[**Martin-Löf**] P. Martin-Löf, *Constructive mathematics and computer programming*, Mathematical Logic and Programming Languages (C.A.R. Hoare and J. C. Shepherdson, eds.), Prentice-Hall, New Jersey, 1985.

DOUGLAS S. BRIDGES
UNIVERSITY OF BUCKINGHAM

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 20, Number 1, January 1989

0273-0979/89 $1.00 + $.25 per page

Holomorphic functions and integral representations in several complex variables, by R. Michael Range. Graduate Texts in Mathematics, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1986, xi + 386 pp., \$49.50. ISBN 0-387-96259-x

Complex function theory of one variable can be developed on the basis of three different approaches.

(a) The (so-called) *Weierstrass* approach, namely the fact that holomorphic functions can be locally represented by their Taylor expansions. Here the basic properties of the ring $\mathscr{O}^{(1)}$ of convergent power series in one variable such as $\mathscr{O}^{(1)}$ being a principal ideal ring become important;

(b) The (so-called) *Riemann* approach, based on the fact that holomorphic functions can be characterized as those differentiable functions $f = g + ih$ in $z = x + iy$ satisfying the Cauchy-Riemann equations:

$$\frac{\partial f}{\partial \overline{z}} := \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right) = 0. \tag{1}$$

Here the "good" properties of the system (1) of partial differential equations are the essential feature. Namely, it is elliptic, linear with constant coefficients and intimately related to the Laplace operator $\Delta = 4\partial^2/\partial z\partial\overline{z}$ with all its wonderful, well-known properties. Furthermore, (1) has as its natural geometric interpretation the conformality of biholomorphic maps.

(c) The (so-called) *Cauchy* approach, based on the Cauchy integral formula for holomorphic functions. The properties of the integral operator(s) with the Cauchy kernel are used as the most powerful tools from this point of view. In particular one has the formula

$$f(z) = \frac{1}{2\pi i}\int_{\partial\Omega}\frac{f(\zeta)}{\zeta - z}\,d\zeta + \frac{1}{2\pi i}\int_{\Omega}\frac{\partial f/\partial\overline{\zeta}}{\zeta - z}\,d\zeta \wedge d\overline{\zeta} \tag{2}$$

which holds for all domains $\Omega \Subset \mathbf{C}$ the boundary of which consists of a finite number of disjoint C^1 Jordan curves, for all $f \in C^1(\overline{\Omega})$ and for all $z \in \Omega$. It can be used very successfully in this approach.

(It should be pointed out that the association of the names of Weierstrass, Riemann and Cauchy with these approaches can be justified only partially from the historical viewpoint. For some interesting details about this see for instance [**27**].)

Most presentations of basic function theory use a pragmatic mixture of the approaches (a)–(c). It can, however, also be quite interesting to see how one can get how far with each of these approaches separately. (Presentations stressing the Weierstrass point of view were, for instance, given in the first edition of [**18** and in **8**]. The Cauchy approach is somewhat stressed in [**19**] (however without any deeper applications to function algebras or other possible topics).)

In the function theory of several complex variables the respective generalisations of the three approaches (a)–(c) play an important role. However, in each of them difficulties arise, which are linked to those new phenomena which are typical of the function theory of more than one variable and make its study so interesting. As only one example we mention the Hartogs extension phenomenon. Namely, let $\Delta := \{(z,w) \in \mathbf{C}^2 \colon |z| < 1, |w| < 1\}$, and put for any ε, $0 < \varepsilon < 1$, $\Omega := \{(z,w) \in \Delta \colon |z| < \varepsilon \text{ or } 1 - \varepsilon < |w|\}$. Then *any* holomorphic function f on Ω can be extended to a holomorphic function on Δ. The new difficulties necessitated the introduction of additional concepts and methods in the pursuit of these approaches. Let us describe in the following some features of each approach.

The Weierstrass approach in some sense dominated the development of a large part of the general theory of complex analysis up to about 1965. The most essential tools are the Weierstrass preparation theorem, the Weierstrass formula and the so-called "Heftungslemma" of H. Cartan (although many proofs of them use Cauchy integrals on polydiscs). As a representative of the new difficulties we mention the fact that the ring $\mathscr{O}$ of convergent power series is no longer a principal ideal domain but only a noetherian ring. This causes, of course, the theory of coherent analytic sheaves to be more complicated and, at the same time, more interesting. Besides, algebraic geometry also local analytic geometry has much influenced the development of modern commutative algebra. The three volumes [**11, 12, 13**] by H. Grauert and R. Remmert give a presentation of some parts of complex analysis in which the Weierstrass philosophy of using algebraic tools as much as possible has been stressed.

The Cauchy-Riemann equations in several complex variables are

$$\frac{\partial f}{\partial \bar{z}_k} = 0, \qquad k = 1, \dots, n.$$

Their inhomogeneous form

$$\frac{\partial f}{\partial \bar{z}_k} = g_k, \qquad k = 1, \dots, n \tag{3}$$

is for $n > 1$ an overdetermined system with the obvious integrability conditions

$$\frac{\partial g_k}{\partial \bar{z}_l} = \frac{\partial g_l}{\partial \bar{z}_k}, \qquad k, l = 1, \dots, n.$$

In the language of differential forms the system (3) leads on any complex manifold M and for any $0 \le p \le n = \dim M$ to the so-called Dolbeault-complex

$$0 \to \Omega^p(M) \to C^{p,0}(M) \xrightarrow{\bar\partial} C^{p,1}(M) \xrightarrow{\bar\partial} \cdots \xrightarrow{\bar\partial} C^{p,n}(M)$$

where $C^{p,q}(M)$ denotes the space of (p,q)-forms with C^∞ coefficients on M, i.e., the forms α which in local coordinates can be written as

$$\begin{aligned}\alpha &= \sum_{\substack{1\le i_1<\cdots<i_p\le n\\ 1\le j_1<\cdots<j_q\le n}} g_{i_1,\dots,i_p,j_1,\dots,j_q}\cdot dz_{i_1}\wedge\cdots\wedge dz_{i_p}\wedge d\overline{z}_{j_1}\wedge\cdots\wedge d\overline{z}_{j_q}\\ &= \sum_{\substack{|I|=p\\|J|=q}}{}' g_{IJ}\, dz_I\wedge d\overline{z}_J\end{aligned}$$

where the $'$ at the $\sum$ means that the sum is taken only over the strictly increasing p-tuples I and q-tuples J, such that the coefficients $g_{IJ}\in C^\infty(M)$ are uniquely determined. The notation $\Omega^p(M)$ is used for the space of holomorphic p-forms on M. The operator $\overline{\partial}$ is then defined by

$$\overline{\partial}\alpha = \sum_{\substack{|I|=p\\|J|=q}}{}'\sum_{k=1}^n \frac{\partial g_{IJ}}{\partial\overline{z}_k}\, d\overline{z}_k\wedge dz_I\wedge d\overline{z}_J.$$

One has $\overline{\partial}^2=0$, and according to the Dolbeault-Grothendieck lemma, the complex (4) is locally exact (replace all $C^{p,q}(M)$ by the space of germs of (p,q)-forms near any given point $z\in M$). The Riemann approach to complex analysis is based on the global study of (4) from the point of view of partial differential equations. It has been developed in a systematic way only since the fundamental works of J. J. Kohn **[20]** and J. J. Kohn and L. Nirenberg **[22]** from 1963–1965 on the one hand, and L. Hörmander **[16]** from 1965 on the other hand. Again a difficulty arises in this context which is typical of the case of more than one variable. Namely, whereas the complex (4) is elliptic near any interior point of M, this is no longer the case near the boundary, not even on any bounded, smoothly bounded domain $D\subset\mathbf{C}^n$, $n>1$. For (p,q)-forms α, $q\ge 1$, on a hermitian complex manifold M with compact support in M one has the elliptic estimate

$$\|\alpha\|_1^2 \le C(\|\overline{\partial}\alpha\|_0^2 + \|\overline{\partial}^*\alpha\|_0^2) \tag{5}$$

where $\overline{\partial}^*$ denotes the L^2-adjoint operator of $\overline{\partial}$, $\|\cdot\|_0$ means L^2-norm on M and $\|\cdot\|_\varepsilon$ means the Sobolev norm of order ε on M (with respect to L^2). In order to deal with the additional difficulties arising at the boundary Hörmander uses in **[16]** L^2-norms with suitable weights which allow to estimate boundary terms arising in typical integrations by part. On manifolds M on which such weights ϕ_1,ϕ_2,ϕ_3 exist (so-called pseudoconvexity properties play an essential role) the basic estimate

$$\|\alpha\|_{\phi_2}^2 \le \|\overline{\partial}\alpha\|_{\phi_3}^2 + \|\overline{\partial}^*\alpha\|_{\phi_1}^2 \tag{6}$$

can be proved for all $\alpha\in C^{p,q}(M)$, $q\ge 1$, for which the adjoint operator $\overline{\partial}^*: L^2_{p,q}(\phi_2)\to L^2_{p,q-1}(\phi_1)$ of $\overline{\partial}$ is defined. (The norm $\|\cdot\|_\phi$ in $L^2_{p,r}(\phi)$ is the L^2-norm with weight factor $e^{-\phi}$.) Existence theorems for (4) follow from (6) in the usual way. In his introductory book **[17]** Hörmander shows how important parts of complex analysis can be deduced from these results or, more generally, in the spirit of the Riemann approach.

J. J. Kohn, L. Nirenberg **[20, 22]** use in their investigation of the Cauchy-Riemann-equations a different approach. Imitating the fruitful connection between the Cauchy-Riemann equations and the Laplace operator in one complex variable they study at first the so-called $\bar\partial$-Neumann problem, namely the L^2-existence and regularity theory of the operator

$$\square := \bar\partial^*\bar\partial + \bar\partial\,\bar\partial^* : L_2^{p,q}(M) \to L_2^{p,q}(M) \tag{7}$$

where $L_2^{p,q}(M)$ are the (p,q)-forms on M with L^2-coefficients. The operator $\square$ is well defined on

$$\begin{aligned}\operatorname{dom}\square_{p,q} := \{f \in L_2^{p,q}(M)\colon f \in \operatorname{dom}\bar\partial \cap \operatorname{dom}\bar\partial^*,\\ \bar\partial f \in \operatorname{dom}\bar\partial^*, \bar\partial^* f \in \operatorname{dom}\bar\partial\}.\end{aligned}$$

On manifolds M with C^∞-smooth boundary ∂M the space $\mathscr{D}^{p,q}(M) := \operatorname{dom}\square_{p,q} \cap C^{p,q}(\overline{M})$ can be characterized by two boundary conditions, the so-called Neumann-conditions. They guarantee that the L^2-adjoint operator $\overline{\mathscr{O}}^*$ of $\overline{\mathscr{O}}$ equals its formally adjoint operator on $\operatorname{dom}\square_{p,q}$ and on $\bar\partial(\operatorname{dom}\square_{p,q})$. It can easily be seen that for the solvability of

$$\square f = g \tag{8}$$

the integrability condition $g \perp \mathscr{H}^{p,q}$ must be satisfied with

$$\mathscr{H}^{p,q} := \{h \in \operatorname{dom}\square_{p,q}\colon \square h = 0\}.$$

And if (8) is solvable for all $g \in (\mathscr{H}^{p,q})^\perp$, we define the Neumann operator N of M as

$$Ng = \begin{cases} 0, & \text{for } g \in \mathscr{H}^{p,q}, \\ f, & \text{for } g \perp \mathscr{H}^{p,q}, \end{cases}$$

where f is the unique solution of (8) with $f \perp \mathscr{H}^{p,q}$. It is easy to see that in this case for any $\alpha \in L_2^{p,q}(M)$, $q \geq 1$, with $\bar\partial\alpha = 0$ the form

$$\beta := \bar\partial^* N\alpha$$

is the unique solution of

$$\bar\partial\gamma = \alpha \tag{9}$$

orthogonal to $\ker\bar\partial$ in $L_2^{p,q-1}(M)$.

The existence of the Neumann operator follows under suitable pseudoconvexity hypothesis already from Hörmander's $\bar\partial$-theory. The regularity properties of N are, in fact, the most important topic of this approach, even already for smoothly bounded domains $\Omega \Subset \mathbf{C}^n$. It is, for instance, one of the most intriguing open questions whether N is hypoelliptic on any such Ω which is pseudoconvex. The local hypoellipticity of N on such domains is a consequence of a subelliptic a priori estimate of the following form: for a given point $z_0 \in \partial\Omega$ there are a neighborhood $U = U(z_0)$, an $\varepsilon > 0$ and a $C > 0$ such that

$$\|\phi\|_\varepsilon^2 \leq C\{\|\bar\partial\phi\|^2 + \|\bar\partial^*\phi\|^2 + \|\phi\|^2\} \tag{10}$$

for all $\phi \in \mathscr{D}^{p,q}(\Omega)$ with $\operatorname{supp}\phi \subset U \cap \overline{\Omega}$, as was shown in **[22]**. This estimate is closely related to very subtle geometric properties of $\partial\Omega$ near

z_0. The question for which domains the estimate holds for $\varepsilon > 0$ has since 1965 given rise to much research. We mention only **[3, 4, 5, 6, 7, 21]**, where the interested reader also can find more references.

Finally, we come to the Cauchy approach to function theory in more than one complex variable. (It is, in fact, this approach which is emphasized in the book by R. M. Range.) Here the situation might at first glance look like being perfect, since by iterating the Cauchy integral formula of one complex variable one obtains a similar formula for certain product domains in $\mathbf{C}^n$, the integration kernel just being a product of Cauchy kernels (formulas of this type were, in fact, generalized by A. Weil to analytic polyhedra). However, also in this approach typical new difficulties arise in more than one complex variable, if one asks for integral representation formulas for (p, q)-forms on smoothly bounded domains $\Omega \Subset \mathbf{C}^n$. In search for a generalization of (2) to this situation one is lead to the Martinelli-Bochner-Koppelman formula, which we will briefly explain here. We denote $\omega(\zeta) := d\zeta_1 \wedge \cdots \wedge d\zeta_n$,

$$(11) \qquad \omega'_{z,\zeta}(\overline{\zeta} - \overline{z}) := \sum_{j=1}^{n} (-1)^{j+1}(\overline{\zeta}_j - \overline{z}_j) \bigwedge_{k \neq j} (d\overline{\zeta}_k - d\overline{z}_k) \wedge \omega(\zeta)$$

and put for any differential form $f \in C^{0,q}(\overline{\Omega})$,

$$(12) \qquad (B_\Omega f)(z) := \frac{(n-1)!}{(2\pi i)^n} \int_{\zeta \in \Omega} f(\zeta) \wedge \frac{\omega'_{z,\zeta}(\overline{\zeta} - \overline{z}) \wedge \omega(\zeta)}{|\zeta - z|^{2n}}, \qquad z \in \Omega$$

and

$$(13) \quad (B_{\partial\Omega} f)(z) := \frac{(n-1)!}{(2\pi i)^n} \int_{\zeta \in \partial\Omega} f(\zeta) \wedge \frac{\omega'_{\zeta,z}(\overline{\zeta} - \overline{z}) \wedge \omega(\zeta)}{|\zeta - z|^{2n}}, \qquad z \in \Omega.$$

One then has for any form $f \in C^{0,q}(\overline{\Omega})$ the representation on Ω,

$$(14) \qquad (-1)^q f = B_{\partial\Omega} f - B_\Omega \overline{\partial} f + \overline{\partial} B_\Omega f$$

(for $n = 1$, $q = 0$ this is exactly (2)). Although this formula is very general, it is only of limited usefulness, in particular, for solving the Cauchy-Riemann equations. This is caused by the fact that for $n > 1$ the terms

$$\frac{\overline{\zeta}_j - \overline{z}_j}{|\zeta - z|^{2n}}$$

in the integral kernel are no longer holomorphic in z.

It was only since 1970 and due to the fundamental work of Grauert and Lieb **[10]**, Henkin **[15]** and Lieb **[24]**, which was based on ideas of Fantappié and Leray, that this difficulty was overcome. The crucial observation was the following. Let $w(z, \zeta) := (w_1(z, \zeta), \dots, w_n(z, \zeta))$ for $z \in \Omega$, $\zeta \in U$, where U is a neighborhood of $\partial\Omega$, be a C^∞ map with

$$\langle w(z, \zeta), \zeta - z \rangle = \sum_{j=1}^{n} w_j(z, \zeta)(\zeta_j - z_j) \neq 0, \quad \text{for all } (z, \zeta) \in \Omega \times \partial\Omega$$

(such a map is called a Leray map) and suppose that $w(z,\zeta)$ is holomorphic in $z \in \Omega$. Put for $0 \leq \lambda \leq 1$,

$$\eta^w(z,\zeta,\lambda) := (1-\lambda)\frac{w(z,\zeta)}{\langle w(z,\zeta), \zeta - z\rangle} + \lambda\frac{\overline{\zeta} - \overline{z}}{|\zeta - z|^2}$$

and for $f \in C^{0,q}(\overline{\Omega})$, $z \in \Omega$,

$$(15) \qquad (R^w_{\partial\Omega}f)(z) := \frac{(n-1)!}{(2\pi i)^n}\int_{\substack{\zeta\in\partial\Omega\\ 0\leq\lambda\leq 1}} f(\zeta)\wedge\omega'_{\zeta,\lambda}(\eta^w(z,\zeta,\lambda))\wedge\omega(\zeta).$$

Then one has with $T_q := (-1)^q(R^w_{\partial\Omega} + B_\Omega)$ the homotopy formula on Ω,

$$(16) \qquad f = \overline{\partial}T_q f + T_{q+1}\overline{\partial}f.$$

If $\overline{\partial}f = 0$ then $u := T_q f$ obviously solves $\overline{\partial}u = f$.

Holomorphic Leray maps were constructed on strictly pseudoconvex domains Ω by Ramirez **[26]**. Therefore, (16) becomes for these domains a tool for solving $\overline{\partial}$. From it—together with many variations of it—in recent years a very extensive theory has been developed with strong links to other fields like singular integrals, analysis on nilpotent groups, pseudodifferential operators (as two recent articles out of a long series we mention **[14** and **25]**, where other references can be found). With respect to many norms it gives the best possible estimates for solutions of $\overline{\partial}$.

The book by R. M. Range is an introductory text to complex analysis based on (16) (or rather an equivalent formula) and its consequences. Since the solution of the Levi-problem is needed to establish (16) globally on strictly pseudoconvex domains, (16) is used at first only locally. This is then put into the machinery of H. Grauert for the solution of the Levi problems and, finally, (16) is obtained globally. As applications Hölder- and L^p-estimates for $\overline{\partial}$ on strictly pseudoconvex domains Ω are given, the Mergelyan property of such Ω is proved, the Gleason decomposition in $A(\Omega)$ is derived and important regularity properties of the Bergman kernel on Ω are shown. They are used for the proof of the Fefferman theorem **[9]** about the boundary behavior of biholomorphic maps between such domains with the method of Webster **[28]**, Bell and Ligocka **[2]** and Bell **[1]**. Furthermore, the book contains the elementary theory of pseudoconvexity and domains of holomorphy.

Ranges' book is very carefully written. The philosophy of what is being done is always explained and extensive notes at the end of the chapters give a good picture of the history of each subject. Many exercises are used to assure a good understanding and to broaden the ideas.

Why does the book in the part treating (16) and its consequences consider only strictly pseudoconvex domains? The reason comes from nature, namely on arbitrary smooth pseudoconvex domains there are in general no holomorphic Leray maps, **[23]**. Also other difficulties arise which altogether explain why the theory of solving $\overline{\partial}$ with estimates on such domains based on integral kernels is still in its beginning. Although the book sometimes hints at this, a chapter on special new phenomena in the weakly pseudoconvex case might have been useful. It can be claimed with only little prophecy that one of the centers of active research during the next years will lie in this area.

The book can be recommended very much as a first text in complex analysis of several variables. But the reviewer would like to repeat here in his own words what the author also says in the book: In modern complex analysis the methods of all three basic approaches have to be put together. Therefore, any student who started in complex analysis with this book should continue by reading other books emphasizing more the Riemann and Weierstrass approaches (and vice-versa).

REFERENCES

1. S. R. Bell, *Biholomorphic mappings and the $\bar{\partial}$-problem*, Ann. of Math. (2) **114** (1980), 103–113.

2. S. R. Belll and E. Ligocka, *A simplification and extension of Fefferman's theorem in biholomorphic mappings*, Invent. Math. **57** (1980), 283–289.

3. D. Catlin, *Necessary conditions of subellipticity and hypoellipticity for the $\bar{\partial}$-Neumann problem on pseudoconvex domains*, Recent Developments in Several Complex Variables, Ann. of Math. Studies no. 100, 1981.

4. ——, *Boundary invariants of pseudoconvex domains*, Ann. of Math. (2) **120** (1984), 529–586.

5. J. D'Angelo, *Real hypersurfaces, orders of contact, and applications*, Ann. of Math. (2) **115** (1982), 615–637.

6. K. Diederich and J. E. Fornaess, *Pseudoconvex domains with real-analytic boundary*, Ann. of Math. (2) **107** (1978), 371–384.

7. K. Diederich and H. P. Pflug, *Necessary conditions for hypoellipticity of the $\bar{\partial}$-problem*, Recent Developments in Several Complex Variables, Ann. of Math. Studies no. 100, 1981, pp. 151–154.

8. K. Diederich and R. Remmert, *Funktionentheorie*. I, Springer-Verlag, Berlin-Heidelberg-New York, 1972.

9. C. Fefferman, *The Bergman kernel and biholomorphic mappings of pseudoconvex domains*, Invent. Math. **26** (1974), 1–65.

10. H. Grauert and I. Lieb, *Das Ramirezsche Integral und die Lösung der Gleichung $\bar{\partial} f = \alpha$*, Bereich der beschränkten Formen, Proc. Conf. Complex Analysis, Rice Univ., 1969, Rice Univ. Studies no. 56, 1970, pp. 29–50.

11. H. Grauert and R. Remmert, *Analytische Stellenalgebren*, Springer-Verlag, Berlin-Heidelberg-New York, 1971.

12. ——, *Theory of Stein spaces*, Springer-Verlag, Berlin-Heidelberg, New York, 1979.

13. ——, *Coherent analytic sheaves*, Springer-Verlag, Berlin-Heidelberg-New York, 1984.

14. P. C. Greiner and E. M. Stein, *Estimates for the $\bar{\partial}$-Neumann problem*, Princeton Univ. Press, Princeton, N.J., 1977.

15. G. M. Henkin, *Integral representations of functions in strictly pseudoconvex domains and applications to the $\bar{\partial}$-problem*, Math. Sb. **82** (1970), 300–308; English transl., Math. USSR Sb. **11** (1970), 273–281.

16. L. Hörmander, *L^2 estimates and existence theorems for the $\bar{\partial}$ operator*, Acta Math. **113** (1965), 89–152.

17. ——, *An introduction to complex analysis in several variables*, 2nd ed., North-Holland, Amsterdam-London, 1973.

18. A. Hurwitz and R. Courant, *Vorlesungen über allgemeine Funktionentheorie und elliptische Funktionen*, 1. Aufl, Springer-Verlag, Berlin, 1922.

19. K. Jänich, *Einführung in die Funktionentheorie*, Springer-Verlag, Berlin-Heidelberg-New York, 1977.

20. J. J. Kohn, *Harmonic integrals on strongly pseudoconvex manifolds*. I, Ann. of Math. (2) **78** (1963), 112–148; II, Ann. of Math. (2) **79** (1964), 450–472.

21. ——, *Subellipticity of the $\bar{\partial}$-Neumann problem on pseudoconvex domains: sufficient conditions*, Acta Math. **142** (1979), 79–122.

22. J. J. Kohn and L. Nirenberg, *Non-coercive boundary value problems*, Comm. Pure Appl. Math. **18** (1965), 443–492.

23. ——, *A pseudoconvex domain not admitting a holomorphic support function*, Math. Ann. **201** (1973), 265–268.

24. I. Lieb, *Die Cauchy-Riemannschen Differentialgleichungen auf streng pseudo-konvexen Gebieten*, Math. Ann. **190** (1970), 6–44.

25. I. Lieb and R. M. Range, *Integral representations and estimates in the theory of the $\bar\partial$-Neumann problem*, Ann. of Math. (2) **123** (1986), 265–301.

26. E. Ramirez, *Ein Divisionsproblem und Randintegraldarstellungen in der komplexen Analysis*, Math. Ann. **184** (1970), 172–187.

27. R. Remmert, *Funktionentheorie*. I, Grundwissen Mathematik no. 5, Springer-Verlag, Berlin-Heidelberg-New York, 1984.

28. S. Webster, *Biholomorphic mappings and the Bergman kernel of the diagonal*, Invent. Math. **51** (1979), 155–169.

KLAS DIEDERICH
BERGISCHE UNIVERSITÄT

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 20, Number 1, January 1989

0273-0979/89 $1.00 + $.25 per page

Lectures on Bochner-Riesz means, by Katherine Michelle Davis and Yang-Chun Chang. London Mathematical Society Lecture Notes, Vol. 114, Cambridge University Press, Cambridge, New York, Melbourne, 1987, ix + 150 pp., $22.95. ISBN 0-521-31277-9

"The theory of functions of several real variables": sounds old-fashioned, doesn't it? But look in your copy of Hewitt and Stromberg [2] or Royden [3]. You'll find plenty of analysis on the real line, and plenty of analysis on abstract topological spaces and measure spaces, and not much in between. This gap is partly a reflection of the temper of the times; but in the early 1960s when these books were written, beyond the level of calculus there really *wasn't* much in between, at least not much that was ready to be transplanted from research journals to books.

In the intervening quarter-century the situation has changed enormously, and analysis on $\mathbf{R}^n$ is now a thriving subject. Among the main lines of development are the following:

(1) Banach spaces of functions and generalized functions defined in terms of various growth or smoothness conditions: L^p spaces, Hardy spaces, Sobolev spaces and their relatives, BMO, and so forth. Closely intertwined with this are the theory of differentiability and the study of approximation of arbitrary functions by suitable types of smooth functions, such as trigonometric polynomials or harmonic functions.

(2) Singular integral operators, oscillatory integral operators, and operators defined by convolutions or Fourier multipliers, and their continuity properties with respect to the Banach spaces mentioned above. These classes of operators include pseudodifferential operators and their generalizations, as well as the Fourier transform.