α is called the meridianal automorphism of π'.

In the two cases where π' is finite and where $\pi' = \mathbf{Z}^3$ or G_6, the positive meridianal automorphisms are discussed at length. The results are essentially complete ("Essentially" meaning up to ideal class group problems in algebraic number theory.)

The book is very pleasant reading (if one accepts the small size of the typography). I have not found any misprint (except the "unavoidable" mutation of meridianal into meridional at a couple of places [on pages 66 and 106] — perhaps for the fun of it?).

This book gives a very complete technical account of the remarkable progress in our understanding of 2-knot groups in the last decade.

It is still difficult at present to give a simple and concise summary of the results, if it ever turns out to be possible; but the author's presentation of such lively and dynamic mathematical research, showing the subject in the process of its development, should certainly be very stimulating for the ambitious student.

MICHEL KERVAIRE
UNIVERSITE DE GENEVE

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 1, January 1991

0273-0979/91 $1.00 + $.25 per page

Capacities in complex analysis, by Urban Cegrell. Aspects of Math., vol. E14, Friedr. Viewag and Sohn, Braunsweig, Wiesbaden, 1988, x+153 pp. ISBN 3-528-06335-1.

Capacities are set functions that can be thought of as nonlinear generalizations of measures. They play an important role in complex analysis, often in connection with giving the correct notion of "small set" for a particular problem. The classic potential theory associated with the Laplace operator and subharmonic functions is a deep and beautiful theory that connects the Dirichlet problem; analytic, harmonic, and subharmonic functions; Brownian motion; and Newtonian or logarithmic capacity. In the last decade, there has been progress in developing analogues of this theory for applications dealing with analytic and plurisubharmonic functions in several complex variables. This short monograph is the first to attempt to bring together some of this material to serve as an introduction for graduate students and mathematicians who wish to get an introduction to some of this recent work.

The most important example of a capacitary set function in potential theory is that associated with Newtonian potential theory in $\mathbf{R}^n$. Formally, the capacity of a compact set K is defined by the relation

$$C(K) = \sup\{\mu(K) : \mu \in \Gamma_K\},$$

where Γ_K denotes the set of nonnegative Borel measures that are supported on K and have potential functions

$$U_\mu(x) = \begin{cases} \int |x-y|^{2-n}\, d\mu(y), & n > 2 \\ -\int \log|x-y|\, d\mu(y), & n = 2 \end{cases}$$

that are bounded by 1 on some fixed open set $\Omega \supset K$. That is, the capacity of K is the largest amount of mass that can be concentrated on the set and still have a potential function bounded by 1. An example of a problem for which this capacity gives the correct notion of "exceptional set" is given by studying removable singularities of bounded harmonic functions. Every function u bounded and harmonic on a bounded open set $\Omega \in \mathbf{R}^n$, except at points in a closed subset E of Ω, has a unique extension to a harmonic function on Ω if and only if E has capacity zero. Newtonian capacity zero also gives the description of the exceptional sets for several other problems in classic potential theory. It describes polar sets, that is, the sets on which a subharmonic function can be identically $-\infty$. Also, the set of points at which the solution u of the Dirichlet problem for the Laplace operator on a bounded open subset Ω of $\mathbf{R}^n$,

$$\begin{aligned} \Delta u(x) &= 0, & x &\in \Omega \\ u(x) &= f(x), & x &\in \partial\Omega, \end{aligned}$$

fails to continuously assume the boundary values given by the continuous function f is a set of capacity zero.

Because of the strong connection between harmonic functions and analytic functions in one complex variable, the case $n = 2$ is especially important. There are many different but equivalent ways to describe the capacity of a compact set in the plane, for example, in terms of Chebyshev constants, transfinite diameter, the Robin constant, and energy. Each of these has connections with problems in analytic function theory, for example, in the theory of conformal mappings and approximation theory.

The Newtonian capacity is defined in $\mathbf{R}^n$ for all $n = 2, 3, \ldots$, so it is natural to consider whether it plays the same role for analytic functions of several complex variables as it does in one variable. Since analytic functions of several variables have harmonic real and imaginary parts, there is still a connection with harmonic functions. However, it is not so strong as in the case $n = 2$, because harmonic functions on $\mathbf{C}^n = \mathbf{R}^{2n}$ are not invariant under analytic changes of coordinates. To obtain results for several complex variables analogous to those in one complex variable, it is natural to replace the classes of harmonic and subharmonic functions with their respective subclasses invariant under analytic changes of coordinates—the classes of pluriharmonic and plurisubharmonic functions—and develop a potential theory based on these classes.

Cegrell's book provides a convenient introduction to this field. It is based on courses given by him in 1985–86. The focus of the monograph is on general properties of capacities as set functions, particularly on those capacities that are envelopes of measures. This choice provides a unifying theme. In the first three sections, properties of general capacitary set functions are studied. The author has done a good job of selecting an important, relevant subset of the general theory of capacities to present. Highlights include a study of K-analytic sets and Choquet's capacitability theorem. The next five sections deal with subharmonic and plurisubharmonic functions and the capacity that is associated to the complex Monge-Ampère operator, $u \to (dd^c u)^n$, in the same way that Newtonian capacity is associated to the Laplace operator: for a compact set K of an open subset Ω of $\mathbf{C}^n$,

$$d(K) = \sup\left\{\int_K (dd^c u)^n | 0 < u < 1\,,\ u \text{ psh on } \Omega\right\},$$

where the supremum is over all negative plurisubharmonic functions u on Ω that satisfy $u(z) \leq -1$ for all $z \in K$. There is also a section on Ronkin's gamma capacity, and in the last three sections the discussion covers a variety of topics related to boundary values of analytic and plurisubharmonic functions on the ball or on strictly pseudoconvex domains.

Most of the material in the notes seems appropriate as an introduction to potential theory in several complex variables for students with a background in real and complex analysis. The best and most complete parts are those dealing with the general theory of capacities and the theory of the capacity associated with the complex Monge-Ampère operator. By first studying general capacity set functions, the author focuses attention on the analytic problems that arise in proving important properties of the capacity, such as continuity under decreasing limits of compact sets. The continuity of $(dd^c u)^n$ under bounded, monotone limits of plurisubharmonic functions is proved, as is the equivalence of negligible sets (with respect to plurisubharmonic functions), pluripolar sets, and sets of capacity zero. Other interesting applications are given to the study of the (pluri-) Green function and Siciak's global extremal function, the analogue of the Green function with pole at infinity in one complex variable.

The major shortcoming of the book is that it does not supply any outline or overview of the subject. There should have been some introductory material in each chapter that calls attention to the main results and the direction one takes to prove them. Also, I did not find any strong connection between the last three chapters and the topics discussed in the first nine chapters. A surprising omission in a book on capacities in several complex variables is that there is no mention of some of the most interesting and important new capacities, such as the projective capacities studied by Sibony and Wong and by H. Alexander, and the capacity associated with the "transfinite diameter," studied by Siciak and Zaharjuta. Of course, it is impossible to have everyone's favorite topics in such a short monograph.

Since these notes are lecture notes from courses given by the author, it is perhaps not surprising that there are many typographical errors in them. However, I found no serious errors. All in all, I think this book is a good source for obtaining an introduction to a new and interesting subject.

B. A. TAYLOR
UNIVERSITY OF MICHIGAN

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 1, January 1991

0273-0979/91 $1.00 + $.25 per page

Computability in analysis and physics, by M. B. Pour-El and J. I. Richards. Perspect. Math. Logic, Springer-Verlag, New York, Berlin, Heidelberg, 206 pp. ISBN 0-387-50035-9

Which processes in analysis and physics preserve computability, and which do not? In order to answer this question, after an