This text will be most useful to those who need a brief and light introduction to modern developments in numerical techniques for initial value odes and for those who wish to explore some of the less widely known techniques for special problems. For a deeper understanding of the subject, the reader may need to turn to one of the other texts mentioned above. There is a good bibliography of over 600 references. Peculiarly, the page numbering in the text does not correspond to that in the list of contents.

REFERENCES

1. J. C. Butcher, *The numerical analysis of ordinary differential equations: Runge-Kutta and general linear methods*, Wiley, Chichester, UK, 1987.
2. C. W. Gear, *Numerical initial value problems in ordinary differential equations*, Prentice-Hall, Englewood Cliffs, NJ, 1971.
3. E. Hairer, S. P. Norsett and G. Wanner, *Solving ordinary differential equations* I, *nonstiff problems*, Springer-Verlag, Berlin and New York, 1987.
4. G. Hall and J. M. Watt (eds.), *Modern numerical methods for ordinary differential equations*, Oxford University Press, Oxford, 1976.
5. P. Henrici, *Discrete variable methods in ordinary differential equations*, Wiley, New York, 1962.
6. ——, *Error propagation for difference methods*, Wiley, New York, 1963.
7. J. D. Lambert, *Computational methods in ordinary differential equations*, Wiley, New York, 1973.
8. L. F. Shampine and M. K. Gordon, *Computer solution of ordinary differential equations: the initial value problem*, Freeman, San Francisco, 1975.

IAN GLADWELL
SOUTHERN METHODIST UNIVERSITY

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 2, April 1991

0273-0979/91 $1.00 + $.25 per page

Infinite crossed products, by D. S. Passman. Academic Press, New York, 460 pp., $84.50. ISBN 0-12-546390-1

Classically, crossed products of arbitrary finite groups over fields were introduced by E. Noether in 1929 in her lectures in Göttingen [vdW]. Earlier, the special case of cyclic algebras was defined by Dickson in 1906 [D1, D2]; the first significant result about them was proved by Wedderburn in 1914 [W]. These crossed products arose naturally as follows: let D be a division algebra finite-dimensional over its center K, and assume D contains a maximal subfield $F \supset K$ such that the extension F/K is Galois with Galois group G. For each $x \in G$, it follows from the Skolem-Noether theorem that the action of x on F becomes inner when extended to D; thus there exists $0 \neq \overline{x} \in D$ such that $a^x = \overline{x}^{-1} a \overline{x}$ for all $a \in F$. The elements $\{\overline{x} | x \in G\}$ are linearly independent over F, and thus since $\dim_K D = (\dim_K F)^2$, $D = \bigoplus_{x \in G} F\overline{x}$. Since $\overline{x}\,\overline{y}$ and $\overline{xy}$ both induce $xy \in G$, it follows that $\overline{x}\,\overline{y}(\overline{xy}^{-1}) \in F$, the centralizer of F in D. Thus we may write $\overline{x}\,\overline{y} = \sigma(x, y)\overline{xy}$, where $\sigma(x, y) \in F^*$, the nonzero elements of F. The map $\sigma : G \times G \to F^*$ satisfies

$$(1) \qquad \sigma(x, yz)\sigma(y, z) = \sigma(xy, z)\sigma(x, y)^z,$$

for all $x, y, z \in G$, and thus is a 2-cocycle for the action of G on F. In this situation D is called a *crossed product* of the group G over the field F. More generally, given any finite Galois field extension F/K with Galois group G, and a 2-cocycle $\sigma : G \times G \to F^*$, Noether constructed a crossed product $A = \bigoplus_{x \in G} F\overline{x} = (F, G, \sigma)$, where A is the F-vector space with basis $\{\overline{x} | x \in G\}$ and multiplication given by

$$(2) \qquad (a\overline{x})(b\overline{y}) = ab^{x^{-1}} \sigma(x, y)\overline{xy},$$

for all $a, b \in F$, $x, y \in G$. Then A is a central simple algebra of dimension $|G|^2$ over its center K.

A fundamental question then arose: Can every central simple algebra A be written as a crossed product? By work of Albert, Brauer, Hasse, and Noether [BHN, AH], it is true if the field K is an algebraic number field. However, the general question was open for 40 more years until Amitsur provided a counterexample in 1972 [Am]. Crossed products are still of great importance for central simple algebras, however, since in the Brauer group any such algebra is equivalent to a crossed product.

Extending the notion of crossed products to allow coefficient rings other than fields was done by Jacobson in the early 1940s; he was motivated by the study of projective representations of groups into vector spaces over division rings. Although he was working over a division ring D, all the essential ingredients for the general case appear in his work. Here is his construction [J2]: Let G be a group, and $\alpha : G \to \operatorname{Aut} D$ and $\sigma : G \times G \to D^*$ be two maps satisfying the condition (1) for $a \in D$ and $z \in G$ (replacing a^z

with $a^{\alpha(z)}$) and a new condition

$$(3) \qquad a^{\alpha(xy)} = \sigma(x, y)a^{\alpha(x)\alpha(y)}\sigma(x, y)^{-1},$$

for all $a \in D$, $x, y \in G$. Then the D-vector space $A = \bigoplus_{x\in G} D\overline{x}$ $= (D, G, \alpha, \sigma)$ is an associative algebra with multiplication defined as in (2); A is a crossed product of G over D. Note that α is not assumed to be a group homomorphism, and so D is not necessarily a G-module; in fact (3) says that D is a G-module precisely when σ has values in the center of D. If one replaces the division ring by an arbitrary ring R with 1, the only additional condition needed to define a crossed product of G over R is that σ take values in $U(R)$, the group of units of R, rather than in R^*. With this change, the product in (2) is associative if and only if (1) and (3) hold. However, this does not seem to have been noticed until 20 years later, by Bovdi [B]. For simplicity, we suppress the particular action and cocycle, and will write a crossed product of G over R as $R * G$.

In fact, a more general construction than a crossed product was defined by Levitzki in 1931 [L]. He considers "normal products" RA of two finite-dimensional algebras R and A: RA is a finite free R-module with a normalizing basis of elements of A. He proves that RA is semisimple if and only if both R and A are semisimple with their centers separable over the base field. However, he does not explicitly describe either the cocycle condition (1) or the "twisted" module condition (3).

Jacobson also was interested in semisimplicity. He proves that if G is finite and H the subgroup of elements of G which are inner on D, then $D * G$ is semisimple if and only if $D * H$ is semisimple; in fact he shows that any nonzero ideal of $D*G$ intersects $D*H$ nontrivially. As a consequence if H is trivial, $D*H$ is simple; moreover if $\sigma \equiv 1$, then $D * G \cong M_n(D')$, $n = |G|$, where $M_n(D')$ is the ring of $n \times n$ matrices over some other division ring D'. These "trivial" crossed products were used earlier in proving a Galois correspondence theorem for a finite group of outer automorphisms of a division ring [J1].

This special case of a crossed product with trivial cocycle is now called a skew group ring, and denoted by RG. Following up on Jacobson's work, the skew group ring was used by Azumaya and Nakayama in the later 1940s to give a Galois correspondence theorem for simple Artinian rings [Az, NA]; there was also work by Hochschild on this problem. Among Azumaya's results is the fact that if R is simple and G outer, then RG is simple. Moreover if R is also Artinian and G is finite, then R^G is a simple ring.

Crossed products over general rings do not seem to have appeared again until the 1960s when they were considered by Bovdi, as mentioned above; his interest in them seemed to be as generalizations of group rings. They also made their appearance in the late 1960s in work of Miyashita [Mi], who made a connection between crossed products and Galois extensions in the sense of Chase-Harrison-Rosenberg, although for noncommutative rings.

Finally in the 1970s crossed products began to be studied more intensively, as a result of two separate developments. One was the study of finite group actions on noncommutative rings; this area had been given a big boost by work of Bergman-Isaacs and Kharchenko proving the existence of fixed elements in some fairly general situation. The skew group ring then proved very useful in studying the fixed ring R^G. In particular when $|G|^{-1} \in R$, RG and R^G are very closely related. For then the element $e = |G|^{-1} \sum_{x \in G} x$ is an idempotent in RG and $e(RG)e \cong R^G$. This elementary fact enables one to study R^G by passing through the skew group ring RG. Although perhaps it was known earlier (and in fact Jacobson observed $e(RG) = eR$), this very useful trick seemed to appear explicitly only about 1975 [ZN, FO]. Even if $|G|$ is not a unit in R, there is still a strong connection between RG and R^G. Thus RG itself became an interesting object of study.

The second development concerned ordinary group rings $R[G]$. A lot of progress had already been made in the 1960s and early 1970s in group rings, much of it by Passman (the best reference is his earlier book [P1]). In some of the remaining difficult problems, it seemed that a more general object might be useful in order to do inductive arguments. That is, consider an arbitrary group G with normal subgroup N. Intuitively, the group ring $R[G]$ is "made up" from the subgrouping $R[N]$ and the quotient group G/N. If we could regard $R[G]$ as a crossed product of G/N over $R[N]$, and prove results for crossed products, then induction could be used to prove results about $R[G]$. In fact the definition of crossed product is just general enough to cover this situation. For, write $S = R[N]$ and $H = G/N$. For each $x \in H$, let $\overline{x} \in G$ be a fixed coset representative. Then $R[G] = \bigoplus_{x \in H} R[N]\overline{x} = \bigoplus_{x \in H} S\overline{x}$, and so $\{\overline{x} | x \in H\}$ is an S-basis for $R[G]$. Since

$N \lhd G$, $\bar{x}^{-1}R[N]\bar{x} = R[N]$ and so $\bar{x}$ induces an automorphism $\alpha(x)$ on S by conjugation. Moreover if $x \in H$ and $s \in S$, then $s\bar{x} = \bar{x}(\bar{x}^{-1}s\bar{x}) = \bar{x}s^{\alpha(x)}$, and if $x, y \in H$, then $(N\bar{x})(N\bar{y}) = N\overline{xy}$, and so $\bar{x}\bar{y} = \sigma(x, y)\overline{xy}$ for $\sigma(x, y) \in N \subseteq U(S)$. Thus $R[G] = S * H = R[N] * G/N$.

Although crossed products were used by Zaleskiĭ in 1971 [Z] to describe quotients of certain group algebras, the first real use as an inductive tool seems to have been by Lorenz and Passman in studying prime ideals in Noetherian group rings [LP2]; they used the technique to lift information from the group algebra of a group of finite index studied by Roseblade. The real triumph of crossed product methods, however, came more recently in the fundamental Induction Theorem of J. Moody [Mo]. This result relates the finitely-generated modules over (for example) a polycyclic-by-finite group G to the modules over a finite set of finite subgroups of G.

These examples show that the two subjects interact in a mutually beneficial way. One can generalize many group ring techniques to prove results about RG and $R*G$; these apply to give results about fixed rings. In the other direction, knowing about group actions enables one to prove results about crossed products and then group rings. Thus in many ways the two subjects have become one, and that is the topic of this book.

The first chapter of the book, while introducing the basic definitions as one would expect, in fact is mostly concerned with the more general situation of graded rings. Observe that a crossed product $S = R*G$ is graded by the group G, where for $x \in G$, the xth component of S is given by $R\bar{x}$. There are several reasons for considering this more general situation. One, of course, is that many arguments work just as well for graded rings as for crossed products and so a more general result can be obtained for free. The less obvious reason is that crossed product results can be applied to prove graded ring results, by means of the "Duality Theorem." Such a result was known for von Neumann algebras [NT] before it was known in ring theory; an algebraic formulation for finite groups was given by M. Cohen and the reviewer [CM]. For a K-algebra S graded by a finite group G of order n, the theorem says the following: if $S\#K[G]^*$ is the (Hopf algebra) smash product of the dual $K[G]^*$ of $K[G]$ with S, then G acts as automorphisms of $S\#K[G]^*$ and the skew group ring $(S\#K[G]^*)G \cong M_n(S)$, the $n \times n$ matrices over S. Now the plan is clear: to prove a result about S, try to prove one about $S\#K[G]^*$, and apply known facts about skew group rings to prove it for $M_n(S)$, which is close to S. The same technique can be used when G is infinite by using an extension of the duality theorem due to D. Quinn. This method is used in Chapter 1 to prove results about induced modules and analogs of Maschke's theorem for graded rings and crossed products. Throughout the book, many other facts are proved for graded rings as well as crossed products.

Chapter 2 demonstrates that some of the basic techniques from group algebras can be used for crossed products, in particular the so-called "delta methods" used with great effect by Passman in group algebras. For any group G, its delta subgroup $\Delta(G)$ is the subgroup of elements with finitely many conjugates in G. The technique is to try to reduce various linear identities in $R * G$ to linear identities in $R * \Delta(G)$, where they may be more tractable. The linear identities of particular importance arise in determining when $R * G$ is semiprime (that is, when R has no nilpotent ideals). The author proves necessary and sufficient conditions for semiprimeness in this chapter, most of the results coming from his own work [P3]. We remark that the crossed product situation is considerably more difficult than that of group algebras. This chapter also begins the study of when a crossed product satisfies a polynomial identity, a topic which is finished at the end of Chapter 5, where complete necessary and sufficient condtions are given.

The third chapter returns to basic definitions and builds up necessary background material on the symmetric quotient ring of a prime ring and on what are called "X-inner" automorphisms. Let R be a prime ring; that is, the product of two nonzero ideals of R is nonzero. The symmetric quotient ring $Q = Q(R)$ is defined for any such R, and can be characterized as the unique (prime) overring with the same 1 satisfying several technical conditions, the most useful being that for any $q \in Q$, there exists a nonzero ideal I of R that such Iq, $qI \subseteq R$. If R is a commutative domain, then $Q(R)$ is simply the usual field of fractions of R. If R is simple, clearly $Q(R) = R$. As a nontrivial example, a theorem of Kharchenko says that if R is a free algebra of rank at least 2 over a field k, then $Q(R) = R$ [Kh2].

The importance of $Q(R)$ lies in its relation to the X-inner automorphisms of R; these are simply automorphisms of R which become inner when extended to Q. These automorphisms are named for Kharchenko and were the ones which caused difficulties in trying to prove his Galois correspondence theorem for semiprime rings [Kh1]. Although Kharchenko never considered cross products, it turns out that these automorphisms are also a major obstruction in trying to pass information between R and $R * G$. For example, in trying to generalize the theorem of Jacobson-Azumaya mentioned above to prime rings, it is false that R prime and G outer implies RG is prime; the automorphisms which are outer on R but inner on Q cause trouble [M1]. The same phenomenon arises throughout the study of crossed products. Thus a basic technical tool (based on [FM]) is the following: If G_{inn} is the set of X-inner automorphisms in G, and we consider G extended to $Q = Q(R)$, then in any crossed product $R * G$, $Q*G_{\text{inn}} = Q \otimes_C C^t[G_{\text{inn}}]$, where $C^t[G_{\text{inn}}]$ is a twisted group algebra over the center C of Q (a twisted group algebra has a cocycle but trivial action). A consequence is that any nonzero ideal of $R * G$ intersects $R * G_{\text{inn}}$ nontrivially. This generalizes the result of Jacobson mentioned above; in his case, where $R = D$, G_{inn} coincides with the usual subgroup of inner automorphisms. Also in this chapter, various results are proved about when a crossed product is symmetrically closed and about the group of X-inner automorphisms of a crossed product.

It is interesting to note that very similar ideas were introduced into operator algebras at about the same time, although there seems to have been no communication going on. If A is a prime C^*-algebra, the role of $Q(A)$ is played by $M^\infty(A)$, the direct limit of the multiplier algebras $M(I)$ of the closed ideals of A [E]; in fact the only difference between $Q(A)$ and $M^\infty(A)$ is that $M^\infty(A)$ uses only the closed ideals. The analog of X-inner automorphisms are called "partly inner"; they were used to study crossed products of C^*-algebras in [Ri].

Chapter 4 and Chapter 5 examine prime ideals in crossed products, with Chapter 4 concerned mostly with finite groups and Chapter 5 with polycyclic-by-finite groups. The basic results in Chapter 4 are due to Lorenz and Passman, and give correspondences between primes in $R * G$ and primes of R, if either G is finite or R is Noetherian and G is polycyclic-by-finite [LP1, P2]. A lovely consequence of this work is Incomparability: That is, if $P_1 \supsetneq P_2$ are prime ideals of $R * G$, then $P_1 \cap R \supsetneq P_2 \cap R$ [LP1]. This result has been generalized by Heinicke and Robson to normalizing extensions and a proof appears in [McR]; however, it is still worthwhile to see here a proof for the important special case of crossed products. Other interesting results in this chapter concern graded rings and conditions for $R * G$ to be semiprime in terms of the Sylow subgroups of G. Chapter 5 begins by summarizing the fundamental work of Roseblade on prime ideals of group algebras of polycyclic-by-finite index [R]; as mentioned earlier, the finite index step was completed by Lorenz and Passman using crossed product techniques. Much of the rest of Chapter 5 is devoted to extending many of these results first to twisted group algebras, and then to crossed products. There is also a section on when $R*G$ is a Jacobson ring, and finally the work on polynomial identities mentioned earlier.

Chapter 6 considers skew group ring applications to fixed rings of finite group actions. Many of these results, particularly those concerning the existence of fixed elements and the relation between trace functions and RG, are already in the monograph by the reviewer [M2]; however, there are a number of newer results which deserve notice. The most striking of these is the solution by D. Quinn [Q] of the integrality question, which had been open for about 10 years: He proved that if G is a finite group acting on R with $|G|^{-1} \in R$, then R is Schelter-integral over R^G. Quinn's proof involved a clever new look at work on integrality by Pare-Schelter and Lorenz-Passman; the necessary background is all included in this chapter. Also contained here are applications of the work of Chapter 4 to the prime correspondence between R and R^G; when $|G|^{-1} \in R$ one obtains a 1-to-1 correspondence between G-orbits of primes in R and a well-defined equivalence class of primes in R^G, where for $P \in \operatorname{Spec} R$, $p \in \operatorname{Spec} R^G$, $\{P^x | x \in G\} \leftrightarrow \{p | p$ is the minimal over $P \cap R^G\}$. We note that further results in this direction are contained in the expository paper [M3].

The theme of group actions is continued in the next chapter, the main point of which is the Galois correspondence theorem of Kharchenko. His general theorem for "N-groups" (after Noether) acting on semiprime rings is quite technical; the presentation here focuses on the simpler case where R is a prime ring and the "algebra of the group" $B(G)$ is a domain. Here $B(G)$ is constructed as follows: Recall from above the symmetric ring of quotients Q of R, which has center C, and the subgroup G_{inn} of X-inner automorphisms in G. Then $B(G)$ is the C-linear span of all units q of Q such that conjugation by q gives rise to some $x \in G_{\text{inn}}$ (we remark that $B(G)$ was introduced by Noether for simple rings in [N]).

The assumption that $B(G)$ is a domain includes the case where either R (and so Q) is a domain or G is X-outer; in particular one recovers as a corollary the old results of Hochschild, Azumaya, and Nakayama on outer Galois theory of simple Artinian rings, and of Jacobson on Galois theory of division rings. In addition these results apply to free algebras, in which case one gets a correspondence between subgroups and intermediate free algebras. Some other interesting facts about algebras are also proved, such as when R^G is finitely generated (almost never, unlike the commutative situation) and the computation of the Hilbert series of R^G. These free algebra results are the work of Kharchenko, Dicks, and Formanek; we note that another exposition of them, by Dicks, recently appeared in [C]; however, the exposition here of the Galois results seems more conceptual. An exposition of Kharchenko's general theorem, including actions of derivations, will appear in his forthcoming book [Kh3].

Chapter 8 contains what may be the high point of the book: an exposition of Moody's theorem on induced modules and Grothendieck groups. Not only is this a deep and lovely theorem in its own right, but also it has some important applications: It solves two outstanding group ring problems concerning zero divisors and Goldie rank. Moreover, it is truly a crossed product theorem, since the proof uses the inductive step $R*G = (R*N)*G/N$ for $N \lhd G$; the results cannot be proved by only considering group rings. The theorem says the following: Let $R * \Gamma$ be a crossed product with R ring Noetherian and Γ a polycyclic-by-finite group. Suppose $G_1, G_2, \dots, G_t$ are representatives of the conjugacy classes of the maximal finite subgroups of Γ. Then the Grothendieck group $\mathbf{G}_0(R * \Gamma)$ is generated by images of the various $\mathbf{G}_0(R * G_i)$ under the induced module map.

The proof given here is a simplification of Moody's original argument, and uses ideas from the subsequent proofs by Cliff and Weiss and by Farkas and Linnell. It is homological in nature; with the author's usual complete style, he proves most of the necessary preliminaries, with only a few references to the literature for standard background material.

The last chapter continues Chapter 8 and gives the applications to group rings mentioned above. The first major application is to the zero-divisor conjecture: That is, if G is a torsion-free group, must $K[G]$ be a domain? The general question is still open, after more than 40 years. As a consequence of Moody's theorem, one can prove: Let $R * G$ be a crossed product with R an Ore domain and G torsion free. If G has a finite subnormal series $(1) = G_0 \lhd G_1 \lhd \cdots \lhd G_n = G$ such that each quotient G_{i+1}/G_i is locally polycyclic-by-finite, then $R*G$ is an Ore domain. In particular, $K[G]$ is a domain if G is a torsion-free solvable group. The previous best-known result in this direction was that the conjecture held if G itself was polycyclic-by-finite, work due to K. A. Brown, Farkas-Snider, and Cliff. The second application of Moody's theorem discussed here is to the Goldie rank conjecture, proposed by Farkas and Rosset about 1980. This is actually a generalization of the zero divisor question, for prime Noetherian group rings $K[G]$; here $K[G]$ has a classical ring of quotients $Q \cong M_n(D)$, the $n \times n$ matrices over a division ring D. The conjecture is that n equals the least common multiple of the orders of the finite subgroups of G. Moody's theorem implies it is true when G is polycyclic-by-finite, the only known situation when $K[G]$ is Noetherian. These applications depend on work of Kropholler, Linnell, and Lorenz as well as that of Moody. The chapter also contains some interesting results of Lorenz and Passman which enable them to explicitly compute $\mathbf{G}_0(R * \Gamma)$ for several groups Γ, using Moody's theorem.

We have only mentioned the highlights of the book; many other interesting topics are discussed. Altogether, there is a wealth of information here, most of which has not appeared in book form before. Moreover, many of the proofs given here are considerably easier than the originals, and are presented in the author's usual clear and complete style. Occasionally some of the technical details may seem daunting, especially in the sections on duality, semiprimeness, and prime ideals of Noetherian group rings; but all the details are there, and with perseverance can be followed.

I would strongly recommend the book to anyone interested in group actions on rings, group algebras, and their interaction.

REFERENCES

[AH] A. A. Albert and H. Hasse, *A determination of all normal division algebras over an algebraic number field*, Trans. Amer. Math. Soc. **34** (1932), 722–726.

[Am] S. A. Amitsur, *On central division algebras*, Israel J. Math. **12** (1972), 408–420.

[Az] G. Azumaya, *New foundation of the theory of simple rings*, Proc. Japan Acad. **22** (1946), 325–332.

[B] A. A. Bovdi, *Crossed products of a semigroup and a ring*, Soviet Math. Dokl. **2** (1961), 438–440.

[BHN] R. Brauer, H. Hasse, and E. Noether, *Beweis eines Hauptsatzes in der Theorie der Algebren*, J. für Math. **167** (1932), 399–404.

[CM] M. Cohen and S. Montgomery, *Group-graded rings, smash products, and group actions*, Trans. Amer. Math. Soc. **282** (1984), 237–258.

[C] P. M. Cohn, *Free rings and their relations*, 2nd ed., Academic Press, London, 1985.

[D1] L. E. Dickson, *Abstract*, Bull. Amer. Math. Soc. (N.S.) **12** (1906), 441–442.

[D2] ——, *Linear associative algebras and abelian equations*, Trans. Amer. Math. Soc. **15** (1914), 31–46.

[E] G. A. Elliot, *Automorphisms determined by multipliers on ideals of a C^*-algebra*, J. Funct. Anal. **23** (1976), 1–10.

[FM] J. W. Fisher and S. Montgomery, *Semiprime skew group rings*, J. Algebra **52** (1978), 241–247.

[FO] J. W. Fisher and J. Osterburg, *Some results on rings with finite group actions*, Proceedings of 1976 Ohio Conference, Lecture Notes in Math., vol. 25, Marcel Dekker, New York, 1976.

[J1] N. Jacobson, *The fundamental theorem of the Galois theory for quasi-fields*, Ann. of Math. (2)**41** (1940), 1–7.

[J2] ——, *Theory of rings*, Math. Surveys, vol. 2, Amer. Math. Soc., Providence, RI, 1943.

[Kh1] V. K. Kharchenko, *Galois theory of semiprime rings*, Algebra i Logika **16** (1977), 313–363 (English transl. 1978, 208–258).

[Kh2] ——, *Algebras of invariants of free algebras*, Algebra i Logika **17** (1978), 478–487 (English transl. 1979, 316–321).

[Kh3] ——, *Automorphisms and derivations of associative rings*, Reidel (to appear).

[L] J. Levitzki, *On normal products of algebras*, Ann. of Math. (2) **33** (1932), 377–402.

[LP1] M. Lorenz and D. S. Passman, *Prime ideals in crossed products of finite groups*, Israel J. Math. **33** (1979), 89–132 (addendum **35** (1980), 311–322).

[LP2] ——, *Prime ideals in group algebras of polycyclic-by-finite groups*, Proc. London Math. Soc. **43** (1981), 520–543.

[Mi] Y. Miyashita, *On Galois extensions and crossed products*, J. Fac. Sci. Hokkaido Univ. Ser. I **21** (1970), 97-121.

[McR] J. C. McConnell and J. C. Robson, *Noncommutative Noetherian rings*, Wiley-Intersceience, New York, 1987.

[M1] S. Montgomery, *Outer automorphisms of semiprime rings*, J. London Math. Soc. **18** (1978), 209–220.

[M2] ——, *Fixed rings of finite automorphism groups of associative rings*, Lecture Notes in Math., vol. 818, Springer-Verlag, Berlin and New York, 1980.

[M3] ——, *Prime ideals and group actions in noncommutative algebras*, Invariant Theory, Contemp. Math., vol. 88, Amer. Math. Soc. Providence, RI, (1989), pp. 103–124.

[Mo] J. A. Moody, *Induction theorems for infinite groups*, Bull. Amer. Math. Soc. (N.S.) **17** (1987), 113-116.

[NT] Y. Nakagami and M. Takasaki, *Duality for crossed products of von Neumann algebras*, Lecture Notes in Math., vol. 731, Springer-Verlag, Berlin and New York, 1979.

[NA] T. Nakayama and G. Azumaya, *On irreducible rings*, Ann. of Math. (2)**48** (1947), 949–965.

[N] E. Noether, *Nichtkommutative algebra*, Math. Z. **37** (1933), 514–541.

[P1] D. S. Passman, *The algebraic structure of group rings*, Wiley-Interscience, New York, 1977.

[P2] ——, *Semiprime crossed products*, Houston J. Math. **11**(1985), 257–268.

[P3] ——, *Prime ideals in polycyclic crossed products*, Trans. Amer. Math. Soc. **301** (1987), 737–759.

[Q] D. Quinn, *Integrality over fixed rings*, J. London Math. Soc. **40** (1989), 206–214.

[Ri] M. A. Rieffel, *Actions of finite groups on C^*-algebras*, Math. Scand. **47** (1980), 157–176.

[R] J. E. Roseblade, *Prime ideals in group rings of polycyclic groups*, Proc. London Math. Soc. **36** (1978), 385–447.

[vdW] B. L. van der Waerden, *A history of algebra*, Springer-Verlag, Berlin and New York, 1985.

[W] J. H. M. Wedderburn, *A type of primitive algebra*, Trans. Amer. Math. Soc. **15** (1914), 162–166.

[Z] A. E. Zaleskiĭ, *Irreducible representations of finitely-generated nilpotent torsion-free groups*, Math. Notes **9** (1971), 117–123.

[ZN] A. E. Zaleskiĭand O. M. Neroslavskii, *On simple Noetherian rings*, Proc. Akad. Bylorussia SSR (1975), 38–42. (Russian)

SUSAN MONTGOMERY
UNIVERSITY OF SOUTHERN CALIFORNIA

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 2, April 1991

0273-0979/91 $1.00 + $.25 per page

Perfect groups, by Derek F. Holt and W. Plesken. Oxford University Press, Oxford, New York, 1989, xii + 364 pp., $70.00. ISBN 0-19-853559-7

Around 1980 the completion of the classification of the finite simple groups was announced (see [G]). Group theorists of a