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Representations of compact Lie groups, by Theodor Bröcker and Tammo tom Dieck, Graduate Texts in Mathematics, Vol. 98, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1985, x + 313 pp., \$39.00. ISBN 0-387-13678-9

Compact groups are marvelous. In many branches of mathematics and physics they arise quite by nature, frequently, but not always in the form of Lie groups. They have a long history—long anyhow by the standards of functional analysis. Yet they still provide stimulation for current research. The field of compact groups offers an excellent testing ground for analysts, topologists, and algebraists alike as soon as they care to verify their results in the case of compact groups.

There are good theoretical reasons for this. For example, the entire topological structure of a connected locally compact group is carried (up to a topological copy of \mathbf{R}^n as direct factor) by a compact subgroup which is unique up to conjugation. In the representation theory of locally compact groups, compact groups are not only a model after which the more delicate results of the general theory are fashioned, but they are frequently a determining element in such processes as the construction of induced representations. The best part of the duality theory of locally compact abelian groups concerns the bijective correspondence of compact abelian and discrete abelian groups, and this situation remains quite significant even in the simplest situations involving compact abelian Lie groups on the one hand and finitely generated abelian groups on the other.

The theory of compact groups pivots around the subclass of compact Lie groups. (Dieudonné in 1973: “*Les groupes de Lie sont devenus le centre des mathématiques. On ne peut rien faire de sérieux sans eux.*” [7]) In this area we find the richest treasures of the whole theory. Compact groups yielded the first partial solution to Hilbert’s Fifth Problem which stipulated that any locally euclidean topological group should be a Lie group [17]. Soon it emerged that every compact group can be approximated in a controlled fashion by compact Lie groups, a fact which was distilled into its modern form by the mid-thirties [19]. It is true that the basic representation theory of compact groups rests on real analysis and functional analysis; but the representation theory of compact Lie groups is at the root of the representation theory of arbitrary compact groups in its more subtle aspects. Compact Lie groups and their homogeneous spaces are prime examples of compact smooth manifolds. It is here that the fundamental concepts of the calculus of several variables and of differential geometry may be tested and brought to good use. Many of the elementary aspects of differential geometry are based on properties of the orthogonal groups. The vector product in \mathbf{R}^3 which pops up in so many elementary mathematics and physics courses remains mysterious until it is recognized as

the Lie bracket of $\mathfrak{so}(3)$. Many of the most significant groups appearing in physics are compact Lie groups, and even the finer aspects of their representation theory are of intense interest to physicists. Mathematicians and physicists alike may justifiably indulge in classifying, cataloguing, and listing compact Lie groups and, in this process, learn that a firm grasp of the mathematical concepts is not only of great help: *It is indispensable*. New and unexpected applications of compact Lie groups appear persistently through their history; for some recent developments in which they provide a surprising link between differential geometry and number theory see [8].

The text by Bröcker and tom Dieck on representations of compact Lie groups is an inspired and mature book on an inspired and mature subject. We have come a long way in the textbook situation on Lie groups since the mid-sixties. In those days, the only modern exposition available was Chevalley's book of 1946; Cohn's and Pontryagin's approach was very classical. The Séminaire Sophus Lie was the most up-to-date source available, an influential forerunner of Bourbaki's book on Lie theory, and one whose impact is still felt in the book before us; but it was certainly not easy fare for most consumers at the time. Meanwhile one had to fall back on various sets of lecture notes which circulated more or less sporadically. I recall notes by Freudenthal (later to flow into his book with de Vries [9]), the Bonn Lecture Notes of Tits of 1963-64 (compiled by Krämer and Scheerer, recently available as a book [18]) or the Tübingen Notes of 1963-64 (compiled in part by Falko Lorenz, [11]). The situation began to change dramatically with the appearance of Hochschild's book in 1965 [10], remarkably, still an up-to-date book over twenty years later, which is rightfully consulted to this day and which also influenced the book under review. From there on the sailing was easier. We could almost speak of a surfeit of attempts to present Lie theory in one form or another.

On the side of Lie algebras, the situation had been better all along. Most of the theory here is based on nothing but a sound knowledge of linear algebra. Thus a good selection of monographs, texts, and lecture notes was available with Jacobson's book of 1962 [16] as something like a model. With Lie groups, however, the situation is different. I believe that a good foundation of Lie group theory is a difficult matter. On a sophisticated level, this is borne out by the fact that it took Bourbaki himself such a long time to get to this point [6]. He is still not finished with this project, but compact Lie groups are so far the last subject covered in this series. On the level of teaching, it is not at all clear to me that anyone has yet found the best way of approaching the subject. Probably there is no ultimate pedagogical solution; there may very well be different locally optimal approach routes depending on the students' background as much as on the teachers' bias. In the past, texts specifically devoted to compact groups as such have been conspicuously absent (save certain sets of lecture notes with some distribution such as [12]). Material on compact groups had to be tracked down in sources either on Lie theory or on harmonic analysis.

Bröcker and tom Dieck introduce global Lie group theory in the first fifty pages of the book. There is no nonsense. They start off with manifolds, tangent bundles, vector fields, flows, differential forms, principal bundles, the lot.

Invariant integration on Lie groups is secured by transporting some fixed volume element in the tangent space at the identity via left translation. Examples accompany the development from the first page. Commendably, the authors alert the physics-oriented readers at an early stage that certain standard notation in the physics literature deviates from that in the mathematical one, which is why we hear so much about hermitian operators in quantum mechanics while, for the mathematician, the infinitesimal generators of a unitary group are skew hermitian. A complete treatment of Clifford algebras and the construction of the spinor groups conclude **Chapter I**.

Armed with an adequate invariant integration for Lie groups, the authors can take up “elementary representation theory” in **Chapter II**. After clearing out the required linear algebra, they define representative functions and characters and develop the complete orthogonality formalism and its first consequences. They hasten to give an immediate survey of the representations of the low-dimensional compact Lie groups $SU(2)$, $SO(3)$, $U(2)$, and $O(3)$ in their concrete manifestations, complete with the link to special functions such as the spherical harmonics. A particularly neat source of reference is §6, in which the theory of real and quaternionic representations is linked to complex representation theory: The former are manifestations of the latter, augmented by additional elements of structure. One may call these matters standard techniques, but they are easily passed over, because so often the focus is on the case of the algebraically closed ground field \mathbb{C} , while the real case which is so prevalent in the applications is either slurred over or treated ad hoc. Even Bourbaki tucks a good portion of this material away in an appendix [6, LIE IX. 103]. I do not know of a better place to send a student to read up on this question than the source before us: It is direct, complete, and detailed, and could easily be isolated as seminar material.

The chapter continues with the introduction of the character ring, complete with its λ -ring structure and the Adams operations. Compact abelian Lie groups and their representations are next in line. Of course, they have to be thoroughly understood because the whole of the fine structure of the representation theory of compact Lie groups is based on the representation of abelian groups, as the reader will learn in the second half of the book. Weights are introduced here. In the pursuit of associating with each of them an infinitesimal weight, in passing, we learn that each representation of a compact Lie group induces one of its Lie algebra. The Lie algebra aspect is expressly de-emphasized in this book, but at any rate, from p. 23 on we know that the assignment of the Lie algebra is functorial. The chapter concludes with the structure and representation theory of $\mathfrak{sl}(2, \mathbb{C})$. If anyone should ask what this had to do with compact Lie groups, then he or she should just remember that this algebra is the complexification of $\mathfrak{su}(2) \cong \mathfrak{so}(3)$, whence the relevance of this algebra for the complex representation theory of $SU(2)$ and $SO(3)$. Moreover, the representation theory of $\mathfrak{sl}(2)$ plays such a crucial role in semisimple Lie algebras at large that readers will be grateful for having access to this material within this book. Again the authors point out the classical nature of the representation theory of the three-dimensional simple algebras through its relation to special functions such as the Legendre polynomials.

Chapter III is entitled “Representative Functions.” Now, the authors are ready to teach us some **Real** analysis and prepare us for the celebrated Theorem of Peter and Weyl. (I find it necessary, when talking about the Peter-Weyl Theorem, to point out to students that Weyl’s first name is Hermann, and I have only evasive answers to their question “Whatever happened to Peter?”) The authors have an excellent selection of immediate consequences, all of them instructive and useful (§4). The first consequence, of course, is that every compact Lie group is a closed subgroup of some unitary group. NOW they are telling us! After a brief discourse on “the Large Peter-Weyl Theorem,” the canonical decomposition of Hilbert modules into isotypic components is given. In general, Hilbert spaces belong to the de-emphasized material in this book. We have a compact course on induced representations and a half-introduction to the duality of compact Lie groups in the form given to it by Hochschild. The missing half is that by which we start from a Hopf algebra and pass to its spectrum; if additional elements of structure are taken into account which model the Haar integral, this spectrum is a compact group whose dual Hopf algebra is isomorphic to the one we started with (see [10]). This discussion sets the stage for a nice introduction to the complexification of compact Lie groups. The necessary reminders of algebraic geometry are provided. At this point we reach the half-way mark of the book and roll up our shirt sleeves for the real stuff.

The real stuff, of course, has to begin with the compact group theoreticians’ Sylow theorem, to wit: Every compact connected Lie group is the union of its maximal torus subgroups, and all of these are conjugate. Everything else that follows evolves from here. The fact that every compact Lie group G contains at least one maximal torus is not hard to grasp. But then, one way or another, we have to come to grips with the function $q: G/T \times T \rightarrow G$ given by $q(gT, t) = gtg^{-1}$. First we want to show that it is surjective and thereby prove the Maximal Torus Theorem, and secondly we want to utilize it in order to establish Weyl’s Integral Formula, which says that Haar measure on G is the forward image under q of the measure $\mu \otimes w(G)^{-1} \delta_G \tau$, with $w(G)$ being the order of the Weyl group, μ the invariant normalized measure on G/T , τ Haar measure on T , and finally $\delta_G(t) = \det(\text{Ad}_{L(G)/L(T)}(t)(t) - 1)$. At this point in the book we finally realize why, in the introductory chapter, we had to know Stokes’ and Sard’s Theorems in order to have full command of the Fundamental Theorem on the mapping degree of smooth maps between compact connected orientable manifolds of the same dimension. This theorem will indeed secure the surjectivity of q , and more. Once the Maximal Torus Theorem is available, a rich harvest can be reaped right away, and so **Chapter IV**, even though being one of the shorter ones with its 25 pages, is packed with powerful information on the structure, the representation ring, the Weyl group, the regular elements, and the Cartan subgroups of a compact Lie group. In the midst of this the authors still tell us how this material looks when it is applied to the classical groups.

At this point, we are ready to go into root geometry and Weyl chambers (**Chapter V**). As soon as we have identified the groups of rank one (the rank is the dimension of a maximal torus!), we enter the basic geometry of root systems. This branch of geometry is elementary, but highly nontrivial. Basi-

cally, there are two ways to come upon these root systems naturally: in the classification of complex semisimple Lie algebras and in the classification of compact connected semisimple Lie groups. (From hindsight, these two approaches are equivalent due to the existence of a real compact form for every complex semisimple Lie algebra; but this is outside the scope of this book.) The study of root systems occupies a portion of Chapter V; the classification of Dynkin diagrams is wisely referred to numerous and accessible sources on the subject. There is one place where compact Lie group theory and complex semisimple Lie algebra theory do part ways; that is where root geometry and Weyl group theory are placed at the service of the algebraic topology of a compact Lie group, that is, where those are used for the description of its fundamental group. Stiefel diagrams and alcoves are discussed for this purpose in §7, in which the fundamental group is computed as a factor group of the kernel of the exponential function of a maximal torus modulo the subgroup generated by the inverse roots. The last section of the chapter deals with the decomposition of a compact connected Lie group into its central component and its commutator group; in particular it is shown that the latter, as a semisimple compact Lie group, has a compact universal covering group. It is not recorded that every element in the semisimple part of a compact connected Lie group is in fact a commutator [6, LIE IX.33], which is useful information.

There remains **Chapter VI**, the climax: irreducible characters and weights with the Weyl Character Formula being at the heart of this chapter and of the whole book (p. 242). The first section takes care of the objective of establishing this formula. This state of affairs allows the completion of the proof that the character ring $R(G)$ is isomorphic to the fixed point ring $R(T)^W$ of the character ring $R(T)$ of a maximal torus under the action of the Weyl group W . Partial orders are introduced on the set of weights, and the dominant weight of an irreducible representation is defined. This allows us to identify the representation ring $R(G)$ (which contains most of the information of the finite-dimensional representations of G) as a factor ring of the integral polynomial ring $\mathbb{Z}[X_\lambda : \lambda \in \Lambda]$, and if G is simply connected and of rank k , we obtain an isomorphism with $\text{card } \Lambda = k$. In §3, Kostant's multiplicity formula is proved. It permits us to compute how often an irreducible representation of a maximal torus T is contained in the restriction of a given irreducible representation of G to T . §4 deals with the real and quaternionic cases. Last but not least, the final sections, 5, 6, and 7, present a thorough discussion and cataloguing of the irreducible representations of the groups in the series $SU(n)$, $U(n)$, $Sp(n)$, $SO(2n + 1)$, $SO(2n)$, $Spin(2n + 1)$, $Spin(2n)$ and the nonconnected groups of the series $O(n)$. The information of the representation rings, among many other things, is complete and explicit.

The style and the flow of presentation of this book are superb. Its individual units are well engineered and fine-tuned. The text benefits from the authors' apparent experience in teaching this material, a portion of which circulated previously as a set of lecture notes. Complicated proofs are broken up into bitesize palatable chunks. To keep the flow of thoughts steady, the authors will occasionally defer suitable portions of an argument for later clearance. From time to time they give hints which offer psychological help. There is no tedium, the pace is brisk, and the style elegant. I expect that some readers

should be prepared to supply details according to their own state of preparation and to their own satisfaction.

Each section has a serviceable collection of exercises. Sometimes they complement the material of the book by providing additional information, usually broken down into manageable partial problems. The bibliography is representative, but makes no claim towards completeness. The symbol index is helpful, the subject index is seven pages long and still remains insufficient for a book of this content and ambition; others may find it adequate. The book is obviously well proof-read and exhibits the grade of typography one has come to expect from Springer-Verlag. The geometric flavor of the book is heightened by a series of excellent illustrations in the form of line drawings. Occasionally, an additional diagram here and there would be helpful, for instance in the section on duality. A Hopf algebra cannot be appreciated without diagrams (p. 147).

This book has competitors on the market, and many of them are quoted by the authors. As a textbook, it is a front runner. As a source of reference it has Bourbaki's Chapter IX [6] as a formidable competitor. My advice is to use the two books side by side, because in their respective aspirations, they are complementary.

Since I made no great efforts to hide my enthusiasm and respect for this book, one might well wonder whether I saw any room for criticisms. I see very little, but some, and that depends largely on personal perspective. I think it is legitimate to have different perspectives in this area.

I am fully prepared to accept the authors' attitude to use whatever mathematics they happen to need in going about the business at hand. Their posture provides the reader with the motivation to delve deeper into the required mathematics. The authors provide specific guidance to the literature which will start the reader on such a treasure hunt. The price they pay is that the book is less self-contained than one might expect. But let us accept this as a basis. Then I find it hard to acquiesce to the fact that the authors failed to provide, in their preparatory chapter, a proof of the existence and uniqueness of Haar measure on compact groups. Certainly *this* much Haar measure theory is available on the level which the authors expect from their readers, and if we buy Sard's Theorem or the exact homotopy sequence of fibrations, then we can afford a few extra pages for such a proof. Perhaps, as a spin-off, a few additional insights into, say, the convolution of measures might result which are not more involved than the convolutions used in a variety of contexts in this book. The entire representation theory of compact groups centers around Haar measure, and for a long stretch has nothing to do with Lie theory on manifolds. I would certainly advise the readers of this book to complement it for themselves by supplementing Haar measure on compact groups. I think the authors have provided for this contingency, because nothing needs to be changed, and most of Chapters II and III becomes instantaneously available for arbitrary compact groups.

Continuing in the same vein, it does surprise me that the authors put off the Theorem of Peter and Weyl till the middle of the book. In fact, the presentation here is excellent, self-contained modulo compact operators which are

always used in one form or another and for which enough background is expressly provided, and modulo the Ascoli Theorem. There are alternative approaches to the Peter-Weyl formalism which get by without the Ascoli Theorem (see for instance [13]), but the discussion here is such that a course on compact groups can very well begin with this part. As it stands, we get a substantial portion of the representation theory of compact Lie groups before we know that in general there are any finite-dimensional nontrivial representations at all, let alone that they separate points. It is really stunning that we should have to wait till p. 136 before we finally realize that in this book we have been dealing with closed subgroups of unitary or orthogonal groups all along, a fact which we could just as easily have had on the first few pages. But perhaps the timely availability of this information might have stolen some of the thunder from the introduction to Lie theory and its manifold analysis, because it would have been apparent that the whole book could have been developed practically without any loss with a much more modest Lie theory involving linear groups only. Because of its pedagogical simplicity, such an approach was advocated by Freudenthal long ago [9] and was re-emphasized afresh in a recent article by Roger Howe [14]. The only place where noncompact Lie groups play a role (apart from the universal covering of a torus) is §III-8 presenting the complexification of a compact Lie group. Remarkably, even here we do not have to leave the class of linear groups. The users of this book should be aware of the possibility of restructuring the global architecture of the first half of the book and of getting by with fewer prerequisites. They should also remember that this book is not, and does not claim to be, an introduction to Lie group theory in general.

Another test that any theory of compact Lie groups has to stand up to is the Maximal Torus Theorem. As we have observed, this theorem is proved in the middle of the book, but its proof throws long shadows back into the first chapter, where the Theorem on Mapping Degrees looms large, consuming Stokes' Theorem as well as Sard's (although for the present purposes we could get by without the latter if we used available ad hoc information; the authors explain this). The essence always is the canonical map $q: G/T \times T \rightarrow G$, and the crux is its surjectivity. The Theorem on Mapping Degrees is a beautiful result on its own, and the application made of it here is splendid. However, I prefer other proofs. In particular, Bourbaki's arrangement [6, LIE IX.7 through LIE IX.9], is elementary by comparison. It is based on the linear algebra required to deal with Cartan algebras to secure the conjugacy statement on the algebra level, and it uses a surprisingly direct argument for the surjectivity of q . The first part of this proof rests on an elementary, but astute lemma (Hunt's Lemma, see [6, LIE IX.7 and 15]). Cartan algebras are not mentioned in this book but Cartan subgroups are. This seems to carry the de-emphasis of the Lie algebra aspect a bit too far, in particular in a context where Cartan algebras are so nearby. Granted, with Bourbaki one always has to watch out for references to earlier material, but I still opt for Bourbaki's choice in a proof of the Maximal Torus Theorem. Of course, there are numerous alternatives. As soon as we have Lemma 4.5 on p. 178 of the present book, we have a proof, because it applies to q . Now this lemma is actually true for continuous maps

between topological manifolds (with f a local homeomorphism at x), and the proof is in fact not hard given some basic homology or cohomology on the level of the Eilenberg-Steenrod axioms. At least by way of exercise, this is on a plane on which this book operates in other spots, say in the line of homotopy theory. Or, the Lefschetz-Dold Fixed Point Theorem can be used, as is done in the book by Adams [1]. One has to consider, however, that the authors almost simultaneously handle the Weyl Integral Formula. Not all of the alternative approaches serve in this capacity. But Bourbaki has a good treatment [6, LIE IX.52ff.].

I feel I have to say a word or two on some remarks in the preface and on the rear cover. This book, one reads there, is a “leisurely paced introduction to the subject” and “is at the level of the advanced undergraduate/graduate student assuming little more than a knowledge of calculus on manifolds and linear algebra”, or that “this book can be read by German students in their third year, or by first-year graduate students in the US”. After having plowed through the material of this book and having formed an impression of the immensity of its scope, when I read these sentences, I began to have serious doubts. We saw the authors assume Stokes’ Theorem (and I mean Stokes’ Theorem $\int_{\partial M} \omega = \int_M d\omega$). They calmly invoke Sard’s Theorem (p. 51, p. 179). When they are in need of covering spaces and the lifting of maps to the universal cover, they use such information (p. 91). Multilinear algebra is, of course, used on a massive scale (see e.g. p. 76, where the exponential laws for the exterior and the symmetric algebras amongst other facts are hailed as “an excellent opportunity for the reader to check his understanding of linear algebra”). It would be a good thing if the readers knew what adjoint functors are, since Frobenius reciprocity is indeed a superb example of an adjoint situation (p. 144). They should also be current on such facts as locally trivial fibrations giving rise to the exact homotopy sequence (p. 187, p. 226). Their algebraic background includes, one hopes, Frobenius’ Theorem on the classical division rings. A knowledge of the basic notions of algebraic geometry up to algebraic varieties are expected of them; how else could one interpret the wording that such information is being “recalled” (p. 152). If I consider the level of mathematical sophistication indicated by these spot checks then I must sadly admit that apparently these incredibly astute and ingenious advanced undergraduates who stroll leisurely through this book have eluded me so far.

Let us just say that this book is for motivated and experienced students, and it will require endurance from them and concentration and judgment from the teacher. Given these prerequisites, it will be immensely rewarding. This text will be used in many a course, and it will enter into the standard references in the field.

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The analysis of linear partial differential operators. III, by Lars Hörmander, Grundlehren der mathematischen Wissenschaften, Vol. 274, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1985, viii + 522 pp., \$48.50. ISBN 3-540-13828-5

The analysis of linear partial differential operators. IV, by Lars Hörmander, Grundlehren der mathematischen Wissenschaften, Vol. 275, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1985, vii + 351 pp., \$45.00. ISBN 3-540-13829-3

These two volumes complete L. Hörmander's treatise on linear partial differential equations. They constitute the most complete and up-to-date account of this subject, by the author who has dominated it and made the most significant contributions in the last decades.