results, such as the Ahlswede-Daykin Inequality, reach their full generality and naturalness in the setting of distributive lattices. I have found it pedagogically helpful to include a unit on distributive lattices before presenting this inequality to students.

It is also possible that, in a long semester, an instructor may not want to work through all the details of extremal set theory, and instead include some related topics with a slightly different flavor. In addition to including some lattice theory, one can move on from the linear extensions of the XYZ Conjecture to discuss dimension theory of posets. Later, after becoming thoroughly familiar with subsets and binary vectors via the Kruskal-Katona Theorem, one can finish the course with a unit on coding theory. I will take this approach in my next graduate course.

In packing a well-developed subject into 250 pages, one must make choices; these will never please everyone, and quibbling wastes time. On balance, one is hard put to find complaints about this well-written and thorough book. It is a valuable addition to the literature, it will make it easy for interested mathematicians to acquire a new specialty, and it brings another area of mathematics into the accessible graduate curriculum.

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Stochastic geometry and its applications, by D. Stoyan, W. S. Kendall, and J. Mecke, with a Foreword by D. G. Kendall. John Wiley and Sons, Chichester, New York, Brisbane, Toronto and Singapore, 1987, 345 pp., $49.95. ISBN 0471 90519 4

As a subject Stochastic Geometry surely existed (albeit anonymously) before this term first appeared in a title of a collection of papers [**1**] edited by E. F. Harding and D. G. Kendall in 1974. Numerous problems (and of course less numerous solutions), which in retrospect should be attributed to this field, have been discussed in countless papers scattered within journals and books too often devoted to nonmathematical applications and therefore obscure from the standpoint of a pure mathematician.

In many cases the authors of these papers were equipped merely with the tools of classical geometrical probability theory among which the notions of uniform distribution and independence were the basic. And yet their objectives were substantially more complicated concepts of what later came to be known as Stochastic Geometry. In the lucky cases the deficiency in tools was compensated by intuition.

Terminological ambiguity was quite widespread. For instance, within a paper considering random finite arrays of points the term "distribution" could simultaneously mean (a) the realization at hand, (b) the distribution of the typical point in the array, (c) a statistical estimate of (b), or (d) the distribution of the underlying point process. This terminological and conceptual mess blocked the way to deeper results and also barred the field from the influx of fresh mathematical forces.

In the seventies came the recognition of the fact that behind this mosaic picture lies an attractive and almost virgin mathematical field of inquiry closely connected with such branches as integral geometry, random processes, and even statistical physics.

A burst of mathematical activity followed, culminating in a number of monographs. However their aim remained restricted, from introducing a refined mathematician into the field (G. Matheron [**2**]) to demonstration of the potential of new combinatorial methods (R. V. Ambartzumian [**3**]). None of them could in fact satisfy the basic needs of the interested applications-oriented people who eagerly look for suitable ready mathematical apparatus for their problems. A need existed for a systematic presentation of the subject based on numerous examples and counter-examples and carefully leading the reader from the simple to the more complicated.

The book of D. Stoyan, W. S. Kendall, and J. Mecke makes an important long-delayed step in this direction.

But let me in a few words introduce the subject as it stands today to the general reader. Empirically given sets of geometrical elements (lines, segments, polygons, etc.) often show no sign of independence of the positions of the elements. On the contrary, to assume that in our set the elements have obtained their position as the result of repeated independent drops will often mean that we actually discard essential properties of interest. This is most obvious in case of mosaics (say planar) where the elements (polygons) are organized so as to cover the plane without gaps and overlappings. The concept of a mosaic is broad enough to permit rather chaotic arrangements of polygons (as contrasted to classic regular arrangements). In such cases it is sometimes preferable to think of the mosaic as a *random* aggregate of polygons. Random mosaics are an example of what can be called a "restricted chaos." The probabilistic description of "restricted chaos" in general is one of the main concerns of Stochastic Geometry.

Another example along these lines has connections with statistical physics. The problem is to describe the "most chaotic" translation invariant random sets of *nonintersecting* balls in R^n, all of the same unit radius. Let us denote by P the probability distribution of the set of ball centers and by Π its Palm distribution. Recall that Π is the conditional distribution of the center process described by P given that "there is a center at the origin 0." The equation (first introduced in [**4**] and solved there for $\mathbf{R}$),

$$\Pi = P|_A * \Delta \tag{1}$$

has a family of solutions which from many points of view deserve to be considered as solutions to our problem. Here A is the event that the ball of radius 2 centered at 0 is void of centers from random set, $P|_A$ is the corresponding conditional distribution of the set, and Δ corresponds to a center at 0. As shown in [**7**], the solutions of (1) coincide (for all $\mathbf{R}^n$) with hard core Gibbs models of

Statistical Physics. Although born in general point process theory, the notion of Palm distribution is now one of the main tools in Stochastic Geometry. Moreover, the theory of stochastic point processes provides rather general means for describing random sets of geometrical elements. In Stochastic Geometry the carrier spaces of the point processes happen to be the manifolds representing the geometrical elements. The best known example is the representation of sensed lines on R^2 by points of a cylindrical surface. Another specific feature of the subject is the importance attributed to groups acting on these manifolds.

Because the study of triads consisting of a manifold, a group and an invariant point process on the manifold forms a rather closed circle of elegant problems, some tend to consider such studies to be the essence of the Stochastic Geometry. Others interpret Stochastic Geometry as a much broader subject that stretches to include the general theory of random sets. However, one of the latest developments here is the theory of random shapes with definition depending on a group acting in the basic space. The Euclidean random shapes have been effectively studied by D. G. Kendall and his group in Cambridge while affine random shapes were considered in Yerevan integral geometry group [**5**].

The scope of the book under review can be considered as intermediate. The reader can find here some introductory facts about general random sets; yet as far as the departures from the point processes are considered the main effort in the book is directed to a rather special (but important) case of random fiberfields. The book includes sections on Euclidean random shapes and on connections with Statistical Physics, but more recent developments like affine random shapes and equations in the style of (I), which still lack full maturity, naturally remained outside the scope. Perhaps under the pressure from applications, Stochastic Geometry now tends to shift to more pragmatic topics as compared with those that prevailed in the beginning of its development when abstract general foundations have been laid down. Studies of concrete models, from derivation of formulae to Monte-Carlo simulations, are now much more popular than proving existence-type theorems and the book fully reflects this pragmatic shift. After describing some mathematical prerequisites in Chapter 1 the authors start with Poisson point processes (Chapter 2) and Poisson Boolean model (Chapter 3), then go to general theory and construction of models of point processes (Chapters 5, 6), random closed sets (Chapter 6), and random measures (Chapter 7). These are the basic tools with which the authors pass to work with the topics of Stochastic Geometry proper: random point processes of geometrical objects (Chapter 8), fiber and surface processes (Chapter 9), random tessellations (Chapter 10). A highly interesting Chapter 2 is devoted to "stereology," the art of drawing inferences about a geometrical structure on the basis of projections, sections, or slices. Many facts of Stochastic Geometry admit dual interpretation depending on the way of their derivation. On one hand, it is possible to view them predominantly as corollaries of the theory of random processes. An alternative to this is to consider them as flowing from integral geometry. Accordingly there can be two ways of presenting the subject. As seen from the above listing of chapter titles, the book propounds mainly the stochastic processes vision of the field. This is a particularly natural frame for presenting results of geometrical statistics. The reader finds here many down-to-earth statistical algorithms, estimates, diagrams, and tables.

Pushing integral geometry to the background certainly has its disadvantages. As an instance of these we mention the failure (because the necessary language is not developed in the book) to adequately describe the solution of the stereological problem of sectioning convex polyhedra by random planes considered in §II.5.3. Meanwhile combinatorial integral geometry solves a much broader class of problems (convexity is irrelevant) in a rather efficient way [**3**]. On p. 100 there is an erroneous statement that the well-known Dobrushin-Koroljuk property of stationary point processes in $\mathbf{R}$ generalizes to $\mathbf{R}^2$ only under assumption of motion-invariance. A piece of integral geometry demonstrates readily that in fact translation-invariance will suffice.

There are some other minor mistakes like that on p. 145, where Gauss-Poisson process is wrongly stated to be Cox, or on p. 212, where the authors wrongly state that motion-invariance forces the Cox line processes to be mixed Poisson. However such misprints do not diminish the feeling that this is a splendid work, a result of many years of creative efforts of leading specialists in the field. More than twenty pages of references substantially complement the lists of relevant literature published earlier (say in [**6**]) especially by presenting recent contributions.

Future prosperity of the subject of Stochastic Geometry will depend on whether the book under review will be followed by others of no less maturity and inspiration level. Such a book stressing by way of balance the integral geometry foundations will now be most welcome.

REFERENCES

1. E. F. Harding and D. G. Kendall (eds.), *Stochastic geometry*, Wiley and Sons, Chichester, 1974.

2. G. Matheron, Random sets and integral geometry, Wiley and Sons, New York/London, 1975.

3. R. V. Ambartzumian, *Combinatorial integral geometry*, Wiley and Sons, Chichester, 1982.

4. R. V. Ambartzumian and G. S. Sukiasyan, *An intrinsic description of processes of nonintersecting noninteracting balls*, Izv. Akad. Nauk Armyan. SSR Ser. Mat. **18** (1983), no. 3, 206–215. (Russian)

5. R. V. Ambartzumian, *Problems of Sylvester type for homogeneous Poisson processes*, Izv. Akad. Nauk Armyan. SSR Ser. Mat. **20** (1985) no. 4, 284–288.

6. L. A. Santalo, *Integral geometry and geometric probability*, Addison-Wesley, Reading, Mass., 1976.

7. R. V. Ambartzumian, *Equations involving Palm distributions*, Izv. Akad. Nauk Armyan. SSR Ser. Mat. (In print).

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