BOOK REVIEWS

BULLETIN (New Series) OF THE
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Group theory. II, by Michio Suzuki. Grundlehren der Mathematische Wissenschaften, vol. 248, Springer-Verlag, New York, Berlin, Heidelberg, and Tokyo, x + 621 pp., 1986, $89.00. ISBN 3-540-10916-1

The last thirty-five years have witnessed revolutionary change in finite group theory in a series of steps each of which completely transformed the subject. The fifties saw a seemingly disconnected diverse collection of new ideas and problems come on the scene, from the first special classifications of simple groups like the Brauer-Suzuki-Wall characterization of the linear fractional groups, the use by Hall and Higman of modular representation theory to derive results on the structure of groups, the systemization of finite groups of Lie type started by Chevalley to the staggering new ideas introduced in Thompson's thesis, among many exciting developments. The sixties led off with the breakthrough solution by Feit and Thompson of the odd order problem which united many hitherto separate ideas into a coherent method of studying simple groups. The decade was one where the field centered on simple groups, something quite different from the diversity of the preceding decade, and there was an explosion of classifications of the smaller simple groups as well as the discovery, after a century, of new sporadic simple groups, the isolated ones which are not related to Lie groups. This led to yet another quite different period, the seventies, the age of the breakthrough in the classification of the general simple groups and the solution of this classical problem. Once again, in the eighties, the subject has changed its emphasis completely with the greatest part of research in finite group theory being in representation theory.

Suzuki did fundamental research through much of these periods and this book, the second of a two-volume treatment, is based on his experience and ideas and leads us into the new era of group theory. The first volume covered basic topics in group theory and was more of a graduate text, but the second is a substantial introduction to finite group theory at the time of the classification. The style of this very well written book is leisurely, with many examples and exercises. It also provides an excellent overview by means of discussions—as well as proofs of special cases—of deep results, which themselves are beyond the range of the book. The three chapters progress from the study of p-groups, the most basic tool, to general local methods, which are the study of the local subgroups, the normalizers of the nonidentity p-subgroups, and the next step beyond p-groups, to simple groups and the machinery needed.

However, as up-to-date as this volume is, the field is moving so quickly that a student will not find enough about the topics of interest in today's research. Representation theory dominates today, with spectacular achievements in the representation theory of the groups of Lie type and whole new areas starting up in general representation theory. The use now of representation theory as a tool for studying structure of simple groups is very minimal, though in the long run one suspects this will not remain the case. The other very active area of finite group theory is the study of more geometrical approaches. These ideas, in particular, the amalgam method introduced by Goldschmidt, have blossomed and have applications to structural questions. Indeed, some of the proofs of the basic "pushing up" theorems in local methods, which Suzuki exposits so well, are fast becoming obsolete due to these much more powerful geometric methods. The quickness of progress in finite group theory will no doubt continue to plague authors of books on the subject.

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Introduction to various aspects of degree theory in Banach spaces, by E. H. Rothe, Mathematical Surveys and Monographs, vol. 23, Amer. Math. Soc., 1986, vi + 242 pp., $60.00. ISBN 0-8218-1522-9

Those who have not seen his name before or know as little about the author as we do will suspect that there must be something special about him, since his manuscript was published in an edition which is one of the finest we have seen in recent years. Golden letters on the cover and paper so innocently white that one hesitates to mark the only thing that has to be, namely the ends of proofs, which are often difficult to find since proofs are long, interrupted by lemmas with proofs, etc. The secret is easily brought to light if one starts reading as usual, i.e., references first. There one finds his first paper **[5]** on the subject, and a look into the original reveals that it was written in 1936. In other words the book appeared just in time to celebrate the golden wedding of author and topological degree.

Digging more into history we see that the fundamental paper **[4]** on degree theory in Banach spaces by J. Leray and J. Schauder was published in 1934, and from the second part of this paper it is obvious that the class of maps they consider was motivated by its usefulness in solving elliptic boundary value problems. In fact it was a revolutionary breakthrough in the treatment of these and other nonlinear problems, studied intensively and solely by the then almighty method of successive approximations. Since any revolution is based on previous evolution, let us note that *Leray-Schauder degree*, as it is called today, had a well-known forerunner, the corresponding concept for continuous maps on $\mathbf{R}^n$, called *Brouwer degree*, since L. E. J. Brouwer's paper of