by Sylvestre Gallot. But this book will clearly be *the* reference for some time to come.

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Amenable Banach algebras, by J.-P. Pier. Pitman Research Notes in Mathematics Series, vol. 172, Longman Scientific and Technical, Harlow and New York, 1988, 161 pp., $47.95. ISBN 0-582-01480-8

The concept of amenability was first defined for locally compact groups having evolved from the idea of a translation invariant mean or average on the bounded L^∞-functions on the real line used by von Neumann. If G is a locally compact group, then (left) Haar measure m induces a left translation invariant continuous positive linear functional on $L^1(G)$, the space of m integrable functions. There is no such translation invariant linear functional on $L^\infty(G)$, or on several other large spaces of bounded functions, for most locally compact groups G. The groups for which there is such a positive invariant mean were called amenable by M. M. Day (1950). The transition of amenability from groups to Banach algebras arose from the transfer of Hochschild cohomology into this setting.

If X is a Banach module over a Banach algebra A, then the first (continuous Hochschild) cohomology group $H^1(A, X)$ is the quotient of the linear space of (continuous) derivations by the space of inner derivations. A derivation D from A into X is a linear operator from A into X such that $D(ab) = aD(b) + D(a)b$ for all a, b in A, and D is inner if there is an x in X such that $D(a) = ax - xa$ for all a in X. B. E. Johnson [7] showed that the amenability of a locally compact group G is equivalent to the first cohomology group $H^1(L^1(G), X)$ being zero for each dual $L^1(G)$-module X. One direction of the proof uses the invariant mean directly to average the function $F(g) = f(g^{-1}D(g))$ for all f in the predual of X so obtaining an element in X inducing D; the converse is proved by considering a particular dual module and derivation. B. E. Johnson then defined a Banach algebra to be amenable if $H^1(A, X)$ is zero for all dual Banach A-modules [7]. Omitting the hypothesis "dual" includes too many pathological modules and forces the algebra to be $\mathbf{C}^n$ with pointwise product for some integer n. The natural extension, quotient and ideal properties follow for amenability sometimes only provided the ideal has a bounded approximate identity. Amenable algebras are intuitively not too large or complicated having a weak form of averaging built into them. The Banach algebra of compact operators on a Banach space is known to be amenable for the l^p spaces when $1 < p < \infty$ and $C([0, 1])$.

The definition of amenability may be modified by changing the class of modules for which $H^1(A, X)$ is to be zero, the manner in which the inner derivations are to be constructed, or the continuity conditions on the derivation. For example, W. G. Bade, P. C. Curtis and H. G. Dales [1] define the Banach algebra A to be weakly amenable if X is taken to be the dual module of A. This restriction on A is probably too weak to be of much use. For C^*-algebras strong amenability is defined in terms of the derivation being determined by an element in a specified compact convex set in the dual module under consideration. J. Cuntz used properties of Kasparov's KK-theory applied to the reduced C^*-algebra $C_r^*(G)$ of a discrete group G to define K-amenability for G [4]. Here K-theory restrictions are replacing those of Hochschild cohomology, and the theory is feeding information back to its foundations in group theory.

The most important advances in amenability arose out of developments in the theory of von Neumann algebras. A. Connes defined a von Neumann algebra to be amenable if the module is restricted to be a dual Banach module with the module operations continuous from the weak operator topology on the algebra to the weak* topology on the module and the derivations are similarly continuous [3]. These are the natural topologies for a von Neumann algebra. With these definitions the algebra and geometry of C^*-algebras and von Neumann algebras are bound together in a deep and fundamental way. The algebraic side of the equivalence being cohomological in nature and the geometrical aspects being the completely positive approximation property, or an equivalent formulation, and the Banach space splitting of $B(H)$ as a direct sum by the algebra with splitting projection of norm one. A C^*-algebra is amenable if and only if its enveloping von Neumann algebra is amenable as a von Neumann algebra. A von Neumann algebra is von Neumann algebra amenable if and only if the algebra is injective; a von Neumann algebra is defined to be injective if there is a projection of norm one from $B(H)$ onto the algebra represented as a von Neumann subalgebra of $B(H)$. Finally a C^*-algebra is amenable if and only if the algebra is nuclear; nuclearity is a geometric condition on the algebra ensuring that it has unique C^*-tensor products with all other C^*-algebras. These results are amongst the deepest in the theory of operator algebras depending on major advances in the theory of von Neumann algebras by A. Connes [2] (see also [3, 5, 6]).

Amenability of operator algebras is a part of Hochschild cohomology by its definition and is joined to the geometrical foundations of operator algebras by the beautiful equivalences mentioned above. There should be a discussion of cohomological matters and stability as well as the geometric properties in a thorough development of amenability. These lecture notes develop amenability from its definition through to the geometric relations with operator algebras mentioned above. The discussion starts at the level of a basic advanced text on Banach algebras, and is relatively self contained given this and some operator algebra theory. However the emphasis is on the metric aspects of the equivalences outlined above with little homological input. This is a set of lecture notes rather than a monograph both in the level of polish on the material and the completeness of the discussion. The bibliography omits several important references (for example, there is a reference to Johnson only in the discussion of stability, which was investigated independently by I. Raeburn and J. L. Taylor) and one would need to consult a survey of A. Ya. Helemskii to obtain a clear view of the Russian contributions. The material covered is proved in detail and there are notes at the end of each chapter on the history and other sources. This is a suitable place to start learning about amenability of Banach algebras but consulting original papers and other surveys will be essential for a balanced coverage.

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Introduction to operator theory and invariant subspaces, by Bernard Beauzamy. North-Holland, Amsterdam, New York, Oxford, Tokyo, 1988, xiv + 358 pp., $84.25. ISBN 0-444-7052-X

The object of study of this monograph is a single continuous linear operator $T: E \to E$, where E is a complex Banach space, and the central question considered is the so-called "invariant subspace problem." We recall that a closed linear subspace $M \subset E$ is invariant for T if $TM \subset M$. The invariant subspace problem asks whether every continuous linear operator T on a Banach space E of dimension ≥ 2 has a nontrivial invariant subspace. (The trivial invariant subspaces are $\{0\}$ and E.) This question was first asked probably by von Neumann in the particular case where E is a Hilbert space, and in this case the problem is still open. When E is a Banach space the answer is negative. Examples of continuous linear operators without invariant subspaces were given first by Enflo [12] on a Banach space built for this purpose. Further examples were given by Beauzamy [6] and Read [16]. Read managed later to produce examples on large families of Banach spaces, including such familiar spaces as l^1 and c (the spaces of summable sequences and convergent sequences, respectively).

One should realize that the invariant subspace problem, basic as it is, was not the only reason for the development of operator theory. In fact, merely knowing that an operator T has nontrivial invariant subspaces does not tell us much about T. Fortunately, when an operator or class of operators is shown to have invariant