

- [Go] D. Goss, *The algebraist's upper half-plane*, Bull. Amer. Math. Soc. (N.S.) **2** (1980), 391–415.
- [H] F. Herrlich, *Nichtarchimedische Teichmüllertheorie*, Habilitationsschrift Bochum, 1985.
- [Ma] Yu. Manin, *p-adic automorphic forms*, J. Soviet Math. **5** (1976), 279–333.
- [M1] D. Mumford, *An analytic construction of degenerating curves over complete local rings*, Compositio Math. **24** (1972), 129–174.
- [M2] ———, *An analytic construction of degenerating abelian varieties over complete rings*, Compositio Math. **24** (1972), 239–272.
- [M3] ———, *An algebraic surface with K ample*, $(K^2) = 9$, $p_g = q = 0$, Amer. J. Math. **101** (1979), 233–244.
- [MD] Yu Manin and V. Drinfel'd, *Periods of p-adic Schottky groups*, J. Reine Angew. Math. **262/263** (1973), 239–247.
- [P] M. van der Put, *Stable reductions of algebraic curves*, Indag. Math. **46** (1984), 461–478.
- [Sch] P. Schneider, *Rigid-analytic L-transforms*, Number Theory, Noordwijkerhout 1983, Proc. Journées Arithm., Springer Lecture Notes in Math., vol. 1068 (1984), pp. 216–230.
- [St] U. Stuhler, *p-adic homogeneous space and moduli problems*, preprint, Wuppertal, 1985.
- [T] J. Tate, *Rigid analytic spaces*, Invent. Math. **12** (1971), 257–289.
- [R] M. Raynaud, *Géométrie analytique rigide d'après Tate, Kiehl, ...*, Table Ronde d'Analyse non archimédienne, Bull. Soc. Math. France, Mém. 39–40, 1974.

LOTHAR GERRITZEN

BULLETIN (New Series) OF THE
 AMERICAN MATHEMATICAL SOCIETY
 Volume 15, Number 2, October 1986
 ©1986 American Mathematical Society
 0273-0979/86 \$1.00 + \$.25 per page

Topology and analysis, the Atiyah-Singer index formula and gauge-theoretic physics, Universitext, by B. Booss and D. D. Bleeker, Springer-Verlag, New York-Berlin-Heidelberg-Tokyo, 1985, xvi + 451 pp., \$34.00. ISBN 0-387-96112-7

“Why is the \hat{A} -genus of a spin manifold an integer?”

The two mathematicians who asked each other this question in Oxford in the early sixties answered it in the form of a theorem which is still being generalized and reproved nearly a quarter of a century later. The question itself concerned a result in algebraic topology drawn from the pages of Borel and Hirzebruch, but the answer the pair sought was an analytical one—the integer ought to be the index of an elliptic operator. Finding that operator, the Dirac operator, was a turning point in their endeavors and added another example to the Gauss-Bonnet theorem, the Riemann-Roch theorem, and the Hirzebruch signature theorem, each of which could now be considered as a special case of one all-embracing theorem—the index theorem of Atiyah and Singer.

In its most basic form the theorem says how to calculate the index of an elliptic operator D on a closed manifold in terms of topological invariants of the manifold. The index of D is the difference $\dim \ker D - \dim \operatorname{coker} D$, and the topological invariants are characteristic classes of the tangent bundle of the manifold and of the vector bundles on which the operator D is defined. In its refinements, the index is interpreted as more than simply an integer. For example, if a group G acts on the manifold, there is an index in the

representation ring of G . If the operator is a member of a whole family of operators parametrized by a space X , then there is an index in the K -theory of X . If the operator is skew-adjoint then, most subtly of all, there is an index in the integers modulo 2. The fact that there is an index to be calculated is not difficult to see. The power of the index theorem is that it provides a formula in all of these cases for its computation.

For the first ten years of its life, the index theorem was applied and known only in the area of topology or number theory. A typical number theoretical application would involve taking a group action on a manifold which was so well analyzed that both the analytical side and the topological side could be calculated independently. The index theorem then produced an identity, sometimes well known, at other times new.

Gradually, however, by attacking index problems in different areas, indices and attitudes towards the theorem became more concerned with analysis or differential geometry than with topology. This was reflected in the proofs themselves. The original proof made heavy use of the topological theory of cobordism—the underlying manifold is even allowed to change its topological type in the course of the proof. The second proof was modelled on Grothendieck's proof of the Riemann-Roch theorem and drew on the ideas of algebraic geometry. The third proof, in the early seventies, aimed for a more direct transition between the analysis and characteristic classes by expressing these as curvature integrals. The link between the two was provided by the heat kernel.

While the mathematicians' ideas were gradually flowing from topology to analysis a parallel flow of thought, in the opposite direction, was also taking place. Not in the thought processes of mathematicians, though often ironically within a few yards of their offices. It was the physicists who were reinventing the index theorem. They were concerned with gauge theories and were interested in calculating the number of zero modes of a differential operator in terms of the topological charge of the gauge field. Translating the language, they wanted $\dim \ker D$ in terms of characteristic classes, and this is precisely what the index theorem, coupled with what mathematicians would call a vanishing theorem for $\operatorname{coker} D$, provides. Naturally, the mathematical physicist is more familiar with eigenvalues of operators and the heat kernel than with characteristic classes, but once the realization came that each side had something to learn from the other, the walls came down and the setting for the most recent phase of index theory was formed.

It is this setting which provides the background for *Topology and analysis: the Atiyah-Singer index formula and gauge-theoretic physics* by B. Booss and D. D. Bleeker. The bulk of the book is a translation from German of a version which was published in 1977. This was the year when, in fact, the barriers between mathematicians and physicists began to break down with the mutual interest of instantons preoccupying both of their attentions. Naturally, in the intervening years a new audience has arisen for a book about the index theorem and this updated version with an added hundred-page postscript on gauge theories is, therefore, to be welcomed.

It demands few prerequisites for the reader and is well-suited for self-study, being full of exercises and hints. This characterizes the work, in the sense that

it is more useful for learning the essentials of the subject than for gaining an overview of the state of the subject at the present time, despite the multitude of asides and context-placing references. The main thrust of the book is to work up from the idea of the index of a Fredholm operator on Hilbert space, through the introduction of pseudo-differential operators to the second proof of the index theorem, yielding the index in K -theory. The organization of the material is generally good, with theories and techniques brought in for the attainment of specific goals and not for their own sake. One slight hiccup in the linear organization is that K -theory gets defined twice, and if we include the gauge-theoretic section, Sobolev spaces on manifolds are defined twice also (and in different ways!). Nevertheless, the book provides a proof of the index theorem and a good description of what it can do.

Times move on, of course, and the more recent proofs of the index theorem which are motivated by supersymmetry provide a rationale for the role which functions like $x/(1 - e^{-x})$ and $x/\tanh x$ play in sorting out the combinations of characteristic classes which occur in the index formula. By now workers in partial differential equations, stochastic processes, Riemannian geometry, algebraic geometry, algebraic topology and mathematical physics all have the index theorem doing something for them. In another twenty years the list will almost certainly be longer.

Like Stonehenge, the theorem stands there as an immovable edifice, with each generation giving its own interpretation. For one it is a computational device, for another a more mystical representation of supersymmetry. Either way, it has created a bridge between mathematics and physics and has given mathematicians and physicists a deeper, or at least more sympathetic, understanding of each other's work. The Dirac operator will never be reinvented a third time!

NIGEL J. HITCHIN

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 15, Number 2, October 1986
©1986 American Mathematical Society
0273-0979/86 \$1.00 + \$.25 per page

Sequence transformations and their applications, by Jet Wimp, Mathematics in Science and Engineering, vol. 154, Academic Press, 1981, xix + 257 pp., \$38.50. ISBN 0-12-757940-0

In this volume the author restricts himself mostly to material on sequence transformations which has not appeared in book form in English. Some of the material is available in French (Brezinski, 1977, 1978), but much of the material has never appeared in book form in any language. Some has not appeared in published papers [the thesis work of Higgins (1976) and Germain-Bonne (1978) for instance], and much is new altogether.

The subject of this book touches virtually every area of analysis, including interpolation and approximation, Padé approximation, special functions, continued fractions, and optimization methods, to name a few.