curriculum of many colleges and universities, the next generation of texts may have a substantially different emphasis. Devaney's book is an excellent choice for professional mathematicians to read as an introduction to the subject. There are ample exercises.

JOHN GUCKENHEIMER

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Markov processes: *Characterization and convergence*, by Stewart N. Ethier and Thomas G. Kurtz, John Wiley & Sons, New York, Chichester, Brisbane, Toronto, and Singapore, 1986, x + 534 pp., $47.50. ISBN 0-471-08186-8

The theoretical side of mathematical probability has long been preoccupied with limit theorems; not surprisingly, since one of the natural interpretations of *probability* is as *long-run frequency*. Traditionally, limit theorems have been divided into two categories: strong limits, where some asymptotic event is asserted to occur with probability one, and weak limits, where the distributions of a sequence of random quantities are asserted to converge to a limit distribution. The prototype weak limit result is the *central limit theorem* (CLT), which says that under mild conditions the sums $S_n = X_1 + \cdots + X_n$ of independent random variables can asymptotically be approximated by Gaussian distributions. This is the key result in elementary mathematical statistics. The average height of a random sample of people is a random quantity whose exact distribution is very complicated, depending on the entire list of heights of the population; but the CLT says that the distribution of the average height of a large sample is approximately a Gaussian distribution with two parameters which depend only on the mean and standard deviation of the population heights. This illustrates the practical purpose of weak convergence theorems, to approximate complicated exact finite distributions by simpler limiting distributions. A more sophisticated example concerns neutral genetic models. Much observed genetic variation within a species (e.g., eye color in humans) confers no apparent selective advantage (i.e., is apparently "neutral"). Is it plausible that such variation is really neutral? To study this question one needs to set up a mathematical model and compare its predictions with observations. In detail, any model will be rather arbitrary and unrealistic, but one can hope that the long-term behavior of a model is insensitive to its details and instead approximates some mathematically natural process with only a few parameters.

Returning to the CLT, a pure mathematician would regard its proof as an easy exercise in Fourier analysis. Modern probabilists look at it differently. For each n, consider not only the single random variable S_n but instead the whole process $(S_m; 0 \leqslant m \leqslant n)$, which can be regarded as a random element of function space. Under the same conditions as the CLT these processes, suitably normalized, converge to the Brownian motion process $(B_t; 0 \leqslant t \leqslant 1)$. Not only is this "functional CLT" mathematically stronger than the basic CLT, but also it leads to quite different methods of proof. Brownian motion can be characterized as essentially the only process with continuous paths and stationary independent increments. It is easy to show that, *if* the normalized processes (S_m; $0 \leqslant m \leqslant m$) converge to a limit process, *then* the limit process has the properties above, and hence must be Brownian motion. Thus the proof of the functional CLT can be reduced to the technical issue of showing that the normalized processes (S_m) are "tight", that is, relatively compact as probability measures on function space. Thus there is a general strategy for proving weak convergence of processes Z^n to a limit process Z:

(a) find a characterization of the limit process Z;

(b) show the approximating process Z^n to have the characteristic property "asymptotically";

(c) show (Z^n) is tight.

At first sight this is a circuitous procedure, but its advantage is that it avoids explicit consideration of distributions of the approximating processes Z^n; whereas Fourier analysis enables these to be studied directly for processes arising as sums of independent variables, such direct analysis is not possible in more complex settings.

Characterization problems occur in many parts of pure mathematics, and so studying characterizations of random processes is theoretically interesting in its own right. The strategy described above links weak convergence theory, which has direct "applied" motivation, with theoretical characterization problems. The book under review studies the interplay between convergence and characterization in the context of Markov processes. Here there are three ways to characterize limit processes: via generators, via martingales, or as solutions to stochastic differential equations. Assuming a background of a standard first-year graduate course in probability theory, the book starts with three concise but clear 50-page accounts of operator semigroups, martingales and related parts of the general theory of processes, and abstract properties of weak convergence in function space. The heart of the book is in Chapter 4, a thorough account of the martingale method for characterizing the Markov process associated with a given generator. From such characterizations it is straightforward to obtain conditions for weak convergence in terms of martingales derived from the approximating processes. This is the first easily accessible but general account of this important topic. After further concise accounts of stochastic differential equations (confusingly, termed *integral equations*) and examples of generators, the final third of the book treats examples of weak convergence. The examples concentrate on what might loosely be called population models: that is, branching processes, genetics models, and their relatives. Within this field both classic topics (e.g., diffusion approximation of the Wright-Fisher model) and recent topics (branching Markov processes, random environments, measure-valued diffusion limits of infinite-alleles models) are treated. A final brief chapter treats random processes driven by converging underlying processes.

There are several ways to view this book. As a reference book, for theoretical probabilists in other fields wanting results about characterizations and their use in establishing weak convergence results of the types mentioned, it is a clear success and deserves to be widely used.

The thought of using it as a textbook raises some interesting speculations. While the content of a first-year graduate course in probability is fairly standard and several good books are available, there is no such consensus on what is the core material in more advanced theoretical stochastic processes; while there are many books on particular topics, there is no satisfactory overall treatment. The book under review, while also focusing on specialized topics, does have fine accounts summarizing different fields; it also has an excellent set of exercises. Its main defect as a textbook is lack of motivation, and lack of explanation of how the background topics relate to other problems in probability theory. Williams [6], whose volume 2 is expected soon, gives the most wide-ranging account of modern stochastic process theory, a brilliant exposition of "the big picture" and the intuitive ideas, but without the rigorous theorem-proof development. In content these books overlap; in style they are complementary; between them, they cover a vast range of modern material; and so they are the reviewer's nomination for the "core material" of modern process theory.

Billingsley [1] popularized weak convergence in probability theory, and that comparatively slim book, containing much of the material known at that time (1968), is still a standard reference. The subject has grown so much that no one would attempt a comprehensive account of the current knowledge of weak convergence. There are several books on subareas: Pollard [5] emphasizes empirical distributions, Kallenberg [3] random measures, Borovkov [2] queueing theory, and Kushner [4] engineering applications, for example. There has been a noticeable gap at Markov processes, which this book does a good job of filling, at least for theoreticians.

Books, like automobiles, cannot be designed to please all customers, and I feel this book will not be so satisfactory for applied workers. Many applications are rather routine; one has a sequence of processes, one sees how to rescale so that conditional means and covariances converge, making it heuristically obvious that the processes converge to the appropriate diffusion. From the applied viewpoint, most of weak convergence is "formalizing the obvious", and the duty of theoreticians is to provide plug-in theorems to justify the limit in the case at hand! This book has all the theoretical tools, but only assembles them for ready use in a few settings (mostly population models, as mentioned before); using the results in different settings would require theoretical facility which an applied worker may well be unwilling to acquire. Thus the reviewer, working on applied problems involving "obvious" but nonstandard limit processes (function-valued diffusions and sequence-valued Ornstein-Uhlenbeck processes) was disappointed to find nothing in the book to improve on bare-hands implementation of the fundamental ideas. A couple of extra chapters of qualitatively different examples would have been welcome.

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DAVID J. ALDOUS

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Scattering by obstacles, by A. G. Ramm, D. Reidel Publishing Company, Dordrecht, Boston, Lancaster, Tokyo, 1986, xiv + 423 pp., $89.00. ISBN 90-277-2103-3.

For well over a hundred years, scattering theory has played a central role in mathematical physics. From Rayleigh's explanation of why the sky is blue, to Rutherford's discovery of the atomic nucleus, through the modern medical applications of computerized tomography, scattering phenomena have attracted, perplexed and challenged some of the outstanding scientists and mathematicians of the twentieth century. Broadly speaking, scattering theory is concerned with the effect an inhomogeneous medium has on an incident particle or wave. In particular, if the total field is viewed as the sum of an incident field u^i and a scattered field u^s then the direct scattering problem is to determine u^s from a knowledge of u^i and the differential equation governing the wave motion. Of equal (or even more) interest is the inverse scattering problem of determining the nature of the inhomogeneity from a knowledge of the asymptotic behavior of u^s, i.e., to reconstruct the differential equation and/or its domain of definition from the behavior of (many) of its solutions. The above oversimplified description obviously covers a huge range of physical concepts and mathematical ideas, and for a sample of the many different approaches that have been taken in this area the reader can consult the monographs of Bleistein [**1**], Colton and Kress [**3**], Jones [**5**], Lax and Phillips [**8**], Newton [**9**], Reed and Simon [**10**], and Wilcox [**12**].

The simplest problems in scattering theory to treat mathematically are those of time harmonic acoustic waves which are scattered by either a penetrable inhomogeneous medium of compact support or by a bounded impenetrable obstacle. In addition to their appearance in realistic physical situations (e.g., acoustic tomography and nondestructive testing) such problems also serve as models for more complicated wave propagation problems involving electromagnetic waves, elastic waves, or particle scattering. To mathematically model these two problems, assume the incident field is given by the time harmonic plane wave

$$u^i(\mathbf{x}, t) = \exp[ik\mathbf{x} \cdot \alpha - i\omega t]$$