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Ω-bibliography of mathematical logic, edited by Gert H. Müller in collaboration with Wolfgang Lenski. Springer-Verlag, Berlin, Heidelberg, New York, 1987,

Vol. I: *Classical logic*, edited by Wolfgang Rautenberg. xxxix+485 pp., $170.00. ISBN 3-540-17321-8.

Vol. II: *Non-classical logics*, edited by Wolfgang Rautenberg. xxxvii+469 pp., $170.00. ISBN 3-540-15521-X.

Vol. III: *Model theory*, edited by Heinz-Dieter Ebbinghaus. xlv+617 pp., $200.00. ISBN 3-540-15522-8.

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Vol. V: *Set theory*, edited by Andreas R. Blass, li+791 pp., $230.00. ISBN 3-540-15525-2.

Vol. VI: *Proof theory and constructive mathematics*, edited by Jane E. Kister, Dirk van Dalen and Anne S. Troelstra. xli+405 pp., $155.00. ISBN 3-540-15524-4.

Oranges. This collection of six hefty, orange volumes is a dream come true for anyone interested in mathematical logic and its history. It contains a remarkably complete bibliography of the field, from 1879, the year of Frege's *Begriffsschrift*, through 1985.

I was delighted to be asked to review this bibliography, since I knew that short of hocking my car or personal computer, it was the only way I would ever own one. And having had the bibliography for some months, I have not been disappointed. It is enormously useful in looking up references to works dimly remembered, in finding one's way to surprising works in fields one thinks one knows, and in gaining a better appreciation for the breadth and depth of work done in mathematical logic over the past hundred odd years.

The six volumes cover classical logic, nonclassical logic, model theory, recursion theory, set theory, and (in one volume) proof theory and constructive mathematics. Each volume has a number of introductory sections, including a general survey of work in the volume, and useful appendices of various sorts. However, the core of each volume consists of three indices: Subject Index, Author Index, and Source Index.

First comes the Subject Index, where items are ordered by means of a modification of the 03Xxx part of the AMS Subject Classification Scheme, the part that covers mathematical logic. (Since only mathematical logic is covered in the bibliography, the "03" part of the notation is dropped.) Within that, items are arranged by year of publication, and within that, alphabetically by author. For example, opening the Subject Index of Volume V (Set Theory) at random, we find section E10 on ordinal and cardinal numbers. For the year 1915 there is only one item:

Hartogs, F., *Ueber das Problem der Wohlordnung.*
◊ E10 E25 ◊

The information between the ◊'s consists of all the subject classifications of the article, in this case E10 and E25, the latter being the section on the Axiom of Choice. To find out more about the article in question, one turns to the Author Index. As one might expect, this index is an alphabetical listing by author of all articles referred to in the Subject Index. Looking up Hartog's article, we find:

Hartogs, F. [1915] *Ueber das Problem der Wohlordnung.*

(J 0043) Math Ann 76*439-460

◊ E10 E25 ◊ REV FdM 45.125 •ID 05705

The reference "J0043" tells one where to find the fuller meaning of "Math Ann" in the Source Index. (It stands for the 43 entry of the Journal section.) Again between ◊'s we find information about the sections of the subject index under which the entry can be found, E10 and E25. Next comes information about where one can find reviews of the article. Finally comes an identification number that corresponds to the entry of the item in the computerized database from which the bibliography has been created.

At first sight, this seems like a fairly complicated scheme, since if one wants the full reference of a paper, or if you want to actually go look up the paper, it can sometimes involve assembling information from three separate places in the volume. However, there is a clear rationale for setting things up this way. In the first place, it reduces the bulk of the volumes by keeping the amount of redundant information to a minimum. And in so doing, it allows the Subject Index to be very readable, that is, pleasing to the eye, and informative. For example, you can quickly browse through the entire section E10 on Ordinal and Cardinal numbers, seeing how the subject grew, what topics came and went, and who the contributors were over the years.

This work has clearly been a labor of love for many people over a number of years, and they have done a marvelous job. So before going on to list some problems, let me applaud them for what they have achieved. It continues the tradition in logic established by Alonzo Church in his efforts over many years of maintaining a bibliography of logic in the *Journal of Symbolic Logic.* This bibliography is appropriately dedicated to Professor Church.

Now let's turn to some of the difficulties. Of course some problems would seem to be inherent in any such project. There are bound to be omissions and errors. And the bibliography is going to be out of date immediately. So let's look at problems more specific to this effort.

The use of the Subject Index as described above is of course highly constrained by the classification scheme itself. To the extent that the classification scheme happens to have an entry that neatly fits the subject you are investigating, fine. In my experience, though, the classification scheme seldom lists what I am looking for. It is, after all, an attempt to classify a huge continuum of work under roughly 100 headings. So I usually find myself using the bibliography in a different way.

Suppose, for example, that you are interested in finding out about treatments of set theory that have axioms guaranteeing the existence of circular or otherwise nonwellfounded sets. There is no entry in the classification scheme for this topic. So how do you find where they are listed? Well, as long as you know one author, say M. Boffa, who has written on this topic, you can find your way. You look up Boffa in the set theory Author Index, find two or three of his articles on alternatives to the Axiom of Foundation, and discover that they are all listed as belonging to section E70. You can then turn to this part of the Subject Index and read it to find other articles on the same topic. So the construction of the Author Index, in particular the cross references with the Subject Index, partially overcomes the constraints imposed by the classification scheme.

An annoying problem in this regard, though, is that there is no place in the bibliography which tells you where you will find a given classification, say E70. Which volume of the bibliography is it in? And given the volume, where in the volume can you find it? The latter problem arises because the editors did not arrange the subject classification alphabetically. In fact, a portion of the subject classification for Volume V runs E75, C55, C62, E30, E70, G30,.... It would have made more sense to me to have each volume contain the items listed under, say E and F, and then have those arranged numerically. Another obvious remedy, given the way things are organized, would be to list in the Subject Classification scheme (which helpfully appears in each volume) the volume and page number where each section is to be found. Even better would be to summarize the Classification Scheme, with page references, on the inside cover of each volume.

Another problem is probably inescapable. Just what *is* mathematical logic, anyway? That is, what should one expect to find covered in the bibliography of mathematical logic? One can imagine various answers. Perhaps mathematical logic is that part of logic that is carried out using tools from mathematics. Or perhaps it is that part of logic that studies mathematics. Or perhaps it is the intersection of these two: that part of logic that uses tools from mathematics to study mathematics. Or perhaps it is just whatever has come to be called mathematical logic by logicians. In fact, none of these characterize the material covered by the volumes. None of the mathematical work done in the logic of computer science, outside of recursion theory, is included, for example. Nor is work like Montague's using mathematical tools to analyze the logic of natural language. So the first definition is not the one being used. On the other hand, work on deontic, modal, and tense logic is included, so none of the other definitions apply either. It really isn't clear what the rationale for inclusion was. The AMS Subject Classification scheme for logic provided a starting point, but some topics were added and others were deleted. As one who currently believes the first definition of mathematical logic is the only sensible one, I applaud the inclusions, and lament the omissions. (And I note that the *Handbook of Mathematical Logic*, which I edited, was regrettably even narrower. So I do understand the problems and sympathize with the editors' dilemma.)

The volumes are beautifully produced, in the best Springer-Verlag tradition. However, they are fantastically expensive. I gather that the editors protested and were even willing to forgo royalties in an attempt to get the price within reach of their colleagues, but failed. This is surely their biggest failing. The price will severely damage the utility of this prodigious effort, by making it available essentially only in libraries, and then only in the most affluent libraries with a logician on hand to argue forcefully for its purchase.

This is a real pity, since libraries without a strong incentive will probably not purchase the work, thus depriving future researchers of this valuable tool. One can only hope that a cheaper edition will be made available at some point in the future.

Apples. I hope I have made it clear how wonderful I think the bibliography is, what a service it is to the logic community, and how delighted I am to own a copy. Because I now want to raise a question about the whole project. Wouldn't it have made a lot more sense to produce something else entirely? In particular, wouldn't a computerized, on-line version of the bibliography have been much more useful?

In the first place, the bibliography was produced from a computerized data base. If one had that at one's disposal it would be much easier to find one's way around. It could be produced at a fraction of the cost. And it could be corrected and updated much more easily. Also, such a format would make the use of such a rigid classification scheme unnecessary. After all, the current classification scheme is really nothing more than a list of roughly 100 key words and phrases, listed in a linear order. In an on-line version, the number of key words and phrases could easily be an order of magnitude greater, and open ended. Furthermore, the need for a linear ordering of the material would be eliminated.

Indeed, this would be a natural for some sort of hypertext format, where information is organized not sequentially, but logically, through various sorts of links. The whole point of hypertext is to arrange information in a graph-theoretic structure according to a wide range of relationships. For example, it would be possible to have such a bibliography where, when one was examining the Hartogs entry, you could ask for a list of papers referred to by Hartogs's article. Similarly, you could ask for a list of papers that refer to Hartogs's article. The variations are literally endless. And if the bibliography were available in something like the Macintosh Hypercard format, users would be free to create such links of their own, as well as update the bibliography. One could even annotate the bibliography with personal notes when one read a given article.

Of course there are problems to be overcome in producing such a computerized version of the bibliography. The main difficulty is the diversity of different computer systems now available, and the rapidity with which such systems come and go. So perhaps it would have been premature in 1985. But surely future versions of the Bibliography will be dinosaurs if they do not take advantage of the technology that is now available, technology that could solve most if not all the problems that seem to be inherent in an undertaking of this sort.

Apples and oranges. In conclusion, it seems to me that the editors of this work have done a wonderful job, as good a job as anyone could have done, given the medium in which they chose to produce the bibliography. In doing so, they have created an important reference work to slightly over one hundred years of mathematical logic. However, it also seems clear that a much more compact, useful, versatile, and inexpensive bibliography could and should eventually be created in a hypertext format, for use in libraries but also at home on personal computers. It really would not be that difficult, starting from the database the editors of this work have painstakingly assembled. In other words, what we have here are six lovely oranges, but what I want is just one Apple.

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Differential geometry of complex vector bundles, by Shoshichi Kobayashi. Publications of the Mathematical Society of Japan, no. 15 Iwanami Shoten Publishers and Princeton University Press, Princeton, N. J., 1987, xi+304 pp., $57.50. ISBN 0-691-08467-x

The book under review is a research monograph laying the foundation for the theory of Einstein-Hermitian structures on holomorphic vector bundles. The concept of an Einstein-Hermitian structure has been introduced by the author in 1978 and has proved to be very fundamental and popular since. Being fundamental usually is not sufficient for being popular; what made this concept so popular? I see at least two principal reasons:

The first is that it provides a link between differential geometry and algebraic geometry, leading to a good problem, the so-called Kobayashi-Hitchin conjecture. This problem has in the meantime been completely solved by Donaldson [**2**, **3**] and Uhlenbeck and Yau [**10**].

The second reason is that the solution of this conjecture in the 2-dimensional case made several spectacular applications possible.

In complex dimension 2 the conjecture ties Yang-Mills theory and algebraic geometry. Together with Donaldson's fundamental work on instanton moduli spaces it led to unexpected results on the differential topology of algebraic surfaces [**4**, **5**, **9**].

What is an Einstein-Hermitian structure? To explain this consider a compact complex submanifold $X \subset \mathbf{P}^N_{\mathbf{C}}$ of some projective space endowed with the induced metric. A holomorphic vector bundle $\mathscr{E}$ on X is a locally trivial fibre space over X with fibres $\mathbf{C}^r$ and holomorphic transition functions. Suppose we want to equip $\mathscr{E}$ with a Hermitian structure h, i.e., a C^∞ family $(h(x))_{x\in X}$ of Hermitian metrics on the fibres. It would then be natural to look for a best or in some sense distinguished structure h. Now it is a fundamental fact that every choice of a Hermitian structure on a holomorphic bundle gives rise to an associated concept of parallelism, in other words, to a compatible connection D_h in $\mathscr{E}$. The mean curvature K_h of this connection is a Hermitian form on $\mathscr{E}$, also depending on the metric on X, which measures how the bundle is twisted. If this form is proportional to the metric h, $K_h = c \cdot h$, one says that h is an Einstein-Hermitian structure or that $(\mathscr{E}, h)$ is an Einstein-Hermitian bundle.

This concept obviously generalizes the notion of a Kähler-Einstein metric. Kobayashi arrived at the definition of an Einstein-Hermitian structure when