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Abelian Faces of State Spaces of C^*-Algebras

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Abstract. Let F be a closed face of the weak* compact convex state space of a unital C^*-algebra A. The class of F-abelian states, introduced earlier by the author, is studied further. It is shown (without any restriction on A or F) that F is a Choquet simplex if and only if every state in F is F-abelian, and that it is sufficient for this that every pure state in F is F-abelian. As a corollary, it is deduced that an arbitrary C^*-dynamical system (A, G, α) is G-abelian if and only if every ergodic state is weakly clustering. Nevertheless the set of all F-abelian (or even G-abelian) states is not necessarily weak* compact.

1. Introduction

In the algebraic model of quantum statistical mechanics, decompositions of the invariant states of a C^*-dynamical system (A, G, α) into ergodic states have become important [5]. Particular interest has centered on the question of whether the weak* compact convex set $S_G(A)$ of invariant states forms a Choquet simplex. Lanford and Ruelle [11] showed that this is the case if every invariant state ϕ is G-abelian in the sense that the restriction of $\pi_\phi(A)''$ to the subspace $\mathscr{H}_\phi^G$ of $u_\phi(G)$-invariant vectors in $\mathscr{H}_\phi$ is an abelian von Neumann algebra [where $(\mathscr{H}_\phi, \pi_\phi, u_\phi)$ is the covariant representation of (A, G, α) associated with ϕ]. (This fact was already implicit in [10].) The converse of this result was subsequently obtained by Dang-Ngoc and Ledrappier [7]. Meanwhile it had also been established that for an ergodic state ϕ, G-abelianness is equivalent to the "weak cluster property", namely

$$\inf\{|\phi(a'b) - \phi(a)\phi(b)|\} = 0$$

for all a and b in A, where the infimum is taken over all a' in the convex hull of the G-orbit of a. This raised the question whether every invariant state is G-abelian if every ergodic state is weakly clustering. Dang-Ngoc [6] used direct integral theory to establish this when A is separable.

Recently the present author [4], interested in the class $S_0(A, \alpha)$ of ground states associated with a (strongly continuous) one-parameter C^*-dynamical system

$(A, \mathbb{R}, \alpha)$, has introduced the class F^{ab} of "F-abelian" states in an arbitrary weak* closed face F of the state space $S(A)$. Amongst others, the following conditions on F were considered:

(Fa) $F^{ab} = F$,

(Fb) F is a simplex,

(Fc) $F^{ab} \cap P(A) = F \cap P(A)$,

where $P(A)$ denotes the set of all pure states in $S(A)$. The implications $(Fa) \Rightarrow (Fb) \Rightarrow (Fc)$ were proved in general, and $(Fc) \Rightarrow (Fa)$ was proved under certain conditions on A and F, automatically satisfied if A is separable or $F = S_0(A, \alpha)$. Although $S_G(A)$ is not in general a face of $S(A)$, it can be canonically identified with a face $F_G(A)$ of $S(G \times A)$, where $G \times A$ is the C^*-crossed product of (A, G, α), so the results of Dang-Ngoc and Ledrappier could be recovered from [4]. However this left unanswered the following conjecture, even for $F = F_G(A)$:

Conjecture 1. Conditions (Fa), (Fb), and (Fc) are equivalent.

Since F^{ab} is convex, an affirmative answer to Conjecture 1 would have followed from the Krein-Milman Theorem and an affirmative answer to the following:

Conjecture 2. F^{ab} is weak* compact.

In the case when $F = F_G(A)$, Conjecture 2 was originally suggested to the author by O. Bratteli (private communication).

Here Conjecture 1 will be answered affirmatively, and Conjecture 2 negatively. The former is established by developing an idea originally used in [3] in a study of ground states of a uniformly continuous system $(A, \mathbb{R}, \alpha)$. In that case, $S_0(A, \alpha)$ is the set of states annihilating the spectral projection of the minimal positive generator in A^{**} of α corresponding to the interval $(0, \infty)$. For general F, the ideal structure theory of C^*-algebras [9, 16] shows that there is some open projection q^F in A^{**} such that

$$F = \{\phi \in S(A) : \phi(q^F) = 0\}.$$

As in [3], Pedersen's non-commutative integration theory [13] now enables it to be shown that (Fa) and (Fc), and hence (Fb), are equivalent to each other and to:

(Fd) $1 - q^F$ is an abelian projection in A^{**}.

Alternatively it can be shown directly that (Fd) is equivalent to (Fb), and this gives a proof of the equivalence of (Fa) and (Fb) which differs essentially from previous ones even in the case when $F = F_G(A)$.

Conjecture 2 is shown to be false by extending some results from [4] to find subsets of $P(A)$ whose closed convex hulls are faces F for which $F^{ab} \cap P(A)$ is not weak* closed in $P(A)$. These examples also show that Conjecture 2 is false even if F is assumed to be of the form $S_0(A, \alpha)$ or $F_G(A)$.

Finally an affirmative answer will be given to the following conjecture originally made in [4]:

Conjecture 3. Every metrisable Choquet simplex is affinely homeomorphic to a face of the state space of some separable C^*-algebra.

2. Abelian Faces

Let A be a unital C^*-algebra with state space $S(A)$, which will be assumed to have the weak* topology. Let F be a closed face of $S(A)$, and put

$$J^F = \{a \in A : \phi(a^*a) = 0 \text{ for all } \phi \text{ in } F\}.$$

Then J^F is a closed left ideal of A, so any increasing approximate unit for J^F converges weakly to a projection q^F in the enveloping von Neumann algebra A^{**} of A. It is well-known [9, 14, 16] that

$$J^F = A^{**}q^F \cap A$$
$$F = \{\phi \in S(A) : \phi(J^F) = 0\} = \{\phi \in S(A) : \phi(q^F) = 0\},$$

where $S(A)$ is identified with the normal state space of A^{**}. Furthermore F may be identified with the normal state space of $p^F A^{**} p^F$, where $p^F = 1 - q^F$.

Let $(\mathcal{H}_\phi, \pi_\phi, \xi_\phi)$ be the Hilbert space, normal representation of A^{**} and cyclic vector associated with a state ϕ in F. Following [4], we shall let $\mathcal{H}_\phi^F$ be the linear subspace of $\mathcal{H}_\phi$ spanned by unit vectors η such that the vector state $a \to \omega_\phi^\eta(a) = \langle \pi_\phi(a)\eta, \eta \rangle$ belongs to F, and p_ϕ^F be the projection of $\mathcal{H}_\phi$ onto $\mathcal{H}_\phi^F$. We shall say that ϕ is *F-abelian* if the von Neumann algebra $p_\phi^F \pi_\phi(A)'' p_\phi^F$ is abelian, and that F is *abelian* if every state in F is F-abelian, i.e. if (Fa) is satisfied.

Proposition 1. *Let F be a closed face of $S(A)$, and U^F be the set of all unitaries u in A with $\phi(u) = 1$ for all ϕ in F. For a in A, let $C(a, U^F)$ be the convex hull of $\{u^*au : u \in U^F\}$. Let ϕ be any state in F. Then*

(i) $\mathcal{H}_\phi^F = \pi_\phi(p^F)\mathcal{H}_\phi = \{\eta \in \mathcal{H}_\phi : \pi_\phi(u)\eta = \eta$ *for all u in* $U^F\}$.

(ii) *ϕ is F-abelian if and only if for any a and b in A and η in $\mathcal{H}_\phi^F$,*

$$\inf\{|\omega_\phi^\eta(a'b - ba')| : a' \in C(a, U^F)\} = 0.$$

(iii) *ϕ is pure and F-abelian if and only if $\mathcal{H}_\phi^F$ is one-dimensional, or equivalently if and only if, for any a and b in A,*

$$\inf\{|\phi(a'b) - \phi(a)\phi(b)| : a' \in C(a, U^F)\} = 0.$$

Proof. For any unit vector η in $\mathcal{H}_\phi$,

$$\eta \in \mathcal{H}_\phi^F \Leftrightarrow \omega_\phi^\eta(q^F) = 0 \Leftrightarrow \eta \in \pi_\phi(p^F)\mathcal{H}_\phi.$$

If η does belong to $\mathcal{H}_\phi^F$, then for any u in U^F,

$$\langle \pi_\phi(u)\eta, \eta \rangle = \omega_\phi^\eta(u) = 1$$

so $\pi_\phi(u)\eta = \eta$. Conversely if η is outside $\mathcal{H}_\phi^F$, there is some a in J^F such that $\omega_\phi^\eta(a) \neq 0$, so $\omega_\phi^\eta(a^*a) > 0$. Spectral theory shows that there is a unitary u such that $1 - u$ lies in the C^*-algebra generated by a^*a, and hence in J^F, but $\omega_\phi^\eta(u) \neq 1$. Then u belongs to U^F, but $\pi_\varphi(u)\eta \neq \eta$. This completes the proof of (i).

Since U^F consists of those unitaries u for which $\psi(au) = \psi(a)$ $(a \in A, \psi \in F)$, it is a subgroup of the unitary group of A and acts on A via inner automorphisms. Now the result of (i) shows that $\mathcal{H}_\phi^F = \mathcal{H}_\phi^{U^F}$. Thus (ii) and (iii) follow immediately from corresponding results about G-abelian states [5, Proposition 4.3.7, Theorems 4.3.17, 4.3.22].

Parts (ii) and (iii) of Proposition 1 can alternatively be proved by simply rewording the proofs in [5].

Theorem 2. *The following conditions on a closed face F of $S(A)$ are equivalent:*
(a) *F is abelian.*
(b) *Every pure state in F is F-abelian.*
(c) *$p^F A^{**} p^F$ is abelian.*

Proof. The implication (a)⇒(b) is trivial, and (c)⇒(a) follows immediately from the fact that $p^F_\phi \pi_\phi(A)'' p^F_\phi = \pi_\phi(p^F A^{**} p^F)$ by Proposition 1(i).

Suppose that (b) is satisfied, and let π be an irreducible representation of A on a Hilbert space $\mathscr{H}$. For any unit vector η in $\pi(p^F)\mathscr{H}$, the corresponding vector state ω^η_π of A is a pure state in F, and is therefore F-abelian. Identifying π_ϕ with π, Proposition 1(i) shows that $\pi(p^F A^{**} p^F)$ is abelian. Thus $\pi(p^F)$ is of rank 0 or 1 for each irreducible representation π. Since p^F is the weak limit of a decreasing net in A, it now follows as in [3, Proposition 2.5(ii)] that $p^F A^{**} p^F$ is abelian.

Corollary 3. *The seven conditions of [4, Theorem 2.5] are equivalent for any closed face F of $S(A)$.*

Proof. This is immediate from Theorem 2 and the partial result obtained in [4, Theorem 2.5].

Corollary 3 gives an affirmative answer to Conjecture 1, and Theorem 2 shows that (Fd) is also equivalent to (Fb). It may be of independent interest to note that there is an alternative proof of this. For the self-adjoint part of $p^F A^{**} p^F$ is order-isomorphic to the Banach dual of the real linear span of F, and is therefore lattice-ordered if and only if F is a simplex [1, Sect. II.3]. But it is well-known [17] that the self-adjoint part of a C^*-algebra B is lattice-ordered if and only if B is commutative.

Corollary 4. *The seven conditions of [4, Corollary 4.4] are equivalent for any C^*-dynamical system (A, G, α).*

Proof. This follows on applying Corollary 3 to the face $F_G(A)$ of $S(G \times A)$ defined in [4, Theorem 4.2].

In [4], the crossed product $G \times A$ considered was that arising when G is taken to be discrete, and this is sufficient to prove Corollary 4. However if G has a locally compact topology in which α is strongly continuous, and the crossed product is taken for this topology, [4, Theorem 4.2] remains valid provided the representations σ of A and θ of G are interpreted as taking values in the multiplier algebra of $G \times A$. Although $G \times A$ may be non-unital, the existence of a unit is merely a convenient convention in [4] and Theorem 2 above. Thus any crossed product could have been used in the proof of Corollary 4.

The method of lifting invariant states to a C^*-crossed product $\mathbb{R}^n \times A$ (with the Euclidean topology on $\mathbb{R}^n$) and considering the corresponding projection in $(\mathbb{R}^n \times A)^{**}$ was used by Kastler and Robinson [10] to show that the invariant states of an asymptotically abelian system $(A, \mathbb{R}^n, \alpha)$ form a simplex.

3. Compactness of the F-Abelian States

Let $\{\phi_i : i \in I\}$ be a family of states of A, and ψ be a state in the σ-convex hull of $\{\phi_i\}$, so that $\psi = \sum \lambda_i \phi_i$ for some real numbers $\lambda_i \geqq 0$ with $\sum \lambda_i = 1$. Put $\mathscr{H} = \oplus \mathscr{H}_{\phi_i}$, $\pi = \oplus \pi_{\phi_i}$, $\xi = \oplus \lambda_i^{1/2} \xi_{\phi_i} \in \mathscr{H}$. Then $\psi = \omega_\pi^\xi$, so π_ψ is unitarily equivalent to the restriction of π to the cyclic subspace $[\pi(A)\xi]$ of $\mathscr{H}$. Furthermore if ψ' is any state in the face of $S(A)$ generated by ψ, then there is an operator x in $\pi(A)'$ such that $\psi' = \omega_\pi^{x\xi}$ [8, Proposition 2.5.1], so $\pi_{\psi'}$ is unitarily equivalent to the restriction of π to $[\pi(A)x\xi]$.

These remarks lead immediately to the following result.

Proposition 5. *Let π be any representation of A, and $S_\pi(A)$ (resp. $S'_\pi(A)$) be the σ-convex (resp. convex) hull of the vector states ω_π^η of A in the representation π. Then $S_\pi(A)$ (resp. $S'_\pi(A)$) is a face of $S(A)$ consisting of the states whose GNS-representations are contained in an arbitrary (resp. finite) direct sum of copies of π.*

The next result shows how new faces of $S(A)$ may be constructed out of given ones, and is an extension of [4, Proposition 3.1].

Proposition 6. *Let $\{F_i : i \in I\}$ be a collection of faces of $S(A)$, and suppose that*

$$\phi \in F_i,\ \psi \in F_j,\ i \neq j \Rightarrow \pi_\phi \text{ and } \pi_\psi \text{ are disjoint}.$$

Then the convex hull of $\{F_i\}$ is a face of $S(A)$. If each F_i is σ-convex, then the σ-convex hull of $\{F_i\}$ is also a face of $S(A)$.

Proof. Consider the σ-convex case (the other is similar). Any state ψ in the σ-convex hull is of the form $\sum \lambda_i \phi_i$ where $\lambda_i \geqq 0$ and ϕ_i belongs to F_i. If $\pi = \oplus \pi_{\phi_i}$, then $\pi(A)' = \oplus \pi_{\phi_i}(A)'$ since the representations π_{ϕ_i} are disjoint [8, Proposition 5.2.4]. If ψ' belongs to the face generated by ψ, then by the remarks at the beginning of this section, $\psi' = \omega_\pi^{x\xi}$ for some x in $\pi(A)'$. If $x = \oplus x_i$ where x_i belongs to $\pi_{\phi_i}(A)'$ and $\phi'_i = \omega_{\phi_i}^{\xi'_i} \in F_i$, where $\xi'_i = x_i \xi_{\phi_i}$, then $\psi' = \sum \lambda_i \phi'_i$. This completes the proof.

Let $\hat{A}$ be the spectrum of A. We shall not distinguish notationally between an irreducible representation π of A and its unitary equivalence class in $\hat{A}$.

Corollary 7. *For each π in $\hat{A}$, let F_π be a face of $S_\pi(A)$. The convex hull of $\{F_\pi\}$ is a face of $S(A)$. If each F_π is σ-convex, the σ-convex hull of $\{F_\pi\}$ is also a face of $S(A)$.*

Proof. Proposition 5 shows that $\{F_\pi : \pi \in \hat{A}\}$ is a family of faces of $S(A)$ satisfying the condition of Proposition 6.

Theorem 8. *Let ϕ_i ($i \in I$) be a net of mutually inequivalent pure states of A which converge to a pure state ψ, and suppose that ψ is not multiplicative and is not equivalent to ϕ_i for any i. Then there is a closed face F of $S(A)$ such that ϕ_i is F-abelian ($i \in I$), but ψ is not F-abelian.*

Proof. Since ψ is not multiplicative, there is a pure state ψ' equivalent to, but distinct from, ψ. Let F' be the face of $S(A)$ generated by ψ and ψ'. Then F' is contained in $S_{\pi_\psi}(A)$ and is affinely homeomorphic to a 3-dimensional Euclidean ball [2, Corollary 3.4]. In particular F' is compact and σ-convex. Let F be the σ-convex hull of F' and $\{\phi_i : i \in I\}$. By Corollary 7, F is a face of $S(A)$. Also the closure $\bar{F}$ of F is the convex hull of F' and the closed convex hull C of $\{\phi_i\}$. But [4, Theorem 3.2] shows that C is the σ-convex hull of $\{\phi_i\}$ and ψ. Hence $\bar{F}$ is contained in the σ-convex hull of F', $\{\phi_i\}$ and ψ, so F is closed.

The extreme points of F are the inequivalent states ϕ_i and the extreme points of F', all of which are equivalent to ψ. It follows immediately from [4, Lemma 2.3] that ϕ_i is F-abelian, but ψ is not F-abelian.

Corollary 9. *Let A be a simple, separable, infinite-dimensional, unital C^*-algebra. There is a uniformly continuous one-parameter C^*-dynamical system $(A, \mathbb{R}, \alpha)$ on A such that $S_0(A,\alpha)^{ab} \cap P(A)$ is not closed in $P(A)$. Furthermore the set of all $\mathbb{R}$-abelian ergodic states is not closed in the set of all ergodic states.*

Proof. There is a sequence of mutually inequivalent pure states ϕ_n of A converging to another inequivalent pure state ψ (see the proofs of [4, Corollaries 3.3, 3.4]). Since ψ is not multiplicative, Theorem 8 shows the existence of a closed face F such that ϕ_n is F-abelian but ψ is not F-abelian, so $F^{ab} \cap P(A)$ is not closed in $P(A)$. Since A is simple and separable, [4, Theorem 5.2] shows that $F = S_0(A, \alpha)$ for some uniformly continuous one-parameter system $(A, \mathbb{R}, \alpha)$.

The final statement follows from the fact that for a ground state ϕ, $\mathscr{H}_\phi^{S_0} = \mathscr{H}_\phi^{\mathbb{R}}$, so ϕ is $S_0(A,\alpha)$-abelian if and only if ϕ is $\mathbb{R}$-abelian (cf. [4, §5]).

Corollary 9 gives a negative answer to Conjecture 2.

4. The CAR Algebra

In this final section, we shall consider some C^*-dynamical systems on the algebra of canonical anticommutation relations to show specific examples of systems with the properties given by Corollary 9, and also to answer Conjecture 3.

For each integer j, let A_j be a copy of the C^*-algebra of 2×2 complex matrices, and D_j be the subalgebra of diagonal matrices in A_j. The infinite C^*-tensor product A of $\{A_j : j \in \mathbb{Z}\}$ is the *CAR algebra* (or *Fermion algebra*) [5, 14], and is simple. The C^*-tensor product D of $\{D_j : j \in \mathbb{Z}\}$ is a commutative C^*-subalgebra of A, whose spectrum may be identified with $\{0,1\}^{\mathbb{Z}}$ in the product topology. Here $\varepsilon : \mathbb{Z} \to \{0,1\}$ corresponds to the restriction to D of the pure state $\phi_\varepsilon = \bigotimes_j \phi_{\varepsilon j}$ where

$$\phi_{\varepsilon j}\begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix} = \begin{cases} \lambda_{11} & \text{if} \quad \varepsilon(j)=0 \\ \lambda_{22} & \text{if} \quad \varepsilon(j)=1 . \end{cases}$$

Note that any state coinciding with ϕ_ε on D is ϕ_ε itself, since the finite subproducts of $\{A_j : j \in \mathbb{Z}\}$ have the same property. Also ϕ_ε and $\phi_{\varepsilon'}$ are equivalent if and only if ε and ε' are equivalent in the sense that $\varepsilon(j) = \varepsilon'(j)$ for all except finitely many j [14, Proposition 6.5.6].

Let $\mathscr{E}$ be a closed subset of $\{0,1\}^{\mathbb{Z}}$. There is a positive operator x in D such that

$$\phi_\varepsilon(x) = 0 \Leftrightarrow \varepsilon \in \mathscr{E} .$$

For example one might take

$$x = \sum_{n=1}^{\infty} 2^{-n} \prod_{\varepsilon \in \mathscr{E}} \left\{ 1 - \prod_{j=-n}^{n} p_{\varepsilon j} \right\},$$

where $p_{\varepsilon j}$ is the projection in D_j (regarded as a subalgebra of D) given by

$$p_{\varepsilon j}=\begin{cases}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} & \text{if } \varepsilon(j)=0\\ \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} & \text{if } \varepsilon(j)=1.\end{cases}$$

If $F=\{\phi\in S(A):\phi(x)=0\}$, then $F=S_0(A,\alpha_x)$, where

$$\alpha_x(t)(a)=\exp(itx)a\exp(-itx) \qquad (a\in A).$$

If $\mathscr{E}$ contains two equivalent functions ε and ε', then ϕ_ε is not F-abelian. If ε in $\mathscr{E}$ is not equivalent to any ε' in the closure of $\mathscr{E}\backslash\{\varepsilon\}$, then for $n\geqq 1$, there is an operator x_n of norm 1 in the tensor product of $\{D_j:|j|>n\}$ (regarded as a subalgebra of D) such that

$$\phi_\varepsilon(x_n)=1$$

$$\phi_{\varepsilon'}(x_n)=0 \qquad (\varepsilon'\in\mathscr{E};\ \varepsilon'\neq\varepsilon).$$

For ϕ in F, the restriction of ϕ to D, regarded as a probability measure on $\{0,1\}^{\mathbb{Z}}$, is carried by $\mathscr{E}$. Thus if $\mathscr{E}$ is countable, there are non-negative real numbers $\lambda_{\varepsilon'}$ $(\varepsilon'\in\mathscr{E})$ with $\sum\lambda_{\varepsilon'}=1$ such that ϕ coincides with $\sum\lambda_{\varepsilon'}\phi_{\varepsilon'}$ on D. Now $\phi(x_n)=\lambda_\varepsilon$, so if ϕ is equivalent to ϕ_ε, then for some unitary u in A (independent of n),

$$|1-\lambda_\varepsilon|=|\phi_\varepsilon(x_n)-\phi(x_n)|=|\phi(u^*x_nu-x_n)|\leqq 2\delta_n,$$

where δ_n is the least distance of u from the C^*-tensor product of $\{A_j:|j|\leqq n\}$. Since $\delta_n\to 0$ as $n\to\infty$, $\lambda_\varepsilon=1$, so ϕ coincides with ϕ_ε on D and hence on A. Thus ϕ_ε is F-abelian.

This discussion shows that in order to exhibit the effects described in Corollary 9, it suffices to choose $\mathscr{E}$ to consist of two equivalent functions ε_0 and ε_0' and a sequence of mutually inequivalent functions ε_n converging in $\{0,1\}^{\mathbb{Z}}$ to ε_0.

There is a natural C^*-dynamical system $(A,\mathbb{Z},\beta)$ on the CAR algebra in which $\beta(n)$ restricts to the identity mapping of A_j onto A_{j+n}. This system is $\mathbb{Z}$-abelian, and the ergodic states are dense in $S_{\mathbb{Z}}(A)$ [5, Example 4.3.26]. Hence $S_{\mathbb{Z}}(A)$ is (affinely homeomorphic to) the Poulsen simplex K [12, 15]. Thus K is realized as the face $F_{\mathbb{Z}}(A)$ of $S(\mathbb{Z}\times A)$. An arbitrary metrizable simplex is a face of K and hence of $S(\mathbb{Z}\times A)$, so Conjecture 3 is established. It is worth noting that $\mathbb{Z}\times A$ is simple [14, Theorem 8.11.12].

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