[WY] Herman Weyl, *Die Idee der Riemannischen Fläche*, Teubner, Leipzig, 1913. The third (1955) German edition was translated into English by G. R. MacLane and appeared as *The Concept of a Riemann Surface*, Addison-Wesley, Reading, 1964.

[WH] Hassler Whitney, *Geometric integration theory*, Princeton Univ. Press, Princeton, N.J., 1957.

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BULLETIN (New Series) OF THE
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0273-0979/84 $1.00 + $.25 per page

Finite groups. II, by B. Huppert and N. Blackburn, Grundlehren der Mathematischen Wissenschaften Series 242, Springer-Verlag, Berlin and New York, 1982, xiii + 531 pp., $68.00. ISBN 0-3871-0632-4

Finite groups. III, by B. Huppert and N. Blackburn, Grundlehren der Mathematischen Wissenschaften Series 243, Springer-Verlag, Berlin and New York, 1982, viii + 454 pp., $59.00. ISBN 0-3871-0633-2

Group theory. I, by Michio Suzuki, Grundlehren der Mathematischen Wissenschaften Series 247, Springer-Verlag, Berlin and New York, 1982, xii + 434 pp., $48.00. ISBN 3-5401-0915-3

After three decades of intensive research by hundreds of group theorists, the century old problem of the classification of the finite simple groups has been solved and the whole field has been drastically changed. A few years ago the one focus of attention was the program for the classification; now there are many active areas including the study of the connections between groups and geometries, sporadic groups and, especially, the representation theory. A spate of books on finite groups, of different breadths and on a variety of topics, has appeared, and it is a good time for this to happen. Moreover, the classification means that the view of the subject is quite different; even the most elementary treatment of groups should be modified, as we now know that all finite groups are made up of groups which, for the most part, are imitations of Lie groups using finite fields instead of the reals and complexes. The typical example of a finite group is $\mathrm{GL}(n, q)$, the general linear group of n dimensions over the field with q elements. The student who is introduced to the subject with other examples is being completely misled.

The volume by Suzuki can stand on its own as an introduction to group theory, emphasizing finite groups, even though it is preparation for the second volume which will deal largely with simple groups and methods of studying them. The style is detailed and leisurely with many examples and exercises. The first chapter is devoted to the rudiments of the subject but closes, very appropriately, with a study of the general linear groups over arbitrary fields. The third chapter is also given mainly to the most important examples of groups and their properties: classical groups, BN pairs and buildings, symmetric and Coxeter groups, finite simple groups. These are exactly the right topics to be discussed early in a treatment on group theory. This chapter, and the preceding one, also contain material on infinite abelian groups, a topic often discussed in group theory texts. This is an anomaly as this subject is the interesting theory of large and exotic modules over commutative rings in the special case of the ring of integers and, as such, has nothing to do with group theory.

The second chapter is devoted to many of the basic theorems of the subject like Sylow's theorem, results on p-groups, various types of series of subgroups, the Krull-Schmidt theorem. But it also contains a good introduction to the study of group extensions by cohomology of groups and, in particular, treats covering groups of finite groups. Here one is examining a finite group G and describing all covers of G, that is, all finite groups H which possess a homomorphism onto G with kernel central in H and contained in the commutator subgroup of H. These arise in two different ways: in Clifford theory which compares the representation theory of a group, its normal subgroups and quotient groups; in proving general theorems on finite groups by reducing to simple groups one is often forced to consider their covers as well. The needed background in cohomology and the results on low dimensional cases are all carefully given. However, as usual, the student will only eventually gain a full appreciation when he has obtained a knowledge of a lot more homological algebra.

The Huppert-Blackburn volumes consist of the continuation of the introductory first volume and are devoted to more advanced topics. The very high standards set in the first volume are maintained; these books are not only very careful and detailed, they are a pleasure to read. It is a real loss for the subject that there will be no further volumes in this series.

There is a fundamental, even though trivial, observation that is the motivation for much of the material developed in Volume II. Suppose that E is an elementary abelian p-group so that E is abelian and every nonidentity element has prime order p. Writing the group operation as addition and letting the field F of p elements operate on E via the taking of powers, we can regard E as a vector space over F. If E is a subgroup of the group X and L is the normalizer of E in X then each element of L induces, by conjugation, a linear transformation of E, and so E becomes a module for the group algebra FL. This allows us to study E, L and X, in fact, by using representation theory and machinery quite outside the group. Even in the case that $L/C(E)$ is cyclic, the eigenvalues on E for a generator of $L/C(E)$ can be used in a decisive manner.

Volume II is devoted to representation theory and its applications to group theory by these means. Therefore, the first part of the book is a module-theoretic treatment of representation theory over fields of arbitrary characteristic. This is followed by a very comprehensive chapter on the applications of these linear methods to nilpotent groups and, in particular, to the topics of Suzuki 2-groups and regular automorphisms of p-groups. The final chapter is devoted to the linear methods application to p-solvable groups and begins quite properly with the famous Theorem B of Hall and Higman which first showed how modular representations could be used to prove structure theorems for p-solvable groups. (In fact, the first version of their theorem showed how to give the structure of finite groups of exponent six by a reduction to examination of specific representations of the symmetric group of degree three and the alternating group of degree four in characteristics two and three, respectively!)

The last volume is devoted to simple groups and begins with the local theory of finite groups. A p-local subgroup of a finite group is a normalizer of a nonidentity p-subgroup, and the local theory is that body of theorems which show how much of the structure of finite groups is captured by these subgroups. These ideas are the main ideas used in the classification. The next two chapters deal with certain permutation groups which are basic to the study of simple groups. First, the Zassenhaus groups are described, and this is the first time in book form. This is one of the first steps of the classification. Second, multiply transitive groups are studied; this is where the sporadic groups first arose, in the work of Mathieu, well over a century ago.

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Factorization of matrix functions and singular integral operators, by K. Clancey and I. Gohberg, Operator Theory: Advances and Applications, Vol. 3, Birkhäuser Verlag, Basel, 1981, x + 234 pp., $17.55. ISBN 3-7643-1297-1

The present book is a review of the current state of the theory of factorization of nonsingular matrix functions along closed contours and systems of singular integral equations. As such it is a natural companion of the Gohberg-Krupnik book [**8**] which deals with scalar equations. To understand the type of factorization considered here, it is necessary to recall its definition. Let Γ be an oriented closed smooth contour on the Riemann sphere with inner domain F^+ and outer domain F^-, and assume $t^{\pm}$ are fixed points in $F^{\pm}$. Let $A(t)$, $t \in \Gamma$, be a nonsingular $n \times n$ matrix whose entries are continuous functions on Γ. A *factorization of A relative to the contour* Γ is a representation of A in the form

$$A(t) = A_-(t)D(t)A_+(t), \qquad t \in \Gamma, \tag{1}$$

where $D(t)$ is an $n \times n$ diagonal matrix,

$$D(t) = \operatorname{diag}\left(\left(\frac{t-t^+}{t-t^-}\right)^{\kappa_1}, \ldots, \left(\frac{t-t^+}{t-t^-}\right)^{\kappa_n}\right), \tag{2}$$

the matrix functions A_+ and A_- are analytic on the inner and outer domain of Γ, respectively, both A_+ and A_- are continuous up to the boundary of Γ, and $\det A_{\pm}(t)$ does not vanish on $F^{\pm} \cup \Gamma$. The integers $\kappa_1 \geqslant \cdots \geqslant \kappa_n$ are uniquely determined by A and called the *partial indices* of A relative to Γ. The factorization is called *canonical* when all indices are zero. The role of the points