

that the Bishop-Phelps theorem holds in *complex* Banach spaces with the (real) Radon-Nikodym property. Similarly, a striking application of the Huff-Morris theorem (concerning the existence of extreme points in any nonempty closed bounded subset of a Banach space with the Radon-Nikodym property) is P. Mankiewicz's proof that complex Banach spaces with the Radon-Nikodym property have unique complex structure; the omission of this result is unfortunate, again because the question of uniqueness of complex structures on complex Banach spaces (the complex Mazur-Ulam problem) is open in general. Almost nothing is said about the role of the Radon-Nikodym property in the study of operator ideals, a subject arguably central to the study of the geometry of Banach spaces. Again, nothing is said about the part played by the Radon-Nikodym property in abstract harmonic analysis, both commutative and noncommutative. All this is nitpicking though since the objective of the monograph is *not* to tell everything there is about the Radon-Nikodym property, but rather to tell about a substantial amount of certain geometric aspects of the Radon-Nikodym property. In this regard, Dick Bourgin has done an admirable job.

JOE DIESTEL

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 11, Number 2, October 1984
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0273-0979/84 \$1.00 + \$.25 per page

Differential geometry of foliations, by Bruce L. Reinhart, *Ergebnisse der Mathematik und ihrer Grenzgebiete*, vol. 99, Springer-Verlag, Berlin, 1983, ix + 194 pp., \$42.00. ISBN 3-5401-2269-9

A structure on a differentiable manifold of dimension m can be defined by requiring that there exist an atlas whose coordinate transformations satisfy a special condition. In the case of a foliated structure of dimension p , the transformations must map the points in their domains lying in a p -plane parallel to some fixed subspace $R^p \subseteq R^m$ into a p -plane of the same type. Thinking of the layers of an onion or the pages of a magazine suggests the right mental picture when $p = 2$ and $m = 3$. A structure defined by an atlas determines a reduction of the structure group of the tangent bundle to the group consisting of the tangent maps to coordinate transformations. (A similar game can also be played with higher order jet bundles.) Integrability problems in differential geometry are concerned with reversing this process, that is, with determining if a given reduction can be realized in the way described, at least up to some kind of equivalence. For instance, Reeb's "Problème Fondamental" in the first monograph on foliations [2] was to determine whether a manifold that admits a continuous field of p -planes can also be given a foliated structure of dimension p .

Reinhart, with the pardonable exaggeration of an enthusiast, subtitles his book *The fundamental integrability problem*. I would have been more comfortable with the indefinite article, but he probably wanted to show both his intention of including much more within foliations than the minimal structure described above and that the field is of fundamental importance as a rich

source of integrability problems that are neither too trivial nor impossibly difficult. The relative tractability of these is explained in part by the fact that the local situation is well understood because of the classical theorem of Deahna, Clebsch, and Frobenius, and, consequently, in foliations one is free to concentrate on global questions.

The book gets off to a good start in the preface and Chapter I. Basic concepts are clearly defined, motivation and well-chosen examples are provided, and several theorems needed for later developments are carefully proved. Among the latter is the classical local integrability theorem mentioned above, which is presented in a refined version that gives unusually detailed information on smoothness. The next chapter begins with a somewhat unexpected, but welcome, treatment of jets and related topics. Reinhart reasonably suggests that the machinery set up will someday prove to be valuable even though none of the results obtained here rates a higher designation than "Proposition". In his words, "...most of the history of jets and truncated polynomial groups will occur in the future".

One would expect to find the topics of the next sections, connections, characteristic classes, secondary classes, and the relation of these to foliations, in any survey of foliations since many of the triumphs of the field are based on them. For instance, Bott's solution to Reeb's problem depended on the construction of a connection that enabled one to find obstructions to foliating in terms of Pontryagin classes. This construction is simple, intuitive, and important, so it comes as a shock to see that it is here buried by largely unmotivated formalism and that there is no explicit mention of the solution to Reeb's problem. The exposition in these sections was much too brutal for my taste in view of the motivation and examples that could have easily been provided. Some readers may want to consult the original works or the survey articles of Lawson [1] and Thomas [3] (which were unaccountably omitted from the bibliography) in order to keep their bearings. On the other hand, Reinhart's organization of this material has the virtue of doing justice to some of the early work on connections that is sometimes overlooked.

A chapter that I wish had been longer is somewhat misleadingly labelled "Singular Foliations". It gives a quick sketch of the classifying space for a topological groupoid and of various cohomologies of Lie algebras. Kinds of foliations with singularities called Frobenius structures are defined and discussed briefly, but very little seems to be known about them. The final chapter deals with various combinations of metric structures and foliations such as Riemannian foliations, bundle-like metrics, the growth of leaves and its geometric implications, and holonomy invariant measures. I found this chapter much more pleasant to read than its predecessors, in part surely because it deals with topics closer to my own interests. However, it also seemed to be less formal, better motivated, and fuller of illuminating examples and geometric insights than those immediately preceding it.

A lot of ground is covered in less than 200 pages, even if one allows for the fact that, as is appropriate in a survey, one is referred to original works for long or difficult proofs. Some compression is also achieved by assuming that the reader not only has a lot of facility with differential geometry, including

Lie groups, bundle theory and sheaves, but also is experienced enough to be able, for example, to figure out what kind of induced map is denoted by an asterisk attached to a symbol, or to which of several structures that happen to be lying around a word like “homomorphism” might refer. Even well-prepared readers might have difficulty in deciphering occasional slips into a cryptic style in the places where there is a small error or omission. A bit more redundancy or explanation might have made it possible to guess what was intended by a passage such as the one comprising the first three sentences of Example 1.20 (p. 8), which was among several that I never understood.

It used to be easy to be an expert on foliations. One only had to read Reeb [2] and a few important papers. Now that so much more is known, it is harder, but thanks to Reinhart it is at least possible to get a good idea of the field in a reasonable time. He has written a book reflecting his own tastes rather than some sort of consensus view among foliators. Even though most will find that some of their favorite topics have been omitted or only given brief mention, I believe that they will agree that the book is more stimulating and interesting because of its individuality. It deserves to be successful and I hope that there will be later editions that will show a little more compulsive attention to detail and pity for the frailties of readers.

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RICHARD SACKSTEDER

BULLETIN (New Series) OF THE
 AMERICAN MATHEMATICAL SOCIETY
 Volume 11, Number 2, October 1984
 © 1984 American Mathematical Society
 0273-0979/84 \$1.00 + \$.25 per page

Function theory on planar domains, a second course in complex analysis, by Stephen D. Fisher, John Wiley & Sons, Inc., 1983, xiii + 269 pp., \$34.95. ISBN 0-4718-7314-4

The most intensively studied spaces of analytic functions are the Hardy spaces on the unit disk D in the complex plane. For each positive number p , the Hardy space $H^p(D)$ is the set of analytic functions f on the unit disk D such that

$$\sup_{0 < r < 1} \int_0^{2\pi} |f(re^{it})|^p dt < \infty.$$

The Hardy space $H^\infty(D)$ is the set of bounded analytic functions on D . A sample of the wealth of information that twentieth century mathematicians have discovered about these spaces can be found in the books of Hoffman [4], Duren [2], Koosis [5], and Garnett [3].