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BULLETIN (New Series) OF THE
 AMERICAN MATHEMATICAL SOCIETY
 Volume 24, Number 1, January 1991
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 0273-0979/91 \$1.00 + \$.25 per page

2-Knots and their groups, by Jonathan Hillman. Austral. Math. Soc. Lect. Ser., vol. 5, Cambridge University Press, New York, 1989, 164 pp., \$19.95. ISBN 0-521-37812-5

Whereas the algebraic characterization of n -knot groups (fundamental group of the complement $S^{n+2} \setminus K^n$ of an n -knot K^n) is easy for $n \geq 3$, and apparently hopeless for $n = 1$, the problem of characterizing algebraically the 2-knot groups is very challenging: It is certainly a difficult one, but with some optimism, may perhaps be viewed as not totally hopeless. This book gives, in rather condensed form, an essentially complete survey of the subject and a very good account of the status of the problem.

After an introductory Chapter 1 and a slick exposition of the classic background (i.e., of the results, which are more than ten years old) in Chapter 2, the book starts for good on page 36 with five chapters on the recent rather prolific developments of the algebraic study of 2-knot groups in the last ten years. Many of these results are in fact due to the author.

Thus, the book is intended for the working research topologist who wants to acquire a comprehensive idea of the status of the

classification of 2-knots. It will also be very useful to the talented student with already a fair amount of background knowledge in topology and homological algebra, and with a serious intention of working on the subject.

The book is not an introduction to the subject of 2-dimensional knots or even a slowly progressing textbook. Many proofs draw heavily on the specialized literature and an inexperienced reader will need much patience and effort in going back to the original papers if he (or she) cares to fill the details of all the proofs. However, the needed references (at least for the less classic results) are carefully provided.

The main prerequisites are from homological algebra: Poincaré duality groups, the classification of finite groups with periodic cohomology, and so on. Some experience in surgery theory is also indispensable for a useful reading of the last two (and probably also the first two) chapters.

The most clear-cut results concern 2-knot groups with finite commutator subgroups. A complete list is given in this case (Chapter 4, Theorem 3, pages 59–60).

If the commutator subgroup is infinite, the author concentrates on the case where the knot group contains an abelian normal subgroup of positive rank. The reason for this assumption is technical; it is explained at the beginning of Chapter 3 and goes back to a theorem of S. Rosset on noncommutative localization. Geometrically, this assumption enables the author to prove that the 4-dimensional manifold obtained by surgery on the knot is “often” aspherical, and thus such 2-knot groups with abelian normal subgroup of positive rank are Poincaré duality groups of dimension 4. However, if the abelian normal subgroup has rank 1 or 2, the hypothesis of the available theorems remain rather technical, and the situation is still somewhat confused.

There is again a clear-cut statement if π is a 2-knot group with an abelian normal subgroup of rank > 2 . In that case the commutator subgroup is either free abelian of rank 3 or the group G_6 with presentation (Chapter 6, Theorem 6, page 96):

$$\langle x, y | xy^2x^{-1}y^2 = yx^2y^{-1}x^2 = 1 \rangle.$$

Of course, if π' is known, the description of π is then completed by specifying the automorphism α of π' determined by $\alpha(x) = txt^{-1}$, where $t \in \pi$ maps to a generator of the infinite cyclic abelian quotient π/π' .

α is called the meridional automorphism of π' .

In the two cases where π' is finite and where $\pi' = \mathbf{Z}^3$ or G_6 , the positive meridional automorphisms are discussed at length. The results are essentially complete (“Essentially” meaning up to ideal class group problems in algebraic number theory.)

The book is very pleasant reading (if one accepts the small size of the typography). I have not found any misprint (except the “unavoidable” mutation of meridional into meridional at a couple of places [on pages 66 and 106] — perhaps for the fun of it?).

This book gives a very complete technical account of the remarkable progress in our understanding of 2-knot groups in the last decade.

It is still difficult at present to give a simple and concise summary of the results, if it ever turns out to be possible; but the author’s presentation of such lively and dynamic mathematical research, showing the subject in the process of its development, should certainly be very stimulating for the ambitious student.

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AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 1, January 1991
©1991 American Mathematical Society
0273-0979/91 \$1.00 + \$.25 per page

Capacities in complex analysis, by Urban Cegrell. Aspects of Math., vol. E14, Friedr. Viewag and Sohn, Braunschweig, Wiesbaden, 1988, x+153 pp. ISBN 3-528-06335-1.

Capacities are set functions that can be thought of as nonlinear generalizations of measures. They play an important role in complex analysis, often in connection with giving the correct notion of “small set” for a particular problem. The classic potential theory associated with the Laplace operator and subharmonic functions is a deep and beautiful theory that connects the Dirichlet problem; analytic, harmonic, and subharmonic functions; Brownian motion; and Newtonian or logarithmic capacity. In the last decade, there has been progress in developing analogues of this theory for applications dealing with analytic and plurisubharmonic functions in several complex variables. This short monograph is the first to