of plant communities. One *defines* as *climax* the community eventually reached and which is in a state of relative equilibrium. This notion certainly owes a lot to observation of plant communities in temperate regions, but probably something also to the *a priori* idea that most things go, after a while, to a "state of relative equilibrium."

Another accepted notion about the fate of a dynamical system is that of eternal return. It is a comforting thought that the laws of the universe will some day bring things to be again what they were before. A sophisticated way to try to justify this would be in terms of Poincaré recurrence, but we are here dealing with unsophisticated notions, and the accepted view is that of cyclic oscillations. One may put several oscillating systems together and try to explain the world in terms of such assemblies of "oscillators" or "modes." This intuitively satisfactory picture has been used since antiquity to describe celestial motions and more recently it has been remarkably successful in the study of some physical systems. It is however not universal: a finite dimensional dynamical system may show a time evolution involving a continuous superposition of frequencies. The finite dimension implies a finite number of oscillators, but the "continuous spectrum" requires an infinity: the "mode" description breaks down. Such a breakdown appears to occur at the onset of turbulence in hydrodynamics. A continuous superposition of frequencies occurs there, which one had tried to explain by an accumulation of "modes" in the same way as Ptolemaic astronomy had accumulated epicycles to explain the motion of planets. The correct explanation appears in both cases mathematically simpler but further away from primitive intuition.

„Nach Leibnitz ist unsere Welt die bestmögliche aller Welten, und daher lassen sich die Naturgesetze durch Extremalprinzipien beschreiben.“ This is the first sentence in Siegel's „Vorlesungen über Himmelsmechanik“ [2]. Siegel then goes on to explain the consequences of extremality for the time evolution of conservative systems. Extremal principles are indeed very useful in the study of dynamical systems, but they exert a fascination which goes beyond their technical usefulness. Much effort has been spent to obtain extremal principles also for dissipative dynamical systems, but this does not seem possible in general. It appears in fact that the time evolutions observed for dissipative systems present the same qualitative features as found in the computer studies of rather arbitrary differential dynamical systems.

It is a natural desire to reconcile our primitive intuitive grasp of the world with our technical knowledge. More ambitiously one may try to attain some scientific truth by clever use of primitive intuition. Sometimes this seems to work, but sometimes it seems that the world in which we are is not quite the world for which our intuition prepared us. Part of the knowledge which we get from the world fits our philosophy, and part is beyond it. The realization of this is not new, for it has been noted long ago that there are more things in heaven and earth than are dreamt of in our philosophy.

Our intuition of time and space has apparently been shaped by evolution — millions of years of African primate ancestry — and by personal experience. This intuition is a remarkably effective tool. That it can be trained to become a sophisticate logical instrument is even more remarkable (and, from an evolutionary viewpoint, somewhat mysterious). Training is however needed. We cannot hope that the old ape in us, clever as he may be, has direct comprehension of abstract physical or mathematical questions.

References

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