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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 19, Number 2, October 1988

0273-0979/88 $1.00 + $.25 per page

Knots, by Gerhard Burde and Heiner Zieschang. DeGruyter Studies in Mathematics, vol. 5, Walter DeGruyter, Berlin, New York, 1985, x+399 pp., $49.95. ISBN 0-89925-014-9

On knots, by Louis H. Kauffman. Annals of Mathematics Studies, vol. 115, Princeton University Press, Princeton, N.J., 1987, xv+480 pp., $50.00 ($18.95 paperback). ISBN 0-691-08434-3

The central problem in knot and link theory is to distinguish link types via computable invariants. Figure 1 shows an example. for 75 years the two knots in Figure 1 were thought to represent distinct knot types, until in 1974 it was discovered that a totally unmotivated but very simple change in the projection takes the left picture to the right [**P**]. If we cannot find such a change, how can we be sure that two knots are distinct?

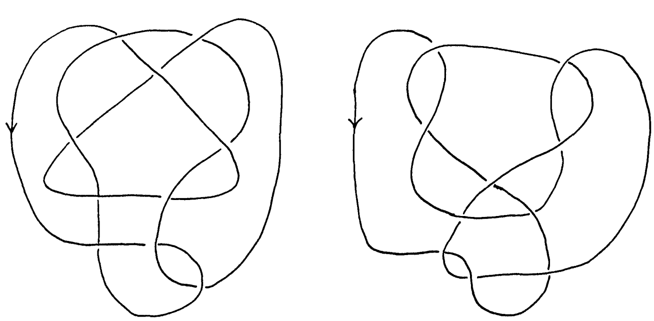


FIGURE 1

This review was written when the author was visiting the University of Paris VII. Partial support and the hospitality of the Mathematics Department during that visit are gratefull acknowledged.

A *link* $\mathbf{K}$ in 3-space is a subset of S^3 which is diffeomorphic to the disjoint union of r copies of S^1. If $r = 1$ it is a *knot*. Two links $\mathbf{K}$, $\mathbf{K}'$ are equivalent if there is a diffeomorphism h of the pair $(S^3, \mathbf{K})$ onto the pair $(S^3, \mathbf{K}')$, the equivalence class $\mathscr{K}$ being a link *type* (Add orientations to S^3 and or $\mathbf{K}$ to get slightly different concepts). The *unlink* $\mathscr{U}(r)$ is the link type which is represented by r disjoint planar circles $\mathbf{U}(r)$. Since knot complements are easily visualized and are at the same time highly nontrivial examples of 3-dimensional manifolds, they are interesting and important objects of study.

This is a particularly good moment for the publication of two new graduate level textbooks on knots and links. The subject has undergone dramatic changes during the past ten years, which both build upon the classical theory and depart from it. I have in mind, first, the introduction of geometry (and in particular hyperbolic geometry) into the subject by Thurston in the early 1980s [**Th1** and **Th2**]. Second, I am thinking of the discoveries in 1984 of vast new families of polynomial invariants of knots and links by Vaughan Jones, via the braid group and Hecke algebras [**J**]. I am also thinking of recent hints (see [**J**, **Ka1**, **F**]) of connections between knot and link theory and physics, in particular statistical mechanics. Finally, I am thinking that questions relating to the *computation* of knot related invariants have received much attention recently, because they are both interesting and accessible (see, for example [**Th3**]). The two books under review will therefore be welcome to graduate students and to research mathematicians who seek entre into the subject and need to learn what is known.

Burde and Zieschang's *Knots* will be prized as a reference source. Its bibliography is scholarly and extensive and is cross-referenced in a manner worthy of the computer era, with separate author and subject indexes that are first rate. The selection of topics is good and the development is interesting and logical. For example, fibered knots are introduced very early in the text. Since the Alexander polynomial of a fibered knot is easily understood, whereas its meaning is somewhat more elusive for arbitrary knots, this seems to me to have been a wise choice. Each section ends with careful historical notes. The tables in the Appendix contain some useful data not readily available elsewhere (e.g., classical signatures and Seifert matrices for the knots up to 10 crossings). The style is, however, sometimes ponderous; for example in many places there are florid Gothic characters decorated with tildes, hats, and multiple subscripts and superscripts, making the book unattractive for browsing.

Kauffman's *On knots* is a completely different sort of book. It is informal and chatty, and very pleasant for browsing. There are lots of wonderful illustrations and a wealth of detail from the author's bag of tricks, gathered over the years, relating to the combinatorics of knot diagrams and also to Seifert pairings, cobordism, signature invariants (several different ones), the Arf invariant, and the ubiquitous Alexander polynomial. There are many challenges to the reader to explore combinatorial patterns, which makes the book stimulating. There is also an Appendix that brings it up to date with a brief discussion of the new knot polynomials and a table giving values of the author's L-polynomial for knots up to 10 crossings. It is not, however, the sort of general reference text that Burde and Zieschang's book will be, for two reasons. First, its style is very informal (for example there isn't even an index); second the coverage reflects very much the author's very particular point of view on the subject, at the center of which lie the beautiful patterns in knot diagrams. Thus the two books really complement each other and have minimal intersection.

We should mention that neither book contains any hint of the role of Thurston's theory of geometric structures in 3-dimensional topology. For example, the Montesinos links that are studied in Chapter 12 of Burde and Zieschang's book are a special case of arborescent links. The latter have a beautiful and simple description (via Thurston's work) in terms of the geometry of their 2-fold covering spaces, but there is no hint of it in the text. It is also a pity that Thurston's study of the complement of the Figure eight knot [**Th1**] is not discussed in either book. A text on knot and link theory that gives an accessible introduction to geometric aspects of the subject is yet to be written.

The historical development of the subject of knots is interesting. Systematic efforts to classify knots began around 1870, the initial motivation coming from physics. The distinct chemical elements had been conjectured by Lord Kelvin to be related to knotting in the vortex lines of the ether, and the scientific papers of the Scottish physicist Peter Guthrie Tait [**Ta**] laid out a program of study based upon these ideas. These ideas seem remarkably sophisticated, and not very far from current thinking about quantum gauge field theory. Tait's hope was evidently that stability requirements (perhaps a restriction to alternating knots?) would impose restrictions on the knot types that could occur. The first step in his program was the collection of empirical data, and for this he enlisted the help of Thomas P. Kirkman [**Ki**], C. N. Little [**L**], and Mary Haseman [**H**]. Together, they assembled the first tables of knots, which have been used ever since, and can be seen, modulo surprisingly few corrections, at the end of both of the books under review. The measure of complexity that they used in their list of distinct knot types is the minimum number of crossings, among all possible planar projections of a knot. Empirical data is of course at the heart of any subject, and so this set of painfully assembled data has had an enormous impact on the subject. For example, a large number of the knots of low crossing number are alternating, have two bridges, and exhibit symmetries, and no doubt one of the reasons that these topics have been studied in depth in the literature is that the available data suggested the existence of structure.

Tait's hope was that he would discover invariants of knot type in the process of assembling data, but to his surprise and disappointment he was unsuccessful. One of the truly remarkable developments during the past few years is that the Jones polynomial [**J**], discovered in 1984, could have been discovered by Tait, Kirkman, Little, and Haseman, if they had only had the help of Louis Kauffman [**Ka2**]! We now explain Kauffman's methods.

We can think of our knot or link type $\mathscr{K}$ (as those earliest knot theorists did) as defined by a diagram, i.e., a projection onto a plane, the multiple points being a finite number of transverse double points, with a marker to indicate the overcrossing strand. A small amount of experimentation (or a proof based upon the use of polygonal representatives) should convince the reader of the reasonableness of a theorem of Reidemeister [**R**]: the three "moves" depicted in Figure 2, applied repeatedly, suffice to take any one diagram of a knot to any other. Thus, if one had a candidate for an invariant of knot type it would suffice to prove that it was invariant under R1, 2, and 3.

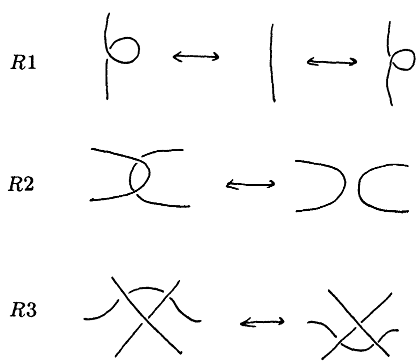


FIGURE 2

With some foresight, Tait might have been led to look for a *polynomial* invariant. We denote the diagram that defines $\mathscr{K}$ by **K** and the hoped-for polynomial by $\langle \mathbf{K} \rangle$. Initially **K** will be assumed to be unoriented. The proposed variables will be denoted $a, b, c, \ldots$. Since crossing number is our measure of complexity, it is natural to ask how the proposed invariant ought to change when we delete crossings. There are two ways to do this, so our first axiom is:

(i) $\langle \times \rangle = a\langle)(\rangle + b\langle \asymp \rangle$,

where the diagrams in question are assumed to be identical everywhere except in the indicated regions, where they differ in the manner shown. Note that rule (i) implies:

(i)$'$ $\langle \times \rangle = b\langle)(\rangle + a\langle \asymp \rangle$,

because we can't distinguish the left-hand sides of (i) and (i)$'$ without adding orientations. It's immediate that $\langle \mathbf{K} \rangle$ will be an integer polynomial in a, b, and c and $\langle \mathbf{U}(r) \rangle$.

Let **O** denote a Jordan curve in the plane and let "$\mathbf{O} \cup \mathbf{K}$" denote its disjoint union with a nonempty diagram **K**. Our next axiom is

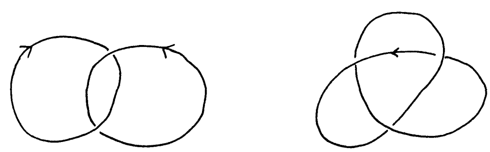
(ii) $\langle \mathbf{O} \cup \mathbf{K} \rangle = c\langle \mathbf{K} \rangle$ if $\mathbf{K} \neq \varnothing$, or 1 if $\mathbf{K} = \varnothing$.

Axioms (i) and (ii) determine $\langle \mathbf{K} \rangle$ unambigously, on all link diagrams.

Let's apply Reidemeister's moves and see what happens. A simple application of the axioms shows that:

$\langle$ $\rangle = ab\langle)(\rangle + (abc + a^2 + b^2)\langle \asymp \rangle$.

This shows that for R1 to hold we'll have to set $b = a^{-1}$ and $c = -a^2 - a^{-2}$. A miracle then occurs, for R3 holds without any further specializations. (The check is left to the reader.) The effect of R1, however, is nontrivial. There are two cases, depending upon the sense of the added loop. In one case $\langle \mathbf{K} \rangle$ is multiplied by $-a^3$, in the other by $-a^{-3}$. To correct for this invariance, we follow Kauffman, and declare the link to be oriented, and regard all previous work as having defined a pre-invariant by simply forgetting the orientation. For an oriented link, the diagram $\mathbf{K}$ has a well-defined algebraic crossing number $w(\mathbf{K})$. It is then a simple matter to show that if one replaces $\langle\mathbf{K}\rangle$by $f_{\mathbf{K}}(a) = (-a)^{-3w(\mathbf{K})}\langle\mathbf{K}\rangle$, then $f_{\mathbf{K}}(a)$ is invariant under all three Reidemeister moves, and so is an invariant of link type! The invariant $f_{\mathbf{K}}(a)$ is (up to a change in variables) the Jones polynomial [**J**], as it was rediscovered in [**Ka2**]. It does a very good job of telling knots and links apart, and would have enabled Tait and his co-workers to reduce years of work into a few days of calculation.



$f(A) = -A^{-2} - A^{-10}$ $\qquad$ $f(A) = A^{-4} + A^{-12} - A^{-16}$

FIGURE 3

Replace $\mathbf{K}$ by its mirror image $\mathbf{K}!$ and you interchange axioms (i) and (i)$'$. From this it follows almost immediately that $f_{\mathbf{K}!}(a) = f_{\mathbf{K}}(a^{-1})$. Since the Hopf link and the trefoil knot (Figure 3) both have polynomials that are not symmetric under the interchange of t and t^{-1}, it follows that neither one is amphicheiral, answering one of the earliest questions in knot theory.

In fact, Jones' invariant was not discovered early in the game, but a somewhat different polynomial invariant was discovered in 1928 by Alexander, using methods (see [**A1**]) that were initially as mysterious as those we just described, and that depended similarly upon the combinatorics of knot diagrams! It's not an exaggeration to say that a fair fraction of the work during the 25 year period that followed Alexander's initial discovery was devoted to explaining "the meaning" of his very effective combinatorial invariant.

To explain Alexander's invariant in a topological setting, we restrict our attention momentarily to knots and pass from knot type to the topological type of the complement $X = S^3 - \mathbf{K}$ and thence to its group $G = \pi_1 X$. The abelianization of G is $\mathbf{Z}$ (generated by the homology class of an oriented loop around the knot), so to learn more we pass to the commutator subgroup G'. Geometrically, G' belongs to the unique infinite cyclic covering space $\mathbf{X}$ of X. We let t denote the generator of the group of covering translations, which acts on $\mathbf{X}$. Now $H_1(\mathbf{X}; \mathbf{Z})$ turns out to be infinitely generated as an abelian group, but it's finitely generated as a $\mathbf{Z}[t, t^{-1}]$ module. The latter is not a module over a PID (a source of much difficulty); however for practical purposes the theory goes through as if it were. The generator of the "order ideal" in our module is the Alexander polynomial $A(t)$. It's a Laurent polynomial, and it is unique up to sign and a multiplicative power of t, although if you wish this ambiguity can be removed [**C**]. It does a pretty good job at distinguishing knots; in the tables mentioned earlier, among the 12965 knot types of at most 13 crossings, 5639 distinct Alexander polynomials occur. Since the group of a knot and its mirror image are isomorphic, they have the same Alexander polynomial. Thus the Alexander polynomial is distinct from the Jones polynomial. (For the same reason, the Jones polynomial is *not* a knot group invariant.) The Alexander and Jones polynomial are known to be specializations of a single 2-variable polynomial [**FYHLMO**]; however they are independent, i.e., each distinguishes knots that the other cannot.

In a spirit similar to that just used in our description of the Alexander polynomial, one can also look at finite-sheeted covering spaces of a knot complement, and at coverings of S^3 branched over the knot. Each such covering produces a 3-manifold, and the homology of the manifolds so obtained is a source of further knot invariants. Other invariants arise when one learns that a knot is the boundary of a (highly nonunique) orientable surface, a *Seifert surface*, embedded in S^3. The surface places structure on the complementary space, and further knot invariants have been obtained by studying such surfaces. The ubiquitous Alexander polynomial in fact has an interpretation in this setting: one associates to each Seifert surface of genus g a $2g \times 2g$ matrix S of integers that records linking numbers of basis curves for the homology of the surface, as they are pushed off the surface into the complementary space, and finds that $A(t) = |S - tS'|$. The matrix S is useful for much more than this, since its study leads naturally to the various signature invariants, and to an investigation of knot invariants which can be understood in the setting of 4-dimensional topology. Kauffman's book gives a beautiful and gentle introduction to this area. Still further invariants arise via representations of knot and link groups onto finite groups and into $\mathrm{PSL}(2, \mathbb{C})$.

We return to the Jones invariant. The Jones polynomial was discovered not by the combinatorial methods that we described above, but by quite different techniques having to do with braid groups. Most of the recent hints at connections between knot theory and other areas of mathematics have to do with braids, so we now describe the braid groups. A knot or link $\mathbf{K}$ is said to be represented as a *closed braid* if there is an unknotted curve $\mathbf{A}$ in $S^3 - \mathbf{K}$ (think of $\mathbf{A}$ as the z axis in R^3) and a product projection $\pi: S^3 - \mathbf{A} \to S^1$ (e.g., use cylindrical coordinates and set $\pi(z, r, \theta) = \theta$), which is monotonic on $\mathbf{K}$. If you choose a closed braid representative, and then split R^3 open along any half-plane through $\mathbf{A}$ you'll obtain an (open) braid representative, which lies between two half-planes as in Figure 4. The equivalence relation should be obvious: two representatives define the same braid if one can be transformed to the other by isotopy in the region between the two copies of our half-plane, keeping the ends fixed and without allowing two strands to cross one another. Braids can be multiplied by concatenation, rescaling, and deletion of the middle plane. This makes braids into a group B_n, in fact into a sequence of groups $B_1, B_2, B_3, \ldots$.

Every knot and link can be represented as a closed braid [**A2**, **Mo**, **Y**]. A pleasant way to obtain such a representative, due to Hugh Morton, is to start with an oriented projection $\mathbf{K}$ for the link in question, and divide it into $2k$ arcs that are alternately "overpasses" and "underpasses." Now let A be a simple closed curve on the plane of projection that separates the set $\mathbf{I}$ of all initial points of overpasses from the set $\mathbf{F}$ of all final points. To obtain a braid axis $\mathbf{A}$ from A simply "thread" A into $S^3 - \mathbf{K}$, using the following rule: traverse

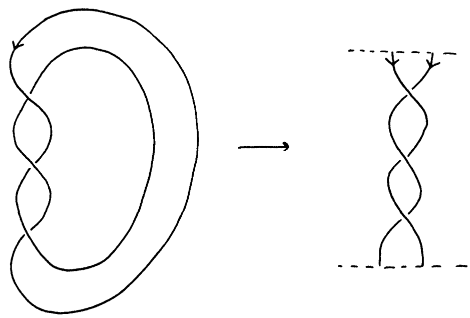


FIGURE 4

K, passing *over A* each time you cross from the **I**-side to the **F**-side, and passing *under A* when you return. (If you're not sure this actually produces a braid axis, see [**Mo**] for further details.) Morton's threading construction has a very nice consequence. By examining the various choices which were made (e.g., the choice of the diagram, of the overpasses, and so forth) he discovered a new and particularly succinct proof of the important theorem of Markov, which follows:

THEOREM ([**Be**, **Bi**, **Ma**, **Mo**]). *Let $\mathscr{B}$ denote the disjoint union of the B_n's, $n = 1, 2, 3, \ldots$. Define a Markov class in $\mathscr{B}$ to be the equivalence class under the equivalence relation generated by conjugacy in each fixed B_n, and the map from B_n to B_{n+1} defined by adding one new unknotted loop about* **A**. *Then there is a one-to-one correspondence between closed braid representatives of oriented link types in oriented S^3 and Markov classes in $\mathscr{B}$.*

Markov's theorem shows that the search for link type invariants can be reinterpreted as a search for class invariants that behave nicely under Markov's map from B_n to B_{n+1}. This had been known for a long time, but could not be implemented before 1984 for lack of knowledge about any such class invariants. All this changed dramatically in 1984, when Jones discovered that the braid groups had representations in an ascending sequence of finite-dimensional matrix algebras, and even more that he already had Markov class invariants on hand, ready to go. His invariants were the Jones polynomial, which we already described earlier via combinatorics. The methods used by Jones are particularly interesting because they generalize to produce other polynomials based upon other representations of the sequence of braid groups, and to suggest many connections between link theory and other areas of mathematics and physics.

Very recently D. Long has shown in [**Lo**] that there is a very general method to produce linear representations of the braid groups. His work makes a systematic study of representations of $\mathbf{B}_n$ now seem possible. To explain his idea, the first thing to note is that $\mathbf{B}_n$ has an interpretation as a subgroup of the automorphism group of a free group $\mathbf{F}_n$. (The free group occurs as the fundamental group of the n-times punctured plane, on which $\mathbf{B}_n$ acts in a natural way.). If one now lets **L** be any Lie group, then a choice of any n-tuple of matrices in **L** gives a representation of $\mathbf{F}_n$. The full representation space

$\mathbf{R}_n$ of $\mathbf{F}_n$ in $\mathbf{L}$ is thus the direct product of n copies of $\mathbf{L}$. The action of $\mathbf{B}_n$ on $\mathbf{F}_n$ induces an action on $\mathbf{R}_n$, so that we can reinterpret $\mathbf{B}_n$ as a subgroup of Diff $\mathbf{R}_n$. If one can find a $\mathbf{B}_n$-invariant subspace of $\mathbf{R}_n$, the action on the tangent space to $\mathbf{R}_n$ at points of this subspace will yield linear representations of $\mathbf{B}_n$. His methods give a truly satisfactory connection between the classical Burau representation (see [**Bu**, **Bi**]) and the new work in [**J** and **B-W**], and should open up a rich new area of investigation.

Clearly there is a great deal of interesting work to be done. It is not impossible that we already know enough about knots and links and braids to be approaching a complete solution to the link problem. The two books under review could play a useful role in such a project.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 19, Number 2, October 1988

0273-0979/88 $1.00 + $.25 per page

The lost notebook and other unpublished papers, by Srinivasa Ramanujan, with an introduction by George E. Andrews, Narosa Publishing House, New Delhi, Madras, Bombay, 1988, xxv + 419 pp., 250 rupies. ISBN 81-85198-06-3, North American and European distribution: Springer-Verlag, ISBN 0-387-18726-X

In the spring of 1976, G. Andrews was looking through a box of Watson's material in the library of Trinity College when he came across about 90 sheets of paper, most of them in Ramanujan's handwriting. In 1957 the Tata Institute for Fundamental Research had published photostatic copies of Ramanujan's early notebooks [**2**], so Ramanujan's writing was well known to Andrews and quite a few others. However very few people would have been able to recognize exactly what was in this box in the Trinity library. Andrews had written a thesis on mock theta functions, so when he saw that some of these sheets contained claims of Ramanujan about mock theta functions, he knew this was a major find. These sheets consist primarily of work Ramanujan did in the last 15 months of his life, after he left England and returned to India. For the last ten years, Andrews has published a number of papers proving results in these sheets, and a few other people have published a little more, but the mathematical community at large has not had access to this fascinating collection. Thanks to Narosa Publishing House, anyone who wants to can now try his or her hand at proving some of Ramanujan's last results.

Many other fascinating things are contained in this book. There is Littlewood's letter to Hardy commenting on Ramanujan's second letter. Among other perceptive comments in this letter is the following: "I can believe that he's at least a Jacobi."

There are some manuscripts of Ramanujan that were not published before, either because of financial problems that the London Mathematical Society had, or because they were unfinished. There is a fascinating sheet (p. 358) which is undated, but was probably written in 1915. It contains four reasons why

$$\begin{aligned}(1)\qquad &1+\frac{x}{1-x}+\frac{x^4}{(1-x)(1-x^2)}+\frac{x^9}{(1-x)(1-x^2)(1-x^3)}+\cdots\\ &=\frac{1}{(1-x)(1-x^6)(1-x^{11})\cdots(1-x^4)(1-x^9)(1-x^{14})}\cdots\end{aligned}$$