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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 2, April 1991

0273-0979/91 $1.00 + $.25 per page

Transformation groups and algebraic K-theory, by Wolfgang Lück. Lecture Notes in Math., vol. 1408, Springer-Verlag, Berlin and New York, 1989, 455 pp., $42.70. ISBN 0-387-51846-0

To understand a manifold, it is necessary to understand its symmetries. This is the basic theme of equivariant topology. Typically, one studies a group acting on a manifold by diffeomorphisms. A basic example of this is when the manifold is R^n, n-dimensional euclidean space. To understand this manifold, one associates the various matrix groups such as $Gl(n, R)$ or the orthogonal group $O(n)$. It is also important to study smaller groups such as the finite subgroups of $O(n)$. A representation is an action by a group of orthogonal matrices on R^n, inducing an action on the unit sphere, or on the unit disk. These are model examples of the kind of group action one considers in equivariant topology. The basic problem is to construct and classify actions with given properties.

In many interesting cases the action of the group is cellular. This means that the manifold has a cellular decomposition, so that the action of the group is given by permuting cells. This is, for instance, the case when the group is finite and the action is smooth. It is, thus, natural to start out studying cellular actions on cell complexes. The most fundamental invariants only depend on the equivariant homotopy type. Equivariant homotopy type is the same thing as homotopy type, but it requires that maps and homotopies commute with the group action. Another reason to study cellular actions on cell complexes is that they are relatively easy to construct. So as a first step in constructing an action on a manifold, one may try to construct an action on a cell complex homotopy equivalent to the manifold.

A special case is when the action of the group G is free. This means that every point in the manifold is mapped nontrivially by every element of the group except the identity element. In this case the cellular chain complex is a chain complex of free ZG-modules. Here ZG denotes the integral group ring of the group G. How to associate K-theoretic invariants to free actions has been studied extensively: finiteness obstructions, Whitehead torsions, Reidemeister torsions, etc.

When the group action is not free, one may still proceed by considering the manifold cut up in various subspaces called strata. A stratum is a subspace on which some subgroup acts freely. One may then try to associate invariants to the group action through invariants associated to these strata. This method has been used quite successfully. The approach does, however, suffer from lack of functoriality.

In the present book the main questions studied are equivariant finiteness obstructions, equivariant Whitehead torsions, and related questions such as product formulae. The main reason to be interested in equivariant Whitehead torsion is the equivariant s-cobordism theorem, which is treated in the book. This theorem is used to get equivariant diffeomorphisms rather than just equivariant homotopy equivalences. The main theme of the book is to use a global replacement of the category of projective ZG-modules. Instead modules over the orbit category are used, an idea that goes back to Bredon. This way the stratified approach is avoided.

If, for example, G is finite, the orbit category $O(G)$ has objects G/H, H being a subgroup. We are to think of G/H as a left G-set. The morphisms are G-maps. This category codifies all information about how the subgroups fit together, and how they behave under conjugacy. It also has the property that every endomorphism is an isomorphism. An R-module over the orbit category is a functor from the orbit category to the category of R-modules. Notice that the object $G/\{1\}$ has endomorphisms labeled by G itself. Hence, a functor applied to this object is in a natural way an RG-module.

The cellular chains of a G-complex X are now chains in the category of R-modules over the orbit category. The theory is made to formally look a lot like it did in the case of free actions. It is, of course, true that when it comes to computations, one still splits up according to strata. It is, however, of great value to have definitions independent of such a decomposition.

The level of the book is very general. Simplifying assumptions are avoided. It has been common to work with assumptions such as fixed sets of all subgroups being connected. An assumption like that is not even satisfied when the group of order 2 acts on a circle by complex conjugation. In this book the interplay between the group action and the fundamental groupoid of the strata is studied in detail. Also, the author does not only consider finite groups. He treats the case where the group comes exhibited with a topology as well. This may make the book difficult to use as a textbook, but it makes it very complete as a standard reference, and it most likely will serve as such in years to come.

There are many payoffs to a functorial approach like the one used in this book. Certain "snags" in the literature were resolved only by these methods. A theory that aims to determine existence and uniqueness of a certain type of manifolds is popularly called a surgery theory, a word referring to methods used. Equivariant surgery theory, developed by many authors, has found a nice form in a subsequent joint work by the author and I. Madsen. Here the formulations from this book are used to deal with the underlying homotopy theory.

The book is divided into three main parts called chapters. These are again divided into sections, each section ending with comments, mostly of a historic nature, and exercises. The comments are well worth reading, even for somebody who is not ready to read the whole book. The first two chapters are concerned with foundational theory of equivariant Whitehead torsion and finiteness obstructions. The third chapter, however, can be read independently by an experienced mathematician, and reads more like a (long) research paper. This is emphasized by the fact that several sections contain reviews of material dealt with earlier in the book. In this chapter several applications are developed. An example is a nice proof of de Rham's theorem, that two representations of a finite group are linearly isomorphic if and only if the unit spheres are equivariantly diffeomorphic. This is proved using a version of Reidemeister torsion defined for a very large class of spaces, finishing off a project started by M. Rothenberg 20 years ago.

Equivariant topology is still a very active area of research. In recent years the main development has been applying controlled methods. This goes beyond the scope of the present book, but combining controlled techniques with those presented here seems a promising area of research. It is difficult to explain controlled methods briefly. Basically, they are methods making it possible to make modifications arbitrarily close to a stratum, but in a "small" way, so the stratum fits as before. Another development, which is mentioned in the book, is relating topologically and analytically defined invariants. The basic object of study is a smooth manifold with a finite group G acting. Using the de Rham complex, it is now possible to define torsion invariant analytically, and relate these invariants to topologically defined invariants. The author has been a very active participant in the development of this area.

One of the virtues of this book is that it is carefully written with few mistakes. A reviewer nevertheless has a certain obligation to find at least one misprint: The signs in the matrices on page 223 are incorrect. The correct signs are obtained by reading the formulae instead.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 2, April 1991

0273-0979/91 $1.00 + $.25 per page

Space mappings with bounded distortion, by Yu. G. Reshetnyak. Translations of Mathematical Monographs, vol. 73, American Mathematical Society, Providence, RI, 1989, 362 pp., $129.00. ISBN 0-8218-4526-8

Quasiconformal (qc) and quasiregular (qr) mappings in euclidean n-space generalize the notions of a plane conformal mapping and of an analytic function of one complex variable, respectively. The systematic study of qc space mappings was begun by F. W. Gehring and J. Väisälä in the early 1960s, whereas the pioneering work on qr space mappings, due to Yu. G. Reshetnyak, appeared a few years later, in 1966–69. During the 1980s these mappings,