At the close of the book the reader will find historical notes that, in the main, seem to be accurate, and a comprehensive bibliography, as well as a Subject Index and an Index of Notation. Stig I. Andersson has done a superb job of translating the Russian text into English.

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A treatise on generating functions, by H. M. Srivastava and H. L. Manocha. Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York, Chichester, Brisbane, and Toronto, 1984, 569 pp., $89.95. ISBN 0-470-20010-3

In the beginning was Laplace, who considered, among other things,

> ... ce que je nomme *fonctions génératrices*: c'est un nouveau genre de calcul que l'on peut nommer *calcul des fonctions génératrices*, et qui m'a paru mériter d'être cultivé par les géomètres. [**2**]

In other words, the idea that the two-variable function F is a *generating function* for the set of one-variable functions $f_0, f_1, f_2, \ldots$ by the way of the *generating function relation*

$$F(x,t) = \sum_{n=0}^{\infty} f_n(x) t^n, \tag{1}$$

was born just over two centures ago. Legendre [**3**] gave, incidentally, one of the first examples: the generating function relation

$$(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x) t^n \tag{2}$$

for the Legendre polynomials $P_0, P_1, P_2, \ldots$, known to most of us.

A relation of the form (1) may well be taken as the *definition* of the set (f_n) of *generated* functions. (Thus the Legendre polonymials may be defined by (2).) But the generating function should not be discarded after having served as a starting-point for the study of the generated functions in question. Indeed, some of their properties may be obtainable from the generating function F itself. For instance, if F is known to depend only on $2xt - t^2$, then the three-term differential-recurrence relation

$$xf_n'(x) - nf_n(x) = f_{n-1}'(x), \qquad n \in \{1, 2, \dots\}, \tag{3}$$

holds. Another, less obvious example is Darboux's theorem on the construction of an asymptotic expression for f_n as $n \to \infty$ with x fixed, from a knowledge of the behavior of F on the boundary of the disc of convergence for the series. (Cf. e.g. [**1**, §19.5].)

Numerous generating functions have been introduced and studied by various methods; and of course the concept itself has been generalized. In the first place, constant factors are allowed for convenience, so that we write $c_n f_n(x)$ instead of $f_n(x)$ in equation (1). A further step consists in studying relations of the form

$$F(x, y, t) = \sum_{n=0}^{\infty} c_n f_n(x) g_n(y) t^n. \tag{4}$$

In this case we say that F is a *bilateral* generating function for the set (f_n) or for the set (g_n). If the two sets are identical, however, we call F a *bilinear* generating function. As an example we mention the well-known *Hille-Hardy formula*

$$\begin{aligned} &(1-t)^{-1}(xyt)^{-\alpha/2} \exp\left[-\frac{(x+y)t}{1-t}\right] I_\alpha \left[\frac{2\sqrt{xyt}}{1-t}\right] \\ &= \sum_{n=0}^{\infty} \frac{n!}{\Gamma(\alpha+n+1)} L_n^{(\alpha)}(x) L_n^{(\alpha)}(y) t^n, \end{aligned} \tag{5}$$

involving the generalized Laguerre polynomials $L_n^{(\alpha)}$ and the modified Bessel function of the first kind I_α. The definition (4) is further extended in a natural way to deal with more variables, multiple summations, and Laurent series.

In addition to classical methods like series manipulations and operational techniques, recent decades have seen some applications of Lie theory to obtain generating function relations. The scheme (when it works) may be briefly explained as follows. In connection with some class of special functions, construct suitable linear differential operators that generate a Lie algebra $\mathcal{G}'$ which is isomorphic to some known Lie algebra $\mathcal{G}$ with matrix group G. Next, construct the multiplier representation T such that for $\mathbf{g}$ near the identity element of G we have an explicitly known operator $T(\mathbf{g})$ on a function space. By a closer study of the differential equations involved, deduce the form of the series expansion of $T(\mathbf{g})f$ for suitable functions f, and from this, finally, construct the generating function relation. For instance, the Lie algebra connected with the generalized Laguerre polynomials is isomorphic to sl_2, and one of the results obtainable via the multiplier representation is the Hille-Hardy formula (5). Lie theory reveals structures and relationships that are not otherwise evident, but it does not furnish *the* easy way of obtaining generating function relations and other results involving special functions.

Whereas the number of research papers dealing with generating functions is very considerable, especially in recent decades, books or chapters devoted primarily to generating functions are rather scarce: Chapter 19 of the Bateman Manuscript Project [**1**], Chapter 8 in Rainville's textbook [**5**], McBride's monograph [**4**] from 1971, and a few other items. There can be no doubt as to the need for more comprehensive, up-to-date expositions, although no one would expect the whole subject to be covered within a single volume. The book under review, however, fills a good deal of the gap. Its scope is adequately described by the authors in the Preface:

> ... this book is written primarily as a reference work for various seemingly diverse groups of research workers and other users of mathematics. It is so designed that it could also be used as a textbook in a beginning graduate course on generating functions.
>
> This book aims at presenting a systematic introduction to and several interesting (and useful) applications of the various methods of obtaining linear, bilinear, bilateral or mixed multilateral generating functions for a fairly wide variety of sequences of special functions (and polynomials) in one, two and more variables.

A few technical features may be mentioned in addition. The exposition of the theory is supplemented by nearly 250 problems, many of which deal with results in recent research papers, and a very extensive bibliography that contains more than 10^3 references. The typography is neat: even the most involved expressions are easy to read, especially because spacing is adequate, and in elegant splittings of long expressions are very rare.

In the reviewer's opinion, the book is a sensibly selected, well-organized, and up-to-date treatment of the subject; it should prove useful to graduate students and indispensable to research workers in the field of special functions.

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