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Quasi-Free Photon States and the Poincaré Group

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Abstract. It is proven that every projectively Poincaré covariant representation of the free photon field defined by a pure quasi-free state is unixarily equivalent to the Fock representation of that field.

1. Introduction

In order to describe photons radiated by a classical external current, coherent and quasi-free states on the quantized free electromagnetic field have been considered by several authors [1–6]. States on the quantized electromagnetic field are, moreover, particularly suited to construct representations of the field that are covariant under a given symmetry group G [3, 7–9[1]]. Quasi-free states also arise in other physical problems, e.g., the vacuum states of generalized free relativistic fields [10], the ground state of the interacting Bose gas at zero temperature [11] and the equilibrium states of the nonrelativistic free Bose gas [12, 13].

Let G be a topological group, $(\mathscr{K}, V)$ a strongly continuous unitary representation of G, E a regular state on the symplectic real-linear space $(\mathscr{K}^r, \sigma)$ induced by the complex Hilbert space $\mathscr{K}$, and $(\mathscr{H}, W, \Omega)$ the Gel'fand-Naimark-Segal (GNS) representation corresponding to E [3]. The Weyl system $(\mathscr{H}, W)$ is called projectively G-covariant if there exists a continuous unitary projective representation $(\mathscr{H}, U)$ of the group G which implements the automorphisms $W(\cdot)\to W(V(s)\cdot)$ $(\forall s\in G)$ of the Weyl system $(\mathscr{H}, W)$. Under additional assumptions for the group G it has been proved in [7, 14] that there exists a bijection from classes of sectors containing coherent states admitting a projectively G-covariant Weyl system, onto the first cohomology group $H^1(G, \mathscr{K}, V)$. In this paper we attempt to extend the foregoing approach to sectors containing quasi-free states.

In Sect. 2 we derive a necessary and sufficient condition for two pure quasi-free states to be in the same sector and we will classify all quasi-free states belonging to

1 During preparation of this paper we received the preprints by Basarab-Horwath, Polley, Reents and Streater dealing with questions treated here

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a fixed sector. In Sect. 3 this condition allows us to deduce the existence of an injection of classes of sectors containing quasi-free states with projectively G-covariant GNS system, into the first cohomology group $H^1(G, \mathcal{T}_2(\mathcal{K}^r), \alpha)$, where $\mathcal{T}_2(\mathcal{K}^r)$ is the Hilbert space of Hilbert-Schmidt operators on the real Hilbert space $\mathcal{K}^r$ and α the representation of G on $\mathcal{T}_2(\mathcal{K}^r)$ defined as $\alpha(s)(A) = V(s)AV(s)^{-1}$ $(s \in G, A \in \mathcal{T}_2(\mathcal{K}^r))$. For the covering group G of the Poincaré group $\mathcal{P}^\uparrow_+$ and its irreducible representations $(\mathcal{K}, V)$ of type $[m, s]$ $(m>0, s=0, \frac{1}{2}, \ldots)$ and $[0, \lambda]$ $(\lambda = 0, \pm\frac{1}{2}, \pm 1, \ldots)$ it is shown in Sect. 4 that the first cohomology group $H^1(G, \mathcal{T}_2(\mathcal{K}^r), \alpha)$ is zero. Combining results of Sect. 3 (Theorem 7) and Sect. 4 (Corollary to Theorem 9) one concludes that every pure quasi-free state associated to a projectively Poincaré covariant representation of the free photon field belongs to the Fock sector.

2. Quasi-Free States and Sectors

Let (L, σ) be a symplectic space, B a real inner product on L and F a real linear functional on L. The functional E_B on L, defined by $E_B(f) = \exp(-\frac{1}{2}B(f, f)) \forall f \in L$, is a state on L if, and only if, $|\sigma(f, g)|^2 \leqq B(f, f)B(g, g) \forall f, g \in L$. A state $E_{B,F}$ on L, defined by $E_{B,F}(f) = \exp(-\frac{1}{2}B(f, f) + iF(f)) \forall f \in L$, is called a quasi-free state on L [15, 16]. In this paper we will restrict our attention to quasi-free states with $F=0$. One establishes straightforwardly the following properties for a quasi-free state E_B, which we mention without proof:

(1) The mapping $\lambda \mapsto E_B(\lambda f + g)$ from $\mathbb{R}$ into $\mathbb{C}$ has derivatives of all orders $(\forall f, g \in L)$; this implies i.a. that E_B is a regular state. For the GNS triple $(\mathcal{H}_B, W_B, \Omega_B)$ corresponding to the state E_B, the mapping $t \mapsto W_B(tf)$ from $\mathbb{R}$ into $\mathcal{B}(\mathcal{H}_B)$ is a strongly continuous one-parameter group of unitary operators; hence there exists a self-adjoint operator $A(f)$ in $\mathcal{H}_B$ with $W_B(tf) = \exp(itA(f)) \forall t \in \mathbb{R}$.

(2) The mapping $f \mapsto W_B(f)$ from L into $\mathcal{B}(\mathcal{H}_B)$ is continuous with respect to the B-norm topology for L and the strong operator topology for $\mathcal{B}(\mathcal{H}_B)$.

(3) $(\Omega_B, A(f_1)\ldots A(f_{2n-1})\Omega_B) = 0 \ \forall n \in \mathbb{N}$ and $\forall f_1, \ldots, f_{2n} \in L$,

$$(\Omega_B, A(f_1)\ldots A(f_{2n})\Omega_B) = \sum_{\pi_{2n}} (\Omega_B, A(f_{i_1})A(f_{j_1})\Omega_B)\ldots(\Omega_B, A(f_{i_n})A(f_{j_n})\Omega_B),$$

where the summation is over all permutations π_{2n} defined by $\pi_{2n}(1, \ldots, 2n) = (i_1, j_1, \ldots, i_n, j_n)$ and satisfying the inequalities $1 = i_1 < i_2 < \ldots < i_n$ and $i_1 < j_1, \ldots, i_n < j_n = 2n$.

Let $\bar{L}^B$ be the real Hilbert space obtained by completing L in the B-norm; the inner product on $\bar{L}^B$ will again be denoted by B. As σ is a bounded $\mathbb{R}$-bilinear functional on L, it can be extended uniquely to a bounded $\mathbb{R}$-bilinear functional $\bar{\sigma}$ on $\bar{L}^B$, which yields a uniquely determined bounded $\mathbb{R}$-linear operator D on $\bar{L}^B$ such that $\bar{\sigma}(f, g) = B(Df, g) \forall f, g \in \bar{L}^B$. It is not hard to show that $\|D\| \leqq 1$ and $D^t = -D$, where D^t is the adjoint operator of D in the real Hilbert space $\bar{L}^B$. Let $D = j|D|$ be the polar decomposition of the bounded normal operator D on $\bar{L}^B$. The initial and final subspaces, $\mathcal{H}_{\text{in}}$ respectively $\mathcal{H}_{\text{out}}$, of the partial isometry j are related by $\mathcal{H}_{\text{in}} = \mathcal{H}_{\text{out}}$. We are now in the position to state the following two lemmata whose proofs can be found in [15]:

Lemma 1. *Let E_B be a quasi-free state on the symplectic space (L, σ). Then the following conditions are equivalent:*

(1) *E_B is a factor state on L.*

(2) *σ can be extended to a nondegenerate bounded $\mathbb{R}$-bilinear functional $\bar{\sigma}$ on $\bar{L}^B$.*

(3) *D is injective.*

If the state E_B satisfies one of these conditions, then $\mathscr{H}_{\text{in}} = \mathscr{H}_{\text{out}} = \bar{L}^B$, i.e., the operator j is orthogonal in $\bar{L}^B$. Moreover, the operator j is a complex structure on $(\bar{L}^B, \bar{\sigma})$: j is symplectic, $j^2 = -\mathbb{1}$ and $\bar{\sigma}(f, jf) \geqq 0$ $\forall f \in \bar{L}^B$. The real Hilbert space $\bar{L}^B$ becomes a complex inner product space L^j if one defines $(i, f) \mapsto jf$ $\forall f \in \bar{L}^B$ and $(f, g)_j = \bar{\sigma}(f, jg) + i\bar{\sigma}(f, g)$ $\forall f, g \in \bar{L}^B$. Let $\mathscr{H}_B$ denote the completion of L^j in the $\bar{\sigma}(\cdot, j\cdot)$-norm.

Lemma 2. *Let E_B be a quasi-free state on the symplectic space (L, σ). Then the following conditions are equivalent:*

(1) *E_B is a pure state on L.*

(2) *E_B is a factor state on L and there exists a complex structure j on $(\bar{L}^B, \bar{\sigma})$ such that $\bar{\sigma}(f, jg) = B(f, g)$ $\forall f, g \in \bar{L}^B$.*

(3) *$|D| = \mathbb{1}$ on $\bar{L}^B$.*

If the state E_B satisfies one of these conditions, then the complex structure in (2) *is uniquely determined, the underlying sets $[\mathscr{H}_B]$ and $[\bar{L}^B]$ of the Hilbert spaces $\mathscr{H}_B$ respectively $\bar{L}^B$ coincide and $\bar{L}^B$ is sequentially σ-complete.*

The σ-topology on a symplectic space (L, σ) is the locally convex topology on L defined by the semi-norms $p_f : g \mapsto |\sigma(f, g)|$ $(f, g \in L)$.

Conversely, starting from a complex Hilbert space $\mathscr{H}$ with inner product $(\cdot, \cdot)$, one can construct a real Hilbert space $\mathscr{H}^r$ with inner product $B_0(\cdot, \cdot) = \text{Re}(\cdot, \cdot)$ and a symplectic space $(\mathscr{H}^r, \sigma)$ with $\sigma(\cdot, \cdot) = \text{Im}(\cdot, \cdot)$. If A is a bounded $\mathbb{R}$-linear operator on $\mathscr{H}^r$, then A^+ denotes its adjoint with respect to σ. If L is sequentially σ-complete then there exists a one-to-one correspondence between pure quasi-free states on L and the complex structures on L with respect to σ. Occasionally we denote a state E_B also by E_j where j is the complex structure on L induced by the state E_B.

Let E_{B_1} and E_{B_2} be two pure quasi-free states on L. Combining the facts that L is sequentially σ-dense in $\mathscr{H}_1 := \mathscr{H}_{B_1}$ and $\mathscr{H}_2 := \mathscr{H}_{B_2}$ and that $\mathscr{H}_1$ and $\mathscr{H}_2$ are sequentially σ-complete, one concludes that the sets $[\mathscr{H}_1]$ and $[\mathscr{H}_2]$ can be identified. This implies the equivalence of the norms B_1 and B_2, hence the dimensions of the Banach spaces $\mathscr{H}_1$ and $\mathscr{H}_2$ are equal and therefore also the dimensions of the Hilbert spaces $\mathscr{H}_1$ and $\mathscr{H}_2$. For this reason there exists an isometric operator T from $\mathscr{H}_2$ onto $\mathscr{H}_1$, i.e.,

$$B_1(Tf, Tg) = B_2(f, g),\ \bar{\sigma}(Tf, Tg) = \bar{\sigma}(f, g)\ \forall f, g \in \mathscr{H}_2^r \quad \text{and} \quad Tj_2 = j_1 T\,.$$

Rephrasing the foregoing we have demonstrated that, given a pure quasi-free state E_{B_1}, for every pure quasi-free state E_{B_2} on L there exists a symplectic isometric $\mathbb{R}$-linear operator T from $\mathscr{H}_2^r$ onto $\mathscr{H}_1^r$ with $j_2 = T^+ j_1 T$. We are now in the position to prove a theorem on which much of the subsequent analysis relies. Compared to the proof in [17, 18, p. 191] ours has been simplified and shortened considerably. $\mathscr{T}_2(\mathscr{H}^r)$ denotes the set of $\mathbb{R}$-linear Hilbert-Schmidt operators on $\mathscr{H}^r$.

Theorem 3. *Let* (L, σ) *be a symplectic space and* E_{B_1} *and* E_{B_2} *two pure quasi-free states on* (L, σ). *Then the states* E_{B_1} *and* E_{B_2} *belong to the same sector if, and only if,* $j_1 - j_2 \in \mathcal{T}_2(\mathcal{K}_1^r) = \mathcal{T}_2(\mathcal{K}_2^r)$.

Proof. We remark that there exists a symplectic operator T on $\mathcal{K}_2^r$ such that $E_{B_2}(f) = E_{B_1}(Tf)\ \forall f \in \mathcal{K}_2^r$. Thus $(\mathcal{H}_{B_1}, W_{B_1}(T\cdot), \Omega_{B_1})$ is a cyclic Weyl system corresponding to the state E_{B_2}. The states E_{B_1} and E_{B_2} belong to the same sector if, and only if, the Weyl automorphism $W_{B_1}(f) \mapsto W_{B_1}(Tf)$ $(\forall f \in L)$ is unitarily implementable in $\mathcal{H}_{B_1}$. By a theorem of Shale [19] the existence of the unitary intertwining operator $U(T)$ is guaranteed if, and only if, $T^t T - \mathbb{1} \in \mathcal{T}_2(\mathcal{K}_1^r)$. Exploiting the equalities $T^t = -j_1 T^+ j_1$ and $T^t T = -j_1 j_2$ one gets the necessary and sufficient condition.

In case E_{B_1} and E_{B_2} belong to the same sector $(\mathcal{H}_{B_1}, W_{B_1}(\cdot), U(T)^{-1}\Omega_{B_1})$ is again a cyclic Weyl system determined by the state E_{B_2}. A formula for the vector $U(T)^{-1}\Omega_{B_1}$ can be given [20, p. 316] but we shall have no occasion to use it. The next theorem characterizes the quasi-free states belonging to a quasi-free sector, i.e., a sector containing a quasi-free state. Although redundant for the proofs in the next sections, Theorem 4 may be nevertheless of some independent interest.

Theorem 4. *Let* $\mathcal{K}$ *be a complex Hilbert space and* E *the Fock state on* $(\mathcal{K}^r, \sigma)$. *If* E_j *is a quasi-free state belonging to the sector fixed by* E, *then*

(1) *there exist a complex linear subspace* $\mathcal{M}$ *of* $\mathcal{K}$ *and a real linear subspace* $\mathcal{K}_0$ *of* $\mathcal{K}^r$ *such that* $\mathcal{K}^r = \mathcal{K}_0 \oplus i\mathcal{K}_0 \oplus \mathcal{M}^r$.

(2) *There exists a symmetric Hilbert-Schmidt operator* h *on* $\mathcal{K}_0$ *with* $\operatorname{Sp} h \subset (-1, 0]$ *and* $0 \notin p\operatorname{Sp} h$ *such that* $j = i + iH$ *with*

$$H = h \oplus h_1 \oplus 0 \quad \text{and} \quad h_1 = i\frac{h}{h+1}i.$$

Conversely, if the conditions (1) *and* (2) *are fulfilled then* $j = i + iH$ *is a complex structure on* L *with respect to* σ *and the state* E_j *belongs to the sector given by* E.

Proof. Theorem 3 assures the existence of a Hilbert-Schmidt operator H in $\mathcal{K}^r$ such that $j = i + iH$. Due to the assumption that j is a complex structure on $\mathcal{K}^r$ with respect to σ one derives that H is a symmetric operator on $\mathcal{K}^r$ with $H \geqq -1$ and $iHiH + iHi - H = 0$. One can easily show that $-1 \notin p\operatorname{Sp} H$ and $\operatorname{Ker} H$ is a complex linear subspace of $\mathcal{K}$. Let $\mathcal{K}_0$ be the real linear subspace of $\mathcal{K}^r$ spanned by the eigenvectors of H with eigenvalues in the interval $(-1, 0)$, then $\mathcal{K}^r = \mathcal{K}_0 \oplus i\mathcal{K}_0 \oplus (\operatorname{Ker} H)^r$. Define $h = H \upharpoonright \mathcal{K}_0$.

A slight change of the preceding argumentation leads also to a designation of the $\mathbb{R}$-bilinear functionals Φ on $\mathcal{K}^r$ such that $\exp(-\frac{1}{2}(B_0 + \Phi))$ is again a pure quasi-free state. Condition (2) is appropriately changed if one drops the Hilbert-Schmidt property of the operator h.

3. Projectively G-Covariant Quasi-Free Sectors and Cohomology

Let G be a topological group, $(\mathcal{K}, V)$ a strongly continuous unitary representation of G and E_B a pure quasi-free state on $(\mathcal{K}^r, \sigma)$. We define for all $s \in G$ the $\mathbb{R}$-bilinear functional $V(s)^{\times}B$ by $(V(s)^{\times}B)(f,g)=B(V(s)f, V(s)g)\,\forall f, g\in\mathscr{K}^r$. It is not hard to show that for all $s\in G$ $E_{V(s)^{\times}B}$ is a pure quasi-free state on $(\mathscr{K}^r,\sigma)$ and $j_s=V(s)^{-1}jV(s)$ the corresponding complex structure on $(\mathscr{K}^r,\sigma)$. In the sequel we shall be concerned with sectors containing a projectively G-covariant pure quasi-free state E_B, i.e., the Weyl system $(\mathscr{H}_B, W_B)$ is projectively G-covariant. If a pure state is projectively G-covariant then all the states from the corresponding sector are projectively G-covariant. On the real Hilbert space $\mathscr{T}_2(\mathscr{K}^r)$ of Hilbert-Schmidt operators on $\mathscr{K}^r$ one defines by $\alpha(s)(A)=V(s)AV(s)^{-1}$ ($A\in\mathscr{T}_2(\mathscr{K}^r)$, $s\in G$) a strongly continuous orthogonal representation of the group G. A 1-cocycle ξ on G with values in $\mathscr{T}_2(\mathscr{K}^r)$ with respect to the strongly continuous orthogonal representation $(\mathscr{T}_2(\mathscr{K}^r),\alpha)$ of G is a continuous mapping $\xi: G\to\mathscr{T}_2(\mathscr{K}^r)$ such that $\xi(st)=\alpha(s)(\xi(t))+\xi(s)\ \forall s,t\in G$. The real linear space of these 1-cocycles ξ will be denoted by $Z^1(G,\mathscr{T}_2(\mathscr{K}^r),\alpha)$. The 1-cocycles ξ for which there exists an $H\in\mathscr{T}_2(\mathscr{K}^r)$ such that $\xi(s)=(\alpha(s)-\mathrm{id})(H)$ for all $s\in G$ will be called 1-coboundaries and will be denoted by $B^1(G,\mathscr{T}_2(\mathscr{K}^r),\alpha)$. The real linear space $H^1(G,\mathscr{T}_2(\mathscr{K}^r),\alpha)=Z^1(G,\mathscr{T}_2(\mathscr{K}^r),\alpha)/B^1(G,\mathscr{T}_2(\mathscr{K}^r),\alpha)$ is the first cohomology group of G with values in $\mathscr{T}_2(\mathscr{K}^r)$ with respect to the representation $(\mathscr{T}_2(\mathscr{K}^r),\alpha)$. If $\xi\in Z^1(G,\mathscr{T}_2(\mathscr{K}^r),\alpha)$ then $\hat{\xi}:=\xi+B^1(G,\mathscr{T}_2(\mathscr{K}^r),\alpha)$ denotes an element of the first cohomology group.

Theorem 5. *Let E_B be a pure quasi-free state on $(\mathscr{K}^r,\sigma)$. Then the state E_B is projectively G-covariant if, and only if, the mapping $\xi: G\to\mathscr{B}(\mathscr{K}^r)$, defined by $\xi(s)=V(s)jV(s)^{-1}-j\ \forall s\in G$, is an element of $Z^1(G,\mathscr{T}_2(\mathscr{K}^r),\alpha)$.*

Proof. Suppose the automorphisms $W_B(\cdot)\to W_B(V(s)\cdot)$ are for all $s\in G$ unitarily implementable in $\mathscr{H}_B$, then E_B and $E_{V(s)^{\times}B}$ belong for all $s\in G$ to the same sector. Appealing to Theorem 3, one concludes that for all $s\in G$, $j_s-j\in\mathscr{T}_2(\mathscr{K}^r_j)=\mathscr{T}_2(\mathscr{K}^r)$. From the work of Shale [19] it follows that the continuity of the projective representation $s\mapsto U(s)$ ($s\in G$) is equivalent to the continuity in the Hilbert-Schmidt norm of the mapping $s\mapsto V(s)^t V(s)-\mathbb{1}$ from G into $\mathscr{B}(\mathscr{K}^r)$. Thus ξ is a 1-cocycle. The proof in the reversed direction is easily accomplished. In fact, the implications we derived were equivalences.

In the following we only consider sectors $\hat{E}_B$ containing a projectively G-covariant quasi-free state E_B.

Theorem 6. *Let E_B, E_{B_1} and E_{B_2} be projectively G-covariant pure quasi-free states with corresponding 1-cocycles ξ, ξ_1, and ξ_2.*

(1) *The mapping $E_B\mapsto\hat{\xi}\in H^1(G,\mathscr{T}_2(\mathscr{K}^r),\alpha)$ is a sector mapping.*

(2) *$\hat{\xi}_1=\hat{\xi}_2$ if, and only if, there exist an $H\in\mathscr{T}_2(\mathscr{K}^r)$ and a $\varphi\in\mathscr{B}(\mathscr{K}^r)$ with $j_1=j_2+H+i\varphi$ and $V(s)\varphi V(s)^{-1}=\varphi$ for all $s\in G$.*

Assuming that the representation V is completely reducible with only infinitely dimensional irreducible subrepresentations, then the operators H and φ are uniquely determined by the given requirements.

Proof. Firstly, let E_{B_1} and E_{B_2} be elements of the same sector with $j_1-j_2\in\mathscr{T}_2(\mathscr{K}^r)$. Then $\xi_1-\xi_2\in B^1(G,\mathscr{T}_2(\mathscr{K}^r),\alpha)$ and $\hat{\xi}_1=\hat{\xi}_2$. Secondly, $\hat{\xi}_1=\hat{\xi}_2$ implies the existence of a 1-coboundary ξ with $\xi_1-\xi_2=\xi$, i.e., there exists an $H\in\mathscr{T}_2(\mathscr{K}^r)$ such that $V(s)(j_1-j_2)V(s)^{-1}-(j_1-j_2)=V(s)HV(s)^{-1}-H$ for all $s\in G$. Define the real bilinear bounded functional Φ on $\mathcal{K}^r$ by $\Phi(f,g)=B_1(f,g)-B_2(f,g)-\sigma(f,Hg)=\sigma(f,(j_1-j_2-H)g)$; it turns out that Φ is invariant under the group G. There exists a uniquely determined bounded $\mathbb{R}$-linear operator φ on $\mathcal{K}^r$ such that $\Phi(f,g)=B_0(f,\varphi g)=\sigma(f,i\varphi g)$ for all $f,g\in\mathcal{K}^r$. From $\Phi(V(s)f,V(s)g)=\Phi(f,g)$ $\forall s\in G$, $\forall f,g\in\mathcal{K}^r$ and the $\mathbb{C}$-linearity of the operators $V(s)$ it follows that $V(s)\varphi V(s)^{-1}=\varphi$ for all $s\in G$. The proof of the converse is obvious. An application of Theorem 10 from the appendix easily proves the third statement of the theorem.

Now we are able to define an equivalence relation among the sectors $\hat{E}_B$. $\hat{E}_{B_1}$ and $\hat{E}_{B_2}$ are called equivalent if, and only if, there exist an $H\in\mathcal{T}_2(\mathcal{K}^r)$ and a $\varphi\in\mathcal{B}(\mathcal{K}^r)$ such that $V(s)\varphi V(s)^{-1}=\varphi$ for all $s\in G$ and $j_1=j_2+H+i\varphi$. This definition is independent of the chosen quasi-free state from a sector. The class containing the sector $\hat{E}_B$ is denoted by $\tilde{E}_B$. The mapping $\tilde{E}_B\mapsto\hat{\xi}\in H^1(G,\mathcal{T}_2(\mathcal{K}^r),\alpha)$ is a class mapping, which is injective.

Theorem 7. *Let G be a topological group containing a one-parameter subgroup G_0, $(\mathcal{K},V)$ a completely reducible strongly continuous unitary representation of G with only infinitely dimensional irreducible subrepresentations. If the representation $V\upharpoonright G_0$ does not contain the trivial representation as a subrepresentation and* $\mathrm{Sp}(V\upharpoonright G_0)\subset[0,\infty)$ *then the class $\tilde{E}_{B_0}$ containing the Fock sector $\hat{E}_{B_0}$ contains only the Fock sector $\hat{E}_{B_0}$.*

Proof. For simplicity we assume that the representation V only has two irreducible subrepresentations. Let $\hat{E}_B$ be a sector belonging to the class $\tilde{E}_{B_0}$. One can write $j=i+i(H+\varphi)$ with $H\in\mathcal{T}_2(\mathcal{K}^r)$ and φ a bounded $\mathbb{R}$-linear operator on $\mathcal{K}^r$ commuting with all operators $V(s)$. Application of Theorem 11 from the appendix yields that φ is a bounded $\mathbb{C}$-linear operator on $\mathcal{K}$. Exploiting the special structure of the representation V, φ takes the diagonal form $\varphi=\begin{pmatrix}\lambda_1\mathbb{1}_1 & 0\\ 0 & \lambda_2\mathbb{1}_2\end{pmatrix}$. (The irreducible subrepresentations are supposed to be defined on Hilbert spaces $\mathcal{K}_1$ and $\mathcal{K}_2$ with $\mathcal{K}=\mathcal{K}_1\oplus\mathcal{K}_2$.) The operator $H+\varphi$ being a symmetric $\mathbb{R}$-linear operator on $\mathcal{K}^r$ allows one to deduce $H^t-H=\begin{pmatrix}(\lambda_1-\bar{\lambda}_1)\mathbb{1}_1 & 0\\ 0 & (\lambda_2-\bar{\lambda}_2)\mathbb{1}_2\end{pmatrix}$. Application of Theorem 10 leads to $H^t=H$ and $\lambda_1,\lambda_2\in\mathbb{R}$. The operator j is a complex structure on $(\mathcal{K}^r,\sigma)$. From the properties $j^2=-\mathbb{1}$ and $\sigma(f,jf)\geqq 0$ one deduces $H+\varphi\geqq -1$ on $\mathcal{K}^r$ and $iHiH+iHi\varphi-\varphi H+iHi-H=\varphi^2+2\varphi$. Again Theorem 10 gives $\varphi(\varphi+2\mathbb{1})=0$. Assume $\lambda_1,\lambda_2\neq 0$, then φ has an inverse and the equation has the solution $\varphi=-2\mathbb{1}$. Substituting this solution into the inequality $H+\varphi\geqq-\mathbb{1}$ one gets $H\geqq\mathbb{1}$. This contradicts H being a Hilbert-Schmidt operator.

Theorem 7 clarifies the structure of the Fock class $\tilde{E}_{B_0}$. The results proven so far for the first cohomology group $H^1(G,\mathcal{T}_2(\mathcal{K}^r),\alpha)$ give only information about other classes if we can show that this cohomology group is trivial. In that case we can draw the conclusion that other classes do not exist beside the Fock class. To elucidate the structure of the real Hilbert space $\mathcal{T}_2(\mathcal{K}^r)$ and the representation α we are compelled to consider an equivalent problem.

Let $\bar{\mathcal{K}}$ be the Hilbert space conjugate to the Hilbert space $\mathcal{K}$. Then every operator $A\in\mathcal{B}(\mathcal{K}^r)$ can be uniquely written as $A=A_1+A_2$ with $A_1\in\mathcal{B}(\mathcal{K},\mathcal{K})^r$ and

$A_2 \in \mathscr{B}(\mathscr{K}, \bar{\mathscr{K}})^r$. [$A_1 = \frac{1}{2}(A - iA\bar{i})$ and $A_2 = \frac{1}{2}(A + iAi)$.] On the complex Hilbert spaces $\mathscr{T}_2(\mathscr{K}, \mathscr{K})$ and $\mathscr{T}_2(\mathscr{K}, \bar{\mathscr{K}})$ one can define strongly continuous unitary representations of the group G by $\alpha(s)(A) = V(s)AV(s)^{-1}$. We can establish an isomorphism from $H^1(G, \mathscr{T}_2(\mathscr{K}^r), \alpha)$ onto $H^1(G, \mathscr{T}_2(\mathscr{K}, \mathscr{K}), \alpha)^r \oplus H^1(G, \mathscr{T}_2(\mathscr{K}, \bar{\mathscr{K}}), \alpha)^r$. It is well known that there exist a uniquely defined isometry I from $\mathscr{K} \otimes \bar{\mathscr{K}}$ onto $\mathscr{T}_2(\mathscr{K}, \mathscr{K})$ such that $I(f \otimes g)(h) = (g, h)f$ $\forall f, h \in \mathscr{K}, \forall g \in \bar{\mathscr{K}}$ and a uniquely defined isometry J from $\mathscr{K} \otimes \mathscr{K}$ onto $\mathscr{T}_2(\mathscr{K}, \bar{\mathscr{K}})$ such that $J(f \otimes g)(h) = \overline{(g, h)}f$ $\forall f, g, h \in \mathscr{K}$. For the representations α one gets $IV(s) \otimes \overline{V(s)} I^{-1}(\cdot) = \alpha(s)(\cdot)$ and $JV(s) \otimes V(s) J^{-1}(\cdot) = \alpha(s)(\cdot)$. [$\overline{V(s)}$ denotes an operator in $\mathscr{B}(\bar{\mathscr{K}}, \bar{\mathscr{K}})$ with the same mapping prescription as the operator $V(s)$ in $\mathscr{B}(\mathscr{K}, \mathscr{K})$.] By this means we have proved the existence of an isomorphism from $H^1(G, \mathscr{T}_2(\mathscr{K}^r), \alpha)$ onto $H^1(G, \mathscr{K} \otimes \bar{\mathscr{K}}, V \otimes \bar{V})^r \oplus H^1(G, \mathscr{K} \otimes \mathscr{K}, V \otimes V)^r$.

4. Cohomology and the Poincaré Group

The study of the first cohomology group of a Lie group G with coefficients in a Hilbert space $\mathscr{H}$ carrying a strongly continuous unitary representation U of G was initiated by Araki [21]. In [22, 23] these results were generalized to comprise arbitrary locally compact groups. Our discussion will rely notably on results of [23]. In calculating the cohomology groups mentioned at the end of Sect. 3 for the Poincaré group $\mathscr{P}_+^\uparrow$ the fact that the Poincaré group is not amenable will play a significant role. For convenience of the reader, we add briefly a few remarks on the terminology and basic results.

Let G be a locally compact group, μ a left invariant Haar measure on G and $L^1(G)$ the Banach*-algebra of the complex-valued μ-summable functions on G. Let $(\mathscr{H}, U)$ and $(\mathscr{K}, V)$ be strongly continuous unitary representations of G, $(\mathscr{H}, \pi_U)$ and $(\mathscr{K}, \pi_V)$ the corresponding nondegenerate representations of the Banach*-algebra $L^1(G)$ and $A(U)$ and $A(V)$ the C^*-algebras generated by $\pi_U(L^1(G))$, respectively $\pi_V(L^1(G))$. Then the representation U of G is said to contain the representation V of G weakly if there exists a homomorphism π from $A(U)$ onto $A(V)$ such that $\pi \circ \pi_U = \pi_V$.

Let $CB(G)$ be the set of complex-valued continuous bounded functions on G. With the usual definition of addition, scalar multiplication, involution and norm, $CB(G)$ becomes a C^*-algebra with unit. A left invariant mean on G is a state M on $CB(G)$ such that $M({}_sf) = M(f)$ $\forall s \in G$ and $\forall f \in CB(G)$. (${}_sf(t) := f(s^{-1}t)$ $\forall s, t \in G$.) The group G is called amenable if there exists a left invariant mean on G. The group G is said to have property R if the (left) regular representation of G contains weakly the one-dimensional trivial representation. The group G is amenable if, and only if, the group G has property R [24, p. 61]. If in addition, the group G satisfies the second countability axiom then the group G has property R if, and only if, the regular representation of G weakly contains every strong operator continuous unitary representation $(\mathscr{H}, U)$ of G with separable Hilbert spaces $\mathscr{H}$ [25, p. 260].

We equip the complex linear space $Z^1(G, \mathscr{H}, U)$ with the topology of uniform convergence on compact subsets of G. For the Poincaré group we aim at proving that the 1-cohomology groups taken into consideration are zero. Clearly, this can be achieved by proving first that $B^1(G, \mathscr{H}, U)$ is dense in $Z^1(G, \mathscr{H}, U)$ and subsequently that $B^1(G, \mathscr{H}, U)$ is closed in $Z^1(G, \mathscr{H}, U)$. The latter problem can be explored using general properties also shared by the Poincaré group. To prove the next Theorem, implicitly contained in [23, p. 329], we need a result of Guichardet-Johnson [23, p. 309]. Let $(\mathscr{H}, U)$ be a strong operator continuous unitary representation of a locally compact group G, which does not contain the trivial representation as a subrepresentation. If, moreover, the representation U does not contain the one-dimensional trivial representation weakly then $B^1(G, \mathscr{H}, U)$ is closed in $Z^1(G, \mathscr{H}, U)$.

Theorem 8. *Let G be a locally compact non-amenable group satisfying the second countability axiom and H a closed amenable subgroup of G. Let $(\mathscr{H}, U)$ be a strong operator continuous unitary representation of G, not containing the trivial representation as a subrepresentation, on a separable Hilbert space $\mathscr{H}$ induced by a strong operator continuous unitary representation U_H of H. Then $B^1(G, \mathscr{H}, U)$ is closed in $Z^1(G, \mathscr{H}, U)$.*

Proof. As H is amenable the regular representation of H contains the representation U_H weakly [24, 25]. As the regular representation of G is induced by the regular representation of H and the representation U of G is induced by the representation U_H of H, the regular representation of G weakly contains the representation U of G [25, p. 260]. The regular representation of G does not contain the one-dimensional trivial representation weakly because G is assumed to be not amenable. Hence the representation U of G does not contain the one-dimensional representation weakly. Applying the Guichardet-Johnson theorem we show that $B^1(G, \mathscr{H}, U)$ is a closed subset of $Z^1(G, \mathscr{H}, U)$.

Now we continue our analysis by taking advantage of the structure of the Poincaré group and its irreducible representations. Let G be the covering group of the Poincaré group; G is the semidirect product $\mathbb{R}^4 \circledS \mathrm{SL}(2, \mathbb{C})$ with respect to the homomorphism Λ from $\mathrm{SL}(2, \mathbb{C})$ onto the proper orthochronous Lorentz group $\mathscr{L}_+^\uparrow$. $\mathbb{R}^4$ is a closed invariant Abelian subgroup of the locally compact group G. Let $(\mathscr{K}, V)$ be a strong operator continuous irreducible representation of G. According to general results of Mackey the representation V is equivalent to one of the standard representations. We only consider the representation $V = V^{\dot{p}, \varrho}$ of G, induced by a representation $V_H^{\dot{p}, \varrho}$ of the closed subgroup $H = \mathbb{R}^4 \circledS G(\dot{p})$ with $\dot{p} = m e_{(0)}$ [$m > 0$ and $e_{(0)} = (1, 0, 0, 0)$] or $\dot{p} = e_{(0)} + e_{(3)}$ ($e_{(3)} = (0, 0, 0, 1)$) and $G(\dot{p}) = \{A \in \mathrm{SL}(2, \mathbb{C}) | \Lambda(A) \dot{p} = \dot{p}\}$ [26, p. 7]. [ϱ is an index to characterize the different equivalence classes of irreducible unitary representations of the little group $G(\dot{p})$, *e.g.* ϱ may denote spin or helicity. Our analysis is independent of the parameter ϱ.] $\mathrm{Sp}(V \upharpoonright \mathbb{R}^4) = \Omega(\dot{p}) := \{\Lambda \dot{p} \in \mathbb{R}^4 | \Lambda \in \mathscr{L}_+^\uparrow\}$.

First we investigate the representation $s \mapsto V(s) \otimes \overline{V(s)}$ $(s \in G)$ on the Hilbert space $\mathscr{K} \otimes \bar{\mathscr{K}}$.

$\mathrm{Sp}(V \otimes \bar{V} \upharpoonright \mathbb{R}^4) = \overline{\mathrm{Sp}(V \upharpoonright \mathbb{R}^4) + \mathrm{Sp}(\bar{V} \upharpoonright \mathbb{R}^4)} = \overline{\mathrm{Sp}(V \upharpoonright \mathbb{R}^4) - \mathrm{Sp}(V \upharpoonright \mathbb{R}^4)} = \overline{\cup \{\Omega(q) | q \in \Omega_{\dot{p}}\}}$ with $\Omega_{\dot{p}} = \{n e_{(3)} \in \mathbb{R}^4 | n > 0\} \cup \{0\}$ for $\dot{p} = m e_{(0)}$ or $\Omega_{\dot{p}} = \{n e_{(3)} \in \mathbb{R}^4 | n > 0\} \cup \{0\} \cup \{e_{(0)} + e_{(3)}\} \cup \{-e_{(0)} - e_{(3)}\}$ for $\dot{p} = e_{(0)} + e_{(3)}$ [26, p. 77]. The representation $\bar{V}$ is unitarily equivalent to a representation of G induced by the representation $\overline{V_H^{\dot{p}, \varrho}}$ of H [27, p. 356].

$\overline{V_H^{\dot{p},\varrho}(a,A)} = V_H^{\dot{p},\varrho}(a,A) = e^{i\dot{p}a} V^{\varrho}_{G(\dot{p})}(A) = e^{j(-\dot{p})a} V^{\varrho}_{G(-\dot{p})}(A) = V^{-\dot{p},\varrho}_{\mathbb{R}^4 Ⓢ G(-\dot{p})}(a,A) = V_H^{-\dot{p},\varrho}(a,A)$ because of $G(\dot{p}) = G(-\dot{p})$ [26, p. 5, 7]. The representation $V \otimes \bar{V}$ is equivalent to $V^{\dot{p},\varrho} \otimes V^{-\dot{p},\varrho}$ [27, p. 343]; these representations $V^{\dot{p},\varrho}$ and $V^{-\dot{p},\varrho}$ of G are induced by the representations $V_H^{\dot{p},\varrho}$, respectively $V_H^{-\dot{p},\varrho}$, of the subgroup $H = \mathbb{R}^4 Ⓢ G(\dot{p})$. The representation $V^{\dot{p},\varrho} \otimes V^{-\dot{p},\varrho}$ of G is unitarily equivalent to a representation of G induced by a representation of $\mathbb{R}^4 Ⓢ H_0$ with $H_0 = \left\{ \begin{pmatrix} \exp\left(\frac{i\varphi}{2}\right) & 0 \\ 0 & \exp\left(-\frac{i\varphi}{2}\right) \end{pmatrix} \middle| \, 0 \leqq \varphi < 4\pi \right\}$ [H_0 is isomorphic to the group SO(2, $\mathbb{R}$)] [26, p. 85]. Writing the tensor product of these two irreducible representations of the group G as a direct integral of irreducible representations of G [26, p. 88] one sees that the spectral measure E of the representation $V \otimes \bar{V} \upharpoonright \mathbb{R}^4$ has the following property: for all Borel sets $\Delta \subset \mathbb{R}^4$ with $\Delta \subset \{p \in \mathbb{R}^4 | p^2 = 0\}$ $E(\Delta) = 0$. The spectral measure E has support $\{p \in \mathbb{R}^4 | p^2 \leqq 0\}$. As the Borel set $\{p \in \mathbb{R}^4 | p^2 = 0\}$ has spectral measure zero, the spectral measure E is supported by the open set $\{p \in \mathbb{R}^4 | p^2 < 0\}$, i.e., $E(\{p \in \mathbb{R}^4 | p^2 < 0\}) = \mathbb{1}$. It is obvious that the set $\{p \in \mathbb{R}^4 | p^2 < 0\}$ can be written as the countable union of closed sets F_n such that each set is invariant under the group $\mathscr{L}_+^\uparrow$ and 0 is not contained in any set F_n. We may now apply a theorem due to Guichardet [23, Theorem 4]. We state this theorem in a form adapted to the group considered here. Let G be the covering group of the Poincaré group $\mathscr{P}_+^\uparrow$ and $(\mathscr{H}, U)$ a strongly continuous unitary representation of the group G. If the spectral measure E of the representation $U \upharpoonright \mathbb{R}^4$ is supported by the set $\bigcup_{n=1}^{\infty} \{F_n \subset \mathbb{R}^4 | F_n$ a closed set, F_n invariant under $\Lambda(A)$ for all $A \in \mathrm{SL}(2, \mathbb{C})$ and $0 \notin F_n\}$ then $Z^1(G, \mathscr{H}, U)$ is the closure of $B^1(G, \mathscr{H}, U)$. Thus application of this theorem leads to the conclusion that $Z^1(G, \mathscr{K} \otimes \bar{\mathscr{K}}, V \otimes \bar{V})$ is the closure of the set $B^1(G, \mathscr{K} \otimes \bar{\mathscr{K}}, V \otimes \bar{V})$.

Next we check the assumptions of Theorem 8 for the group G. Let f be an element of $\mathscr{K} \otimes \bar{\mathscr{K}}$ with $V(s) \otimes \overline{V(s)} f = f$ for all $s \in G$. Then $V(s)(If)V(s)^{-1} = If (s \in G)$ with $If \in \mathscr{T}_2(\mathscr{K}, \mathscr{K})^r \subset \mathscr{T}_2(\mathscr{K}^r)$. Recalling Theorem 10 we see that $If = 0$ and so $f = 0$. ($V \otimes \bar{V} \upharpoonright \mathbb{R}^4$ already does not contain the trivial representation as a subrepresentation.) As the group G is not amenable and the closed subgroup $\mathbb{R}^4 Ⓢ H_0$ of G is amenable, Theorem 8 guarantees that $B^1(G, \mathscr{K} \otimes \bar{\mathscr{K}}, V \otimes \bar{V})$ is a closed subset of $Z^1(G, \mathscr{K} \otimes \bar{\mathscr{K}}, V \otimes \bar{V})$.

With a slight variation on the arguments just given one gets an analogous result for the representation $(\mathscr{K} \otimes \mathscr{K}, V \otimes V)$ of G. In this case $\mathrm{Sp}(V \otimes V \upharpoonright \mathbb{R}^4) = \overline{\cup \{\Omega(q) | q \in \Omega_{\dot{p}}\}}$ with $\Omega_{\dot{p}} = \{n e_{(0)} \in \mathbb{R}^4 | n \geqq 2m\}$ for $\dot{p} = m e_{(0)}$, respectively $\Omega_{\dot{p}} = \{n e_{(0)} \in \mathbb{R}^4 | n > 0\} \cup \{e_{(0)} + e_{(3)}\}$ for $\dot{p} = e_{(0)} + e_{(3)}$. The representation $V^{\dot{p},\varrho}$ with $\dot{p} = m e_{(0)}$ can also be treated by a different albeit standard method [7, p. 201].

The preceding arguments show:

Theorem 9. *Let G be the covering group of the Poincaré group and $(\mathscr{K}, V)$ a representation of type $[m, s]$ $(m > 0, s = 0, \frac{1}{2}, 1, \ldots)$ or $[0, \lambda]$ $(\lambda = 0, \pm\frac{1}{2}, \pm 1, \ldots)$ of G. Then $H^1(G, \mathscr{T}_2(\mathscr{K}^r), \alpha) = 0$.*

Corollary. *If $(\mathscr{K}, V)$ is a representation of type $[0, \lambda] \oplus [0, -\lambda]$ $(\lambda = 0, \frac{1}{2}, 1, \ldots)$ of G then $H^1(G, \mathscr{T}_2(\mathscr{K}^r), \alpha) = 0$.*

Appendix

Theorem 10. *Let $(\mathscr{H}, V)$ be a completely reducible unitary representation of a group G which only has infinitely dimensional subrepresentations. If A is an $\mathbb{R}$-linear compact operator on $\mathscr{H}^r$ with $V(s)AV(s)^{-1}=A$ for all $s\in G$ then $A=0$.*

Proof. First we assume the irreducibility of the representation V. Then $B:=A-iAi$ is a $\mathbb{C}$-linear compact operator on $\mathscr{H}$ commuting with all $V(s)\,(s\in G)$. From the irreducibility one infers $B=\lambda\mathbb{1}(\lambda\in\mathbb{C})$. The infinite dimension of the Hilbert space $\mathscr{H}$ and the compactness of the operator B entail $\lambda=0$. Let A^* be the adjoint of the antilinear operator A. The bounded $\mathbb{C}$-linear operator A^*A commutes with all $V(s)\,(s\in G)$, so $A^*A=\lambda\mathbb{1}(\lambda\in\mathbb{C})$. Applying the same argument as before one concludes $A^*A=0$, so $A=0$.

Next we assume for convenience that the representation V only has two irreducible subrepresentations. We can write $\mathscr{H}=\mathscr{H}_1\oplus\mathscr{H}_2$, $A=\begin{pmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix}$ with A_{jk} $\mathbb{R}$-linear compact operators from $\mathscr{H}^r_k\to\mathscr{H}^r_j$ $(k,j\in\{1,2\})$ and $V(s)=\begin{pmatrix}V_1(s) & 0\\ 0 & V_2(s)\end{pmatrix}$. For the operator A_{12} one obtains $V_1(s)A_{12}A^t_{12}=A_{12}A^t_{12}V_1(s)\,(s\in G)$. Again one concludes $A_{12}A^t_{12}=0$, so $A_{12}=0$.

Theorem 11. *Let $s\mapsto V(s)$ be a strong operator continuous one-parameter group of unitary operators on a Hilbert space $\mathscr{H}$. If the representation V does not contain the trivial representation as a subrepresentation and* $\operatorname{Sp}V\subset[0,\infty)$ *then every $\mathbb{R}$-linear bounded operator on $\mathscr{H}^r$ commuting with all $V(s)$ is $\mathbb{C}$-linear.*

Proof. Every $\mathbb{R}$-linear bounded operator A on $\mathscr{H}^r$ can be uniquely written as $A=A_1+A_2$ with $A_1\in\mathscr{B}(\mathscr{H},\mathscr{H})^r$ and $A_2\in\mathscr{B}(\mathscr{H},\bar{\mathscr{H}})^r$. ($\bar{\mathscr{H}}$ is the Hilbert space conjugate to the Hilbert space $\mathscr{H}$.) If the operator A commutes with all $V(s)$ then the operators A_1 und A_2 also commute with all $V(s)$. For $f\in\mathscr{S}(\mathbb{R})$ let $I(f)$ be the spectral integral of f with respect to the spectral measure E on $\mathbb{R}$ corresponding to the one-parameter group $V(s)$. Let $\tilde{f}$ be the Fourier transform of $f\in\mathscr{S}(\mathbb{R})$; $\tilde{f}(p)=(2\pi)^{-1/2}\int_{-\infty}^{\infty} f(t)\exp(itp)dt$. Then $I(\tilde{f})=(2\pi)^{-1/2}\int_{-\infty}^{\infty} f(t)V(t)dt$ and $A_2I(\tilde{f})=I(\bar{\tilde{g}})A_2$ with $\tilde{g}(p)=\tilde{f}(-p)\,(p\in\mathbb{R})$. Let $(\tilde{f}_n)$ be a sequence of real-valued, uniformly bounded functions converging pointwise to the characteristic function $\chi_{(a,b)}$ of the open interval $(a,b)\subset\mathbb{R}$ $(f_n\in\mathscr{S}(\mathbb{R}))$. Then $I(\tilde{f}_n)\to I(\chi_{(a,b)})$ and $I(\tilde{g}_n)\to I(\chi_{(-b,-a)})$ in the strong operator topology. One deduces that $A_2I(\chi_{(a,b)})=I(\chi_{(-b,-a)})A_2$ and therefore $A_2E(\Delta)=E(-\Delta)A_2$ for every Borel set $\Delta\subset\mathbb{R}$. Let A be the generator of the group with $\operatorname{Sp}A\subset[0,\infty)$ implying $E(\Delta)=0$ for all Borel sets $\Delta\subset(-\infty,0)$. Because the one-parameter group does not contain the trivial representation as a subrepresentation $E(0)=0$. Combining this result with the spectral property of A we get $E((0,\infty))=\mathbb{1}$. Taking $\Delta=(0,\infty)$ we have proved $A_2=0$.

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References

1. Glauber, R.J.: Phys. Rev. **84**, 395–400 (1951)
2. Kibble, T.W.B.: J. Math. Phys. **9**, 315–324 (1968)
3. Roepstorff, G.: Commun. Math. Phys. **19**, 301–314 (1970)
4. Reents, G.: J. Math. Phys. **15**, 31–34 (1974)
5. Provost, J.P., Rocca, F., Vallée, G.: J. Math. Phys. **16**, 832–833 (1975)
6. Kraus, K., Polley, L., Reents, G.: Ann. Inst. Henri Poincaré **26**, 110–162 (1977)
7. Basarab-Horwath, P., Streater, R.F., Wright, J.: Commun. Math. Phys. **68**, 195–207 (1979)
8. Kraus, K., Streater, R.F.: J. Phys. A **14**, 2467–2478 (1981)
9. Polley, L., Reents, G., Streater, R.F.: J. Phys. A **14**, 2479–2488 (1981)
   Basarab-Horwath, P., Polley, L.: J. Phys. A **14**, 3029–3038 (1981)
10. Greenberg, O.W.: Ann. Phys. **16**, 158–176 (1961)
11. Robinson, D.W.: Commun. Math. Phys. **1**, 159–174 (1965)
12. Araki, H., Woods, E.J.: J. Math. Phys. **4**, 637–662 (1963)
13. Lewis, J.T., Pulè, J.V.: Commun. Math. Phys. **36**, 1–18 (1974)
14. Hochstenbach, W.J.M.A.: Commun. Math. Phys. **80**, 217–221 (1981)
15. Manuceau, J., Verbeure, A.: Commun. Math. Phys. **9**, 293–302 (1968)
16. Manuceau, J., Rocca, F., Testard, D.: Commun. Math. Phys. **12**, 43–57 (1969)
17. Daele, A. van, Verbeure, A.: Commun. Math. Phys. **20**, 268–278 (1971)
18. Daele, A. van: Commun. Math. Phys. **21**, 171–191 (1971)
19. Shale, D.: Trans. Am. Math. Soc. **103**, 149–167 (1962)
20. Reed, M., Simon, B.: Methods of modern mathematical physics, Vol. 3. Scattering theory. New York, London: Academic Press 1979
21. Araki, H.: Publ. RIMS **5**, 361–422 (1970)
22. Guichardet, A.: Bull. Sc. Math. **95**, 161–176 (1971): **98**, 201–208 (1974)
23. Guichardet, A.: Bull. Sc. Math. **96**, 305–332 (1972)
24. Greenleaf, F.P.: Invariant means on topological groups. New York, Toronto, London, Melbourne: Van Nostrand-Reinhold Company 1969
25. Fell, J.M.G.: Can. J. Math. **14**, 237–268 (1962)
26. Schaaf, M.: The reduction of the product of two irreducible unitary representations of the proper orthochronous quantummechanical Poincaré group. Berlin, Heidelberg, New York: Springer 1970
27. Gaal, S.A.: Linear analysis and representation theory. Berlin, Heidelberg, New York: Springer 1973

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