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Barrelled locally convex spaces, by P. Pérez Carreras and J. Bonet. North-Holland Math. Stud., vol. 131, (L. Nachbin, ed.), North-Holland, Amsterdam, New York, Oxford, Tokyo, 1987, xv +512 pp., $80.00. ISBN 0-444-70129-X

The book targeted by this review is truly a landmark, projecting barrelledness as the major force which disciplines the general theory of locally convex topological vector spaces (lctvs); it is the definitive study of strong and weak barrelledness structure theory, with a novel choice of applications (Chapters 10–12); it is a unique assimilation of half a century's scholarship, comprehensive, coherent, current, with a marvelous collection of open problems (Chapter 13). In his fanciful flight over an area he has roosted in for two decades, the reviewer finds the book the monument which most powerfully stimulates and facilitates fresh contributions (see below), contributions particularly urgent in view of the fertile open problems and certain creatively correctable mistakes (a modest price for such timeliness). A clearer bird's-eye view of strong barrelledness unfolds (see Figures 1–4).

The recent demise of the beloved patriarch, Prof. Dr. G. Köthe, recalls the historically grand German tradition in topological vector spaces (tvs). Now Spain has emerged a leader with the advent of Prof. M. Valdivia and his prolific followers, including the authors P. Pérez Carreras and J. Bonet. Unfortunately, some of their important results appear in Spanish publications not widely available. Fortunately, the book under review redresses this situation beautifully, transporting *to* the New World a cargo richer than the plunder of conquistadors. As in a Goya painting, the book's subject is robust and beguilingly composed. Although some knowledge of tvs's is requisite, to most in the area the book is necessary and sufficient. Chapters 10, 11, and 12, respectively, make the case that those in $C(X)$ spaces, tensor products, and infinite holomorphy may also profit from the beautiful structure theory of Chapters 1–9, where a cohesive account of strong and weak barrelledness conditions is given, including a parallel development of quasibarrelled conditions, with copious relevant material on (trans)separability and minimality; Baire, Mackey, Fréchet, and Banach spaces; B- and B_r-complete spaces; (ultra)bornological spaces; (DF)- and (LF)-spaces; quojections; closed graph theorems; etc. Obversely, structure theorists have a nicely segued introduction to applications in Chapters 10–12. The earlier informed remarks of a more applications-oriented reviewer [3] are weighted heavily in the book's favor. The tables, the index, and the lists of symbols and abbreviations all enhance the unprecedented accessibility of the latest and best information distilled from the reference section's more than 500 articles and books.

There are three fundamental principles of functional analysis. One of them, the Hahn-Banach theorem, holds in *any* lctvs. The other two are supported in Fréchet spaces (complete metrizable lctvs) by the Baire category theorem. But they also hold in (LF)-spaces, a larger applications-rich class of lctvs, though proper (LF)-spaces are never Baire. Ultimately, via a form of the Hahn-Banach theorem, Bourbaki and Dieudonné defined and characterized barrelled ("tonnelé") spaces as the largest class of lctvs in which the fundamental Uniform Boundedness Principle holds, and A. P. and W. J. Robertson, Pták, and Mahowald showed it is also the largest class for which the fundamental Open Mapping/Closed Graph Theorem holds vis-à-vis Fréchet spaces. Clearly, this is a "fundamentally" important class.

A Hausdorff lctvs is *barrelled* if every barrel (closed, absolutely convex, absorbing set) is a neighborhood of 0. From the Bipolar Theorem, E is barrelled if and only if, given any set B of continuous linear functionals ($B \subset E'$) such that, for each $x \in E$, the scalar set $\{f(x) : f \in B\}$ is bounded, there exists a neighborhood U of 0 in E such that $\{f(x) : x \in U, f \in B\}$ is bounded. That is, pointwise boundedness implies uniform boundedness (equicontinuity). Thus, the pointwise limit of a sequence in E' is also in E', a basic property which, by [4], characterizes barrelledness in metrizable lctvs's. In general, the property is strictly weaker than ℓ^∞-barrelledness, which is strictly weaker than barrelledness. (A Hausdorff locally convex space E is ℓ^∞-*barrelled* if every *countable* pointwise bounded set in E' is equicontinuous.)

A number of such "weak barrelledness conditions" are discussed in Chapter 8, and the study of five conditions between Baireness and barrelledness occupies Chapter 9. An imperative for the study of weak and strong barrelledness conditions is the need for "good" permanence properties, i.e., for classes of spaces with desirable defining properties preserved under the formation of quotients, Cartesian products, countable-codimensional subspaces, etc. As amply demonstrated in the book, Baire and Fréchet spaces have "bad" (i.e., few) permanence properties, while barrelled spaces are notoriously "good." All of the strong barrelledness conditions of Chapter 9 exhibit excellent permanence properties, and to varying extents this is the case for the weak barrelledness conditions in Chapter 8. A rich array of distinguishing examples is provided.

A barrelled space is [*quasi-Baire* (QB)] ⟨*Baire-like* (BL)⟩ if it is not the union of an increasing sequence of nowhere dense [subspaces] ⟨absolutely convex sets⟩. Clearly, BL $\Longrightarrow$ QB $\Longrightarrow$ barrelled, and by Amemiya-Kōmura [1], [metrizable $\wedge$ barrelled] $\Longrightarrow$ BL. So, by [1] and [4]: *Every metrizable ℓ^∞-barrelled space is not only barrelled but also* BL. The formal study of degrees of Baireness and barrelledness often provides, as does barrelledness itself, the precise class in which a theorem or technique is valid, and good permanence properties extend applicability. For example, Theorem 4.7.1 of [9], predating BL spaces [1, 18], is proved separately for E either a Baire space or a metrizable barrelled space, whereas the natural general setting is for E a BL space. Moreover, large products, say, of either locally convex (lc) Baire or metrizable barrelled spaces need not be of the same type, but are still BL, since BL spaces are permanent under products [18]. Let us add to the sizable list of permanence properties enjoyed by ℓ^∞-barrelled spaces and see more of their interaction with QB and BL spaces. Corollary 8.2.20 states: *Every separable ℓ^∞-barrelled space is barrelled.* This interesting result is uncredited, as is Proposition 9.1.5, which should read: *If F is a dense ℓ^∞-barrelled subspace of a* BL *space E, then F is* BL (Theorem 1.1 of [36]). We may replace "BL" by "QB" (same proof) or by "barrelled" (easy proof). The "barrelled" version of 9.1.5 is crucial in [19,34]. Propositions 8.2.31 and 8.2.33 tell us that: *The class of ℓ^∞-barrelled spaces is stable (closed) under the formation of separated quotients, direct sums, inductive limits, completions, products, and countable-codimensional subspaces.* We add [20] the answer to Question 13.8.16: *The ℓ^∞-barrelled spaces affirm the three-space problem;*

i.e., if E is a Hausdorff locally convex space with a closed subspace M such that M and E/M are both ℓ^∞-barrelled, then so also is E [the third space]. The proof is surprisingly simple.

A stimulating Notes and Remarks section ends each chapter and assigns credit, of particular importance in a book which so greatly reduces the need for original sources. The authors insightfully credit Eidelheit [7] for proving in 1936, long before its unattributed appearance in Köthe's Volume I [11], the basic result that: *Every non-normable Fréchet space has a quotient isomorphic to $\omega(\equiv K^{\mathbb{N}})$*. They show (page 331) that the reviewer was not the first to discover noncomplete metrizable (LF)-spaces; that other (4.7.1(i), 4.6.6) ideas of his [16,36] were first Köthe's [11] and S. Dierolf's [5]. The reviewer welcomes such enlightenment with the hope of forgiveness where needed. Forgiveness may, on occasion, be extended as well to the authors. Proposition 4.5.6 on barrelled countable enlargement (BCE) is an important uncredited result of Robertson, Tweddle, and Yeomans [17]. Proposition 4.5.22, the Saxon-Levin result [30] that: *Every algebraic complement of a closed countable-codimensional subspace of a barrelled space is a topological complement with its strongest locally convex topology*, is likewise unattributed, perhaps because it is now well known. Related is one of Saxon's first strong barrelledness results: *A barrelled space is QB if and only if it does not contain a complemented copy of φ* [$\equiv K^{(\mathbb{N})}$, a denumerable-dimensional vector space with its strongest locally convex topology]. This result is presented as Propositions 8.8.3 and 9.1.12, and credited both times to "Bonet, Perez Carreras, (5)", with no regard to its prior publication (sans proof) on both pages 88 and 98 of [18]. The original, simple proof [32] has had limited circulation since 1980 in a preprint of [32], since 1970 in mimeographed notes of Saxon's University of Florida seminar. But the idea for this easy proof goes back at least as far as Karlin's 1948 paper ([10], Theorem 8, Sufficiency), and seems less contrived than the authors' proof. (To simplify the authors' proof, page 316, one need only observe that the subspace $\bigcap_{n=1}^{\infty} v_n^{\perp}$ of F is closed and denumerable-codimensional, so that the above Saxon-Levin result applies.)

A "major highlight" [3], the weighty Chapter 8 triply confuses (LF)-spaces. The definition (pages 2 and 3) does not require the inductive sequence of Fréchet spaces to cover the entire space E. This would mean that *every* vector space with its strongest lc topology is an (LF)-space (take each $E_n = \{0\}$), denying the

Köthe-Grothendieck Open Mapping/Closed Graph Theorems (1.2.20 and 8.4.11), for example. Next consider Hausdorffness: if there exists a non-Hausdorff (LF)-space, then Proposition 8.4.10 and the Köthe-Grothendieck Open Mapping Theorem 8.4.11 fail, as does the proof of 8.4.6 (cf. [33]); a mapping has closed graph only when the range is Hausdorff. Although it is not until §8.8 that the authors declare "All (LF)-spaces here are assumed to be Hausdorff," the most practical recourse is to assume Hausdorffness throughout, as did the theorems' original authors. Be it here defined that a Hausdorff lctvs (E, τ) is a [*proper*] (LF)-*space* if there exists a [properly] increasing sequence of Fréchet spaces (E_n, τ_n) such that (i) $E = \bigcup_{n=1}^{\infty} E_n$, (ii) each (E_n, τ_n) dominates (E_n, τ_{n+1}), and (iii) τ is the finest Hausdorff lc topology for which each (E_n, τ_n) dominates (E_n, τ). (One lctvs F (*strictly*) *dominates* another, G, if F and G coincide as vector spaces and the topology of F is (strictly) finer than that of G.) Obviously, in this context the open problem 13.8.12 "Find conditions under which an (LF)-space is Hausdorff" would need rephrasing. The third confusion concerns properness of (LF)-spaces: [31–33] required (LF)-spaces to be proper, and failure to account for this makes Propositions 8.6.15 and 8.8.13 false.

Proposition 8.8.10 holds only if "infinite-dimensional" is deleted (cf. [32, 33]), but still applies in the proof of 8.8.12, the most general version of which is [21–23, 32]: *Every countable-codimensional subspace of an infinite-dimensional* [*non-normable*]⟨*proper*⟩*metrizable* (LF)-*space has an infinite-dimensional* ⟨*separable*⟩ *Fréchet quotient* [*isomorphic to* $\omega(\equiv K^{\mathbb{N}})$]. This also extends Eidelheit's initial result [7] and the authors' [12], in answer to Question 13.2.2. Valdivia's Lemma 6.3.1 and proof are creatively correctable [29].

Whether every infinite-dimensional Banach (Fréchet) space has a properly separable (Hausdorff with separable dense proper subspace [14]) quotient is a venerable still-open question from Banach's era. Saxon-Wilansky [35] showed that a Banach space has a properly separable quotient if and only if it has a dense non-barrelled subspace (Proposition. 4.6.5), hence Question 13.4.2. Juxtaposed is Question 13.4.1: "Let E be a barrelled space not endowed with the strongest lc topology. Does E have a BCE?" Chapter 4's pioneering partial answers by Robertson, Tweddle, and Yeomans [17, 37] include Proposition 4.5.6: *If* E *has a dense barrelled subspace* F *with* $\dim(E/F) \geq c$, *then* E *has a* BCE.

In both questions, then, a dense subspace of codimension $\geq c$ is desirable (cf. [30] and below), nonbarrelled in the one case, barrelled in the other! More partial answers to 13.4.1 include [34]: *Every properly separable barrelled space has a* BCE, *asssuming the continuum hypothesis* (CH). The proof shows that such a space E is barrelledly fit. (E is *(barrelledly) fit* [24, 34] if there is a dense (and barrelled) subspace F with $\dim(E/F) = \dim(E)$.) Assuming a condition weaker than the generalized CH, we can prove [25] that: *Every barrelled space E is the direct sum of two subspaces, one fit and the other with its strongest lc topology.* This quickly suggests an alternate to Question 13.4.1: Is every barrelled, fit space barrelledly fit?

Complete and partial answers to more of the open questions are forthcoming and there is the exciting report [3] that Taskinen has very recently solved "some of the most interesting" problems, including 13.11.7. The short answer to 13.11.4, "Is there a Fréchet space E such that $E \otimes_\pi E'_b$ is barrelled?", is clearly "Yes, apply Proposition 11.2.2." Surely, the authors intended the question for *non-normable* Fréchet spaces. Similarly, one could give a trite negative answer to 13.1.1: "Does every Banach space have a dense non-Baire hyperplane?" Obviously, the authors intend the question for *infinite-dimensional* Banach spaces, and the question becomes very interesting indeed. Assuming Martin's Axiom (MA), Arias de Reyna [2] answered "Yes" for properly separable Banach spaces, thereby denying the Wilansky-Klee conjecture, and Valdivia [38] is credited with the more general Theorem 1.2.12 which says: *Every properly separable Baire space has a dense non-Baire hyperplane, assuming* MA. The authors present a proof with substantial technical deficiencies. Nevertheless, [27] generalizes Theorem 1.2.12 and provides the nontrite positive answer to the (intended) Question 13.1.1, assuming MA.

The study of weak Baire and strong barrelledness conditions evolved from the original Dieudonné-Bourbaki view:

$$\text{Baire} \Longrightarrow \text{barrelled}$$

to the vision espoused by the authors (pages 348 and 507, Table 5; cf. [17, 18, 31, 36, 39, 42]):

$$\text{Baire} \Longrightarrow \text{UBL} \Longrightarrow \text{TB} \Longrightarrow \text{SB} \Longrightarrow \text{BL} \Longrightarrow \text{QB} \Longrightarrow \text{barrelled}.$$

(Here, as in the book, UBL is unordered Baire-like, TB is totally barrelled, SB is suprabarrelled; see below.) Distinguishing examples are given in Chapter 9. All permanence properties obtain except inductive limits, and closed graph/ open mapping theorems play an important role, as do the separable quotient problem and the classification of (LF)-spaces. The introduction [8] of quasi-suprabarrelled (QSB) spaces occasions here (and in [26]) a comprehensive view inspired by [15, 17]. We say that a barrelled space E is a *db space*, or is *db*, if given any increasing sequence of subspaces covering E, at least one of the subspaces must be dense and barrelled. We define *d spaces* (resp., *b spaces*) as above, with the deletion of "and barrelled" (resp., "dense and"). We define *udb*, *ud*, and *ub* just as we do *db*, *d*, and *b* above, respectively, with "increasing" deleted. The *db* spaces were introduced in [17] as (*db*)-spaces, in [39] as SB spaces; $d \equiv$ QB, $b \equiv$ QSB, and by Theorem 2.2 of [36], $udb \equiv$ UBL. Also, "*ub*" is merely a new designation for "condition (G)" of [13], and *ud* spaces are just those barrelled spaces that are Baire-hyperplane [40]. The new unified nomenclature suggests the unforgettably clear relationships depicted in Figure 1. We also have (1) $[ud \wedge ub] \Rightarrow udb$ by Theorem 4.1 of [36], and (2) $[d \wedge b] \Rightarrow db$, trivially.

No other nonapparent relationships exist. For example, $[ud \wedge b] \Rightarrow db$ from (2), but $[ud \wedge b] \not\Rightarrow u(d)b$, and barrelled $\not\Rightarrow [d \vee b]$. While advantages of a perfectly symmetric, highly mnemonic notation are obvious, the older terms evoke a still-warm history of human endeavor we are loathe to ignore.

A barrelled space E is *totally barrelled* (TB) [42] if, given any covering sequence of subspaces, one of the subspaces must be barrelled and have a finite-codimensional closure. In Figure 2 we readily see how TB and BL spaces fit into the general scheme. Again, all valid relationships follow immediately from this and (1) and (2), with a possible exception: We do not know whether $[ub \wedge d(b)] \Rightarrow$ TB. The authors' Proposition 9.3.3 (cf. [13])

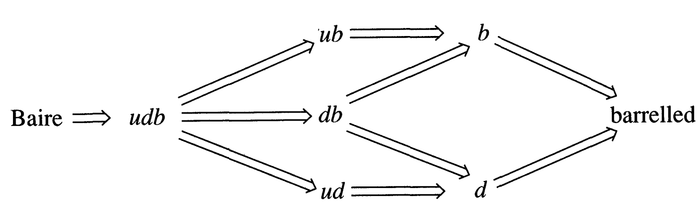

FIGURE 1. General barrelled spaces.

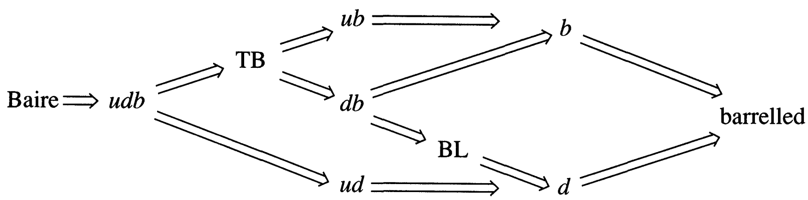

FIGURE 2. General barrelled spaces.

offers partial answers: *If* E *is ub and d(b), then* E *is* TB *provided either* (i) E *is separable, or* (ii) E *does not contain* φ. Of the three classes outside the authors' linear scheme, the *ud* spaces possess all the permanence properties one could expect. However, the *b* and *ub* spaces are problematical: Here questions remain for the three-space problem, completions, and products, although the question of products is reduced to finite products (cf. [8]). The answers [26] are again positive in cases (i) and (ii).

For metrizable barrelled spaces we have no such questions (cf. [26]).

Many Banach spaces contain dense proper (LF)-subspaces (cf. 8.7.9 and [33]), partially answering 13.8.14. By Theorem 9.1.30 (cf. [26, 31–33, 35]): *a Banach space has a properly separable quotient if and only if it has a dense subspace dominated by a proper* (LF)*-space.* Proposition 8.8.13 (corrected) states: *Every barrelled space dominated by a proper* (LF)*-space has an infinite-dimensional separable Hausdorff quotient.* (LF)-spaces and the larger class of barrelled spaces dominated by (LF)-spaces have identical relationships all indicated below, excepting fact (1)/(2).

Since the nonproper (LF)-spaces [$(\mathrm{LF})_4$-spaces] coincide with the Fréchet spaces, Figures 3 and 4 on p. 432 indicate that a proper (LF)-space is BL but not *(d)b* if it is metrizable. The converse holds [32]: *For* E *a proper* (LF)*-space, the following are equivalent:* (i) E *is metrizable;* (ii) E *is* BL [*and not (d)b*]; (iii) E *does not contain* φ. Metrizable proper (LF)-spaces are called $(\mathrm{LF})_3$-*spaces.* $(\mathrm{LF})_2$- and $(\mathrm{LF})_1$-spaces are also defined, characterized, and illustrated in Chapters 8 and 9 and [31–33] so that the $(\mathrm{LF})_i$-spaces ($i = 1, 2, 3, 4$) partition the (LF)-spaces into rich, strong barrelledness-precise subclasses.

As mentioned in [30], techniques of Saxon's dissertation prove, without use of CH, that every subspace of codimension $< c$ in a Fréchet space E is barrelled. Valdivia [41] subsequently obtained this result for more general E

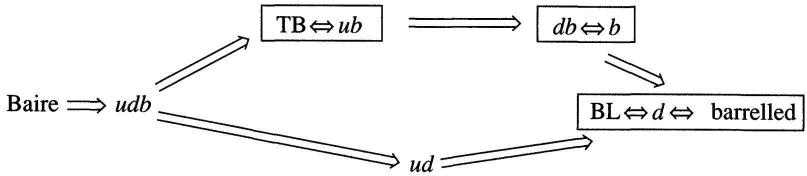

FIGURE 3. Metrizable barrelled spaces.

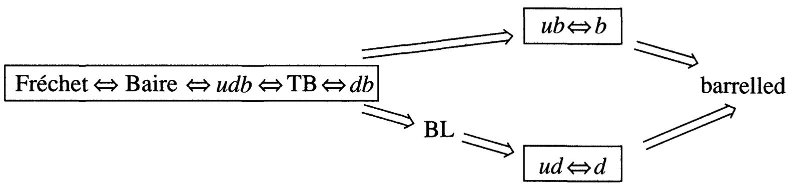

FIGURE 4. [Barrelled spaces dominated by] (LF)-spaces.

(Proposition 4.3.11) by first proving Lemma 4.3.9 (cf. [6]), a recent version [26] of which affirms Question 13.4.3 and has the corollary: *Every subspace of codimension* $< c$ *in a Fréchet space is* TB. One may greatly improve Proposition 8.6.8(iii) to read: *Every subspace of codimension* $< c$ *in an* (LF)*-space is a limit subspace* (Definition 8.6.5). These combine: *Every subspace of codimension* $< c$ *in an* (LF)*-space is the inductive limit of an increasing sequence of* TB *spaces*, which serendipitously yields, via standard closed graph techniques, a maximal generalization of Corollary 8.4.13 (Köthe's homomorphism theorem), going from finite codimension to that $< c$ [28].

Thus Question 13.4.3 has a TB space answer with "practical" (LF)-space results! We may similarly generalize Proposition 8.6.15; in particular: *A subspace of codimension* $< c$ *in an* (LF)*-space is itself an* (LF)*-space if and only if it is closed, and then in fact it must be countable-codimensional* [29].

Historically, *udb* spaces [36] are a consequent of BL spaces and the proof of the Robertson-Robertson Closed Graph Theorem [15]. The QSB ($\equiv b$) space form is by Ferrando and López-Pellicer [8]. Such would routinely shorten the argument in Example 6.4.5, and benefit the Observation 9.1.29 of a metrizable, barrelled, non-*(d)b* space G which is not an (LF)-space, although dominated by one. Even with earlier closed graph theorems, however, Ferrando and López-Pellicer could have expeditiously shown the existence of *a metrizable barrelled space* E *that is not (d)b and is not dominated by an* (LF)*-space*: Let E be any dense hyperplane of an $(\mathrm{LF})_3$-space. E is also *ud* (Figures 3 and 4). Noting [26] that Ferrando and López-Pellicer's original E [8, §3] is not *ud* but is, by Figure 3, *d*, we see with startling ease that E is not dominated by an (LF)-space, from the (elementary) bottom of Figure 4, which requires no closed graph theorem at all.

Undeniably, the book fosters fundamental advances, a most happy fact celebrated by this review. Those who join the celebration will find their own treasures. Others may simply suppose that, with such praise and superlatives, the reviewer must surely owe the authors a very great sum. In fact he does, more than he can tell, and so do all who care about barrelled spaces.

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Encyclopaedia of mathematical sciences, vol. 7, Several complex variables I (A. G. Vitushkin, ed.), Springer-Verlag, New York, Berlin, Heidelberg, 1990, 241 pp., $59.00. ISBN 0-387-17004-9

The present volume is the first of four devoted to the theory of functions of several complex variables by the Soviet encyclopaedia. This section of the encyclopaedia appears under the general editorship of A. G. Vitushkin; each of the four volumes consists of several articles written, with two exceptions by leading Soviet experts. The whole project is massive, totalling about a thousand printed pages, but for the most part the articles are not detailed expositions of their subject, being instead summary outlines of their subjects with rather full commentary but generally without proofs. These four volumes are convincing evidence of the great development seen by multidimensional function theory in the postwar era.

The first volume, the volume under review, is devoted to mainly analytic topics as opposed, say, to the theory of coherent sheaves or the relations of function theory with algebraic geometry. For these subjects, see subsequent volumes. In this volume, we find an introductory essay entitled "Remarkable Facts of Complex Analysis" by Vitushkin, which gives a brief overview of the contents of all four of the volumes. This is followed by articles by G. M. Khenkin on integral formulas in complex analysis, by E. M. Chirka on complex analytic sets, by Vitushkin on the geometry of hypersurfaces and by P. Dolbeault, on the theory of residues in several variables.

Vitushkin's introductory article is written in a style that is accessible to a broad variety of mathematicians. At the beginning