generates a strongly continuous group of operators, he can use methods from the theory of semigroups, instead of those of Banach algebras no longer available. Versions of the operational calculus and spectral decompositions, localized to some linear manifolds, for operators with real spectrum conclude this exposition.

The material is well written, the style is alert and attractive, despite the unavoidable technical portions. Many proofs are nice pieces of fine analysis. The author presents an original, interesting and consistent point of view concerning the spectral theory of linear operators, especially of those having real spectrum. The reviewer has several reasons to believe that the spectral theory of linear operators has much to gain from the systematic study of operators with "thin" spectrum, in particular of those with real spectrum. The present work is a remarkable illustration of this assertion.

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The Cauchy problem, by H. O. Fattorini, Encyclopedia of Mathematics and its Applications, Volume 18, Addison-Wesley, Reading, MA, 1983, xxii + 636 pp., $69.96. ISBN 0-201-13517-5

About three hundred years ago Isaac Newton taught us that the motion of a physical system is governed by an initial value problem or *Cauchy problem* for a differential equation, and the notion of Cauchy problem has been developing ever since. Here the phrase *differential equation* should be interpreted broadly so as to include systems of partial differential equations, integrodifferential equations, delay differential equations, and other kinds of equations. Most, but not all, of the Cauchy problems that arise "naturally" are *well-posed problems* —that is, problems for which a solution exists, is unique, and depends continuously on the ingredients of the problem. These requirements often necessitate imposing auxiliary conditions, such as boundary conditions, on a given Cauchy problem.

Of special interest are *linear* equations. There are two reasons for this. Firstly, many equations, such as the Schrödinger equation of nonrelativistic quantum mechanics, arise naturally as linear equations. Indeed, the linearity of the Schrödinger equation can be taken as an axiom of quantum theory. Secondly, many inherently nonlinear equations (such as those describing heat conduction) can be well approximated in important special cases by linear equations. Understanding the associated linear equations gives one an understanding of the relevant special cases as well as providing one with insight into what is going on in general.

In the nineteenth century mathematicians focused attention on finding explicit solutions to a handful of equations together with developing the separation of variables techniques. The latter led to the spectral theory of selfadjoint and, more generally, normal operators on a Hilbert space, which has proved to be an invaluable tool in the solution of Cauchy problems for differential equations.

By the 1930s it was clear to many that a useful way to treat Cauchy problems was to cast them as ordinary differential equations for Hilbert-space-valued functions of a real variable, namely time. Thus one considers abstract Cauchy problems such as

$$\frac{du(t)}{dt} = Au(t) \quad (t \geqslant 0), \qquad u(0) = u_0,$$

where t is the time, $u(\cdot)$ takes values in a Hilbert space E, u_0 is the (given) initial value, and A is a linear operator from its domain $D(A)$ in E to E (so that $du(t)/dt = Au(t)$ makes sense), and $D(A)$ incorporates the boundary conditions. For example, any vector in $D(A)$ may be thought of as a function satisfying a spatial Dirichlet boundary condition.

Nonselfadjoint problems were also clearly of importance, and spaces other than Hilbert spaces arose naturally. Thus, for instance, if $u(t, x)$ represents the solution of a heat-type equation (where $t \geqslant 0$ and $x \in \Omega \subset \mathbb{R}^n$) then $\|u(t, \cdot)\|_{L^1}$ [respectively, $\|u(t, \cdot)\|_{L^\infty}$] might represent the total heat content [respectively, the maximum temperature] at time t. Thus at some point it became clear that one should look at linear differential equations in Banach space or perhaps in more general topological vector spaces, especially those connected with the L. Schwartz theory of distributions.

The theory of one-parameter semigroups of operators, as developed by E. Hille, K. Yosida, R. Phillips, and others in the 1940s and thereafter, constituted a significant extension of spectral theory and led to many new applications. Semigroup theory led to other advances in the theory of differential equations in abstract spaces, including cosine functions, approximation theory (which explains how solutions depend continuously on the ingredients of the problem), approximation by difference schemes, perturbation theory, equations involving operators depending explicitly on time, improperly posed problems, and so on. To this day the field continues to be an active one. Since significant applications continue to appear, the field, as of today, must be regarded as a healthy one.

This book by Fattorini covers all of the topics mentioned above. It is self-contained and should be accessible to a wide audience of mathematical scientists. Equations treated in some detail include the diffusion equation, the

Schrödinger equation, the neutron transport equation, Maxwell's equations, and the Dirac equation. A notable feature of the book is the treatment of second-order elliptic and parabolic problems in L^2 and L^p spaces. Fattorini does a nice job of explaining the Agmon-Douglis-Nirenberg elliptic machinery (in the second-order case), making it accessible to a wide audience. An important feature of the book is its extensive and useful bibliography occupying more than a hundred pages.

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Bayes theory, by J. A. Hartigan, Springer Series in Statistics, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1983, xii + 145 pp., $16.80. ISBN 0-387-90883-8

A basic problem of statistics is to infer something about a parameter or state of nature θ after observing a random variable x whose distribution p_θ depends on θ. A neat, but controversial, solution to this problem of inference is provided by the Bayesian approach. Assume that θ is a random variable with distribution π *prior* to observing x. The inference is made by calculating q_x, the conditional or posterior distribution of θ, given x. If p_θ and π have probability density functions $f(x|\theta)$ and $g(\theta)$, respectively, then q_x has density $h(\theta|x)$ given by Bayes's formula

$$h(\theta|x) = \frac{f(x|\theta)g(\theta)}{\int f(x|\varphi)g(\varphi)\,d\varphi} \tag{1}$$

or, briefly,

$$h(\theta|x) \propto f(x|\theta)g(\theta). \tag{2}$$

(For simplicity, assume the densities are with respect to Lebesgue measure. However, any σ-finite dominating measure will do.) There is no disagreement about Bayes's formula. The controversy is about its application and its interpretation.

The two major interpretations of the probability of an event E, both of which can be traced back to the seventeenth-century origins of the subject, are as the limiting relative frequency of E in a sequence of trials, or as a measure of the degree of belief in the occurrence of E. For the past half century the majority of probabilists and statisticians have accepted the frequency interpretation, even though it is of limited application and seems somewhat circular in its "dependence" on the law of large numbers. The frequency view is disastrous for Bayesian inference because it rarely happens that prior probabilities make sense as frequencies. They do make sense when viewed as degrees of belief, and this explains why Bayesians are often identified with subjective probability (de Finetti (1974), Savage (1954)). However, there have been, and are, prominent Bayesians who advocate the use of logical or canonical prior