chapters of the book carry us through that time at Cornell. We hear of his colleagues, of travels, and adventures, and achievements, of his work with the Radiation Laboratory at MIT, and through all of this of his never-ending struggle with mathematics. His publications, only modestly referred to, went on clarifying fundamental concepts of probability theory, extending them to number theory, opening new approaches in theoretical physics.

This is not a book on mathematics, but a life story of a mathematician. There are few places devoted entirely to mathematics, or where even some formulas are displayed. One is in Chapter 3, "The Search for the Meaning of Independence," that in a near-popular manner discusses some work initiated jointly with Steinhaus and then continues to show how justified Henri Poincaré was in saying that the normal probability law is considered "*by mathematicians to be a fact of observations and by observers a theorem of mathematics*." There is just one formula displayed in an amusing discussion of Ehrenfest's "dog-flea" model, and there are some isolated graphs and formulas scattered elsewhere.

After Cornell came a first exhilarating and later on disappointing affiliation with Rockefeller University (1961–1981). The last five years of his life Kac spent at the University of Southern California.

The charm of *Enigmas of chance* cannot be even hinted at by surveying its contents. There is in it the spirit of a warm human being possessed by driving curiosity, by an urge to understand and clarify. There is an account of going through a stormy period in history, with personal tragedies and times of happiness. And there is the picture of a mathematician who, instead of clinging to mathematics as an abstract game, treated it as a bridge to reality; a mathematician who, as quoted by Gian-Carlo Rota, warned that "*axioms will change with the whims of time, but an application is forever*." To Kac the problem often was the reason for the theory; he admitted that "*almost everything new in mathematics I learned after getting my doctoral degree has been by being forced to learn it in trying to solve a problem*."

In the Introduction to his book, Kac expressed the hope to be able to impart to the reader some feeling for the thrill that comes with getting a new idea, as well as for the frustrations and disappointments in the life of a scientist. He did it with charm and grace. He also succeeded in carrying out his other wish: the book gives a moving account of a rich life, and the way it was shaped by family, teachers, collaborators, history, and last but not least, by "*that powerful but capricious lady Chance*."

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Derivations, dissipations and group actions on C^*-algebras, by Ola Bratteli, Lecture Notes in Mathematics, vol. 1229, Springer-Verlag, Berlin, Heidelberg, New York, London, Paris, Tokyo, 1986, vi + 277 pp., $23.60. ISBN 0-387-17199-1

The study of derivations is one of the early disciplines in operator algebra theory with roots back in the beginnings of the subject (see, e.g., **[13]**).

Let $\mathfrak{A}$ be a C^*-algebra. Then a derivation δ is defined to be a linear endomorphism of $\mathfrak{A}$ satisfying the familiar Leibniz rule

$$\delta(ab) = \delta(a)b + a\delta(b), \qquad a, b \in \mathfrak{A}. \tag{1}$$

A milestone in the study of such maps is the Derivation Theorem (1966) which states that every derivation in a von Neumann algebra $\mathfrak{A}$ is inner. (We refer the reader to [7, pp. 310–312] for details and for references to the original papers.) We say that a derivation is *inner* if it is implemented by some element $h \in \mathfrak{A}$, that is,

$$\delta(a) = [h, a] = ha - ah, \qquad a \in \mathfrak{A}. \tag{2}$$

It is known [**19a**] that if a derivation δ is defined everywhere on a C^*-algebra $\mathfrak{A}$, then δ is automatically bounded, i.e., the supremum

$$\sup\{\|\delta(a)\|: a \in \mathfrak{A}, \|a\| = 1\} \tag{3}$$

is finite. The number in (3) is called the norm of δ. During the seventies, the subject continued with the study of bounded derivations in C^*-algebras, and culminated in 1978 with contributions by Elliott [**8**] and Akemann-Pedersen [**1**], and, at the same time (in the early to mid-seventies), the study of unbounded derivations was starting.

An unbounded derivation is defined only on a subalgebra (the domain of δ), $D(\delta) \subset \mathfrak{A}$, and it maps into $\mathfrak{A}$. We shall assume that $D(\delta)$ is dense in $\mathfrak{A}$, and that (1) holds for $a, b \in D(\delta)$. (The case when $D(\delta)$ is not assumed dense is interesting and important but not treated in the book.) In view of applications, we shall assume in addition that $D(\delta)$ is a $*$-subalgebra of the C^*-algebra $\mathfrak{A}$, and that

$$\delta(a^*) = \delta(a)^*, \qquad a \in D(\delta). \tag{4}$$

We will restrict attention to this case and refer to δ as an *unbounded derivation*. (A few of the results described below will hold alsô for derivations which are not $*$-derivations, i.e., which do not verify condition (4), but we shall skip this technical point.)

The interest in unbounded derivations originated in different parts of the world but roughly at the same time. The early contributions were motivated however by distinct considerations and applications. Sinai [**21**] in the U.S.S.R. was motivated by applications to ergodic theory, while Bratteli and Robinson [**5a, b**] in Marseille were motivated by a broader class of problems in mathematical physics, most notably in statistical mechanics. Powers and Sakai in Philadelphia were motivated by similar considerations, originating in work by Hugenholtz et al. [**9**] and Segal [**20**], among others. Bratteli begins his present book with a list of names of the researchers who were influenced by these early developments and who subsequently took up the study. (The reviewer is one of them, and he worked in Philadelphia at the time!)

On the mathematical side, the list of sources of problems includes cohomology [**12a**], differentiable structures [**15**], and Hilbert's fifth problem [**23**]. From the point of view of cohomology, the Derivation Theorem amounts precisely to the statement $H^1(\mathfrak{A}) = 0$ when $\mathfrak{A}$ is a von Neumann algebra. During the eighties, Connes developed cyclic cohomology **[6b]** and noncommutative differential geometry **[6a]**, and the derivations with dense domain continue to play a prominent role.

As it turned out, the important algebra is not the initial C^*-algebra $\mathfrak{A}$ itself, but rather a preordained dense *-subalgebra $\mathfrak{A}_0$ of smooth elements in $\mathfrak{A}$, and the derivations are defined only on $\mathfrak{A}_0$ and not on $\mathfrak{A}$. Consider a Lie group G which acts continuously on $\mathfrak{A}$ as a group of automorphisms

$$\tau: G \to \mathrm{Aut}(\mathfrak{A}). \tag{5}$$

If the action is given in terms of $\mathfrak{A}$ in a "natural" way, then we may take for the subalgebra $\mathfrak{A}_0$ the algebra of C^∞-elements (or smooth elements) for the action, i.e., the elements a in $\mathfrak{A}$ such that the orbit function, $g \to \tau_g(a)$, is C^∞ from G to $\mathfrak{A}$. If $\mathfrak{g}$ denotes the Lie algebra of G, then the infinitesimal form of (5) is a Lie algebra of derivations. For $X \in \mathfrak{g}$ the derivation $d\tau(X)$ is defined by

$$d\tau(X)(a) = \frac{d}{dt}\bigg|_{t=0} \tau(\exp tX)(a), \qquad a \in \mathfrak{A}_0, \tag{6}$$

and

$$d\tau([X_1, X_2])(a) = [d\tau(X_1), d\tau(X_2)]a$$

holds.

Bratteli's book begins with a discussion of bounded derivations, and turns quickly to the active and rich field of unbounded derivations. There are two parts in the book: *general theory*, and *noncommutative vector fields*. Part 1 is structured as follows.

1. Analytic properties
   1.1. Closability
   1.2. Generators
2. Classification
   2.1. Nonabelian C^*-algebras with special emphasis on the simple ones
   2.2. Abelian C^*-algebras (i.e., differentiable structures)

The organization of Part 2 is modelled on that of Part 1 but the specific results are much more detailed in the more specialized case of a noncommutative vector field, by which is meant a derivation mapping one class of smooth elements to another.

A derivation δ is said to be *closed* if the graph $G(\delta)$ is closed in $\mathfrak{A} \times \mathfrak{A}$, and δ is said to be *closable* if the closure of $G(\delta)$ is the graph of some linear operator, or equivalently if $G(\delta)$ is contained in the graph of some closed derivation δ_c say. We then say that δ_c is an *extension* of δ.

A derivation δ is said to be a *generator* if there is a strongly continuous one-parameter group of *-automorphisms, $\{\alpha_t : t \in \mathbf{R}\} \subset \mathrm{Aut}(\mathfrak{A})$, such that

$$\delta(a) = \frac{d}{dt}\bigg|_{t=0} \alpha_t(a), \qquad a \in D(\delta), \tag{7}$$

i.e., the elements a in $D(\delta)$ are precisely those for which the limit $\lim_{t\to 0} t^{-1}(\alpha_t(a) - a)$ exists. Generators are closed, and we want to study closed derivations to be able to isolate differentiable structures. An important problem for infinite dynamical systems is to decide when a given derivation δ is a generator, or when it has an extension which is a generator. We refer to [**5b**, vol. II] for some of the models from physics.

Let K be a totally disconnected compact space, and let $C(K)$ be the abelian C^*-algebra of all continuous functions on K. Then Sakai [**19b**] observed that $C(K)$ does not have any nonzero closed derivations with dense domain. We therefore note that K does not carry any differentiable structure. (It is known that $C(K)$ has derivations which are not closed.)

In Part 1 of the book, differentiable structures are studied in terms of closed derivations. The existence of *functional calculi* (i.e., various forms of a generalized chain rule) for closed derivations plays a central role in results from the general theory. This functional calculus is developed efficiently in Part 1 for various algebras of regular functions. The treatment is especially useful since the research papers in this area are scattered over many journals, and some of the details are somewhat technical (with some surprises), although the general ideas are clear. The functional calculus is used in, e.g., K-theory [**2**], and often without explicit references to research papers on the subject.

The one-parameter groups, $\alpha\colon \mathbf{R} \to \mathrm{Aut}(\mathfrak{A})$, are applied to the study of dynamical systems in quantum mechanics, both in quantum field theory [**20**], and in statistical mechanics [**5b**, vol. II]. We also refer to [**10b**] and [**11b**] for additional details and references.

Physicists wish to construct a self-adjoint (generally unbounded) Hamiltonian operator H in some Hilbert space $\mathscr{H}$, and consider the unitary group $\{e^{itH}\}_{t\in\mathbf{R}}$ on $\mathscr{H}$ in order to solve the corresponding Schrödinger equation. Let $\mathfrak{A}$ be a C^*-algebra of operators on $\mathscr{H}$ satisfying

$$e^{itH}\mathfrak{A}e^{-itH} = \mathfrak{A}, \qquad t \in \mathbf{R}. \tag{8}$$

Then there is a one-parameter group $\{\alpha_t\} \subset \mathrm{Aut}(\mathfrak{A})$, given by

$$\alpha_t(a) = e^{itH}ae^{-itH}, \qquad a \in \mathfrak{A},\ t \in \mathbf{R}, \tag{9}$$

and describing the familiar correspondence between the Schrödinger picture and the Heisenberg picture of dynamics. An important and basic problem is to determine the automorphism group

$$\alpha\colon \mathbf{R} \to \mathrm{Aut}(\mathfrak{A}). \tag{10}$$

This is nontrivial because H generally does not exist as an operator, but only as a formal expression. From the outset, it is not clear even what Hilbert space to choose. The symbol $H = H_0 + V$ in quantum field theory is, a priori, only a formal expression, as is the infinite sum

$$H = \sum J_{ij}\sigma_i\sigma_j \tag{11}$$

(summation over a given graph G in $L \times L$ for some configuration or spin lattice L) in quantum lattice models. (We refer to [**5b**] and [**16**] for details.) The mathematical difficulty, which can be overcome by introducing a suitable derivation δ, is that H is not given, or defined, as an operator, let alone a self-adjoint one. But it is possible in important examples from physics to introduce δ and thereby give a precise meaning to H (the Hamiltonian) as a self-adjoint operator in some Hilbert space. Suppose for example that H (formal expression) is given by (11). Then define a derivation δ by

$$\delta(a) = i[H, a] \tag{12}$$

for elements a in the dense subalgebra $D(\delta)$ of finite tensors in the Pauli spin variables $\{\sigma_i\}_{i \in L}$, $\sigma_i = (\sigma_i^x, \sigma_i^y, \sigma_i^z)$. (The C^*-algebra is constructed to be simple, and it is the completion of the finite tensors, so $D(\delta)$ is automatically dense.) We wish to stress that the commutator on the right-hand side of (12) involves only a finite number of terms in the expression (11), although the number of terms does depend on the particular element a in $D(\delta)$. It follows that δ is perfectly well defined as a derivation. The problem is now to show, in concrete examples, that δ generates a one-parameter group $\{\alpha_t\} \subset \mathrm{Aut}(\mathfrak{A})$, or has a generator extension. For the quantum lattice models, the problem is already solved, but not so for other physical models. (The derivation given by (11) and (12) is approximately inner, and if $\sup_i \Sigma_j |J_{ij}| < \infty$, where the summation, for fixed i, is over the set $\{ j \in L : (i, j) \in G\}$, then the closure of δ is a generator [**5b**, vol. II].) In the final analysis, a genuine hermitian operator H satisfying (8), (9), and (12) enters the picture only "through the back door," when the GNS-construction is applied to some invariant state, e.g., ground state, or KMS temperature state.

A number of papers in the seventies were concerned with the Powers-Sakai conjecture [**17**] which arose in connection with the existence problem for ground states and KMS-states. During the eighties, an important role was played by a second problem, raised by Sakai at the 1980 Kingston Symposium [**11b**, vol. II, p. 326]. Sakai asked for simple C^*-algebras with nontrivial differentiable structures. While the Powers-Sakai conjecture was never solved completely, but only in special cases, Sakai's problem was solved fully: first in a special case by Takai [**22**], and then in complete generality (and for all dimensions) by Bratteli, Elliott, and the reviewer [**3**].

One might think of Sakai's problem much more broadly stated as: "understand differentiable structures on (simple) C^*-algebras"! This "generalized" problem is certainly not solved, but a variety of aspects of the broader program are currently being investigated by many researchers.

We need two definitions. A derivation δ with dense domain $D(\delta)$ in a C^*-algebra $\mathfrak{A}$ (resp., an automorphism group $\{\alpha_t\} \subset \mathrm{Aut}(\mathfrak{A})$) is said to be *approximately inner* if there is a sequence of elements $h_n = -h_n^* \in \mathfrak{A}$ such that

$$\delta(a) = \lim_{n \to \infty} [h_n, a], \qquad a \in D(\delta), \tag{13}$$

respectively,

$$\alpha_t(a) = \lim_{n \to \infty} e^{th_n} a e^{-th_n}, \tag{14}$$

where the limit is relative to the norm on $\mathfrak{A}$. Compare formula (13) with (2), and similarly (14) with (9). The conjecture states that every continuous automorphism group $\{\alpha_t\} \subset \mathrm{Aut}(\mathfrak{A})$ is approximately inner if $\mathfrak{A}$ is UHF, i.e., a matricial C^*-algebra in the terminology of [**12b**]. The approximately inner one-parameter groups $\{\alpha_t\}$ are especially important in quantum statistical mechanics since these groups have ground states, and (whenever $\mathfrak{A}$ has a trace) they have KMS-states for all values of inverse temperature $\beta = (kT)^{-1}$, as proved by Powers and Sakai [**17**].

It follows from general theory [**14**] that (13) implies (14). But, so far, (13) has only been established for elements a in various subalgebras of $D(\delta)$. While these subalgebras are dense in $\mathfrak{A}$, they are not known to be dense in $D(\delta)$ relative to the graph norm. The remaining problem is called the *core problem*, and, so far, no one has been able to solve it or to supply a counterexample.

To understand how a differentiable structure can be realized on a C^*-algebra it is necessary to factor out by the approximately inner derivations. Sakai asked [**11b**] for a class of *simple* C^*-algebras which admit a nontrivial differentiable structure. The paper [**3**] supplied the differential analysis of such a class, and the C^*-algebras are now called "pseudo tori", or "noncommutative tori" since they are generalizations of $\mathbf{T}^n$ for $n = 1, 2, \ldots$. Although they are simple C^*-algebras, they share differentiable structure with $\mathbf{T}^n$. In the special case $n = 2$, they were introduced in [**6a**] and [**18**] under the name "irrational rotation algebras." The more general "pseudo tori" may be defined on arbitrary compact abelian groups G. If $\hat{G}$ is the corresponding dual discrete group of characters, and ρ is a nondegenerate antisymmetric bicharacter on $\hat{G}$, then the simple C^*-algebra $\mathfrak{A}(\rho)$ is defined as a functor from the category of bicharacters (properties as above) to the category of simple C^*-algebras. It is known that $\mathfrak{A}(\rho)$ carries a canonical ergodic action of G, τ: $G \to \mathrm{Aut}(\mathfrak{A}(\rho))$. Let $\mathfrak{A}_\infty(\rho)$ be the corresponding ring of C^∞-elements for the action. Let $\mathscr{L}$ be the ∞-dimensional Lie algebra of all derivations in $\mathfrak{A}_\infty(\rho)$. Then a result in [**3**] states that

$$\mathscr{L} = \mathscr{L}_0 +) \, \mathscr{L}_1 \quad \text{(semidirect product)}, \tag{15}$$

where $\mathscr{L}_0$ is just the infinitesimal Lie algebra of the action τ, obtained by differentiation as in (6), and $\mathscr{L}_1$ consists of *all* approximately inner derivations. In particular, $\mathscr{L}_1$ is an *ideal* in $\mathscr{L}$, and nonzero elements in $\mathscr{L}_0$ are not approximately inner. If G has dimension n, $1 \leqslant n \leqslant \infty$, it follows that the abelian Lie algebra $\mathscr{L}_0$ is of dimension n. We say that it is a differentiable structure. (An earlier result of Sakai can be reformulated (modulo the unsolved core problem) as saying that the UHF algebras have no differentiable structure, i.e., they are the noncommutative (quantized) versions of Cantor sets.)

If it is further assumed that ρ satisfies a certain generic diophantine property, then, as we proved in [**3**], $\mathscr{L}_1$ consists of inner derivations. In particular, approximately inner derivations, cf. (13), mapping $\mathfrak{A}_\infty(\rho)$ into itself, are automatically inner, and therefore bounded. For the irrational rotation algebras $\mathfrak{A}_\theta$ (irrational rotation angle θ), the generic condition is known to hold if θ is irrational algebraic.

It was shown later in [**10b**, Connes-Rieffel] that Yang-Mills functionals are defined for $\mathfrak{A}_\theta$ in terms of the differentiable structure. Connes [**6a**] had already defined the differential geometric concepts for $\mathfrak{A}_\theta^\infty$: connections, curvature, and Chern character; and he had proved an Atiyah-Singer index theorem in this setting. It was shown in [**10b**, Connes-Rieffel] that constant curvature connections on $\mathfrak{A}_\theta^\infty$ are extremal functionals for the Yang-Mills problem.

The constant curvature connections turned up recently in a further analysis of noncommutative vector field theory. The authors of **[4]** classified, using results from **[10a]**, all the smooth Lie group actions on the simple C^*-algebras $\mathfrak{A}(\rho)$ when ρ is assumed to satisfy the generic condition. The Lie groups which act as groups of diffeomorphisms were classified, and the actions determined. Let τ: $G \to \mathrm{Aut}(\mathfrak{A}_\infty(\rho))$ be such a Lie action, and let $\mathscr{M}_\tau$ be the Lie algebra of derivations obtained from τ by differentiation, cf. (6). We proved that every finite-dimensional real Lie subalgebra $\mathscr{M} \subset \mathscr{L}$ arises this way from a Lie action of some G, and we thereby linearized the classification problem.

Let $\mathscr{M} = \mathscr{S} +)\mathscr{R}$ (semidirect product) be the Levi decomposition of some given $\mathscr{M}$ with $\mathscr{L}$ semisimple, and $\mathscr{R}$ the solvable radical. Then we showed **[4]** that $\mathscr{S}$ must be compact (i.e., have negative definite Killing form), and consist of inner derivations. All compact semisimple cases may be realized. Moreover $\mathscr{R}$ must necessarily be step 2 and of a very special form. If $n = 2$, then $\mathscr{R}$ may contain a copy of the 3-dimensional Heisenberg Lie algebra, and we show that the classification of the possible smooth actions of the Heisenberg group is closely connected with the analysis of constant curvature connections: The classification of the Heisenberg actions is given in terms of the moduli space for the Yang-Mills problem.

The book under review gives an excellent background on the theory of noncommutative vector fields, along with recent applications, including some of those mentioned above. While this review has focused on derivations, the book also treats dissipations, alias semiderivations. The derivations (or rather *-derivations) describe the dynamics in conservative systems, and the semiderivations are used to make explicit the dynamics for (infinite) dissipative systems.

Bratteli's book is very clear and well written, and it should prove useful for graduate students and researchers in functional analysis and related fields. So far, there has been no systematic treatment of recent results in this very active area of operator algebras. We finally note that a book by Sakai is in preparation. It covers a rather different (but equally interesting) side of the subject.

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