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Rational homotopy theory and differential forms, by Phillip A. Griffiths and John W. Morgan, Progress in Mathematics, vol. 6, Birkhäuser, Boston, Basel, Stuttgart, 1981, xi + 242 pp., $14.00.

Differential forms have motivated algebraic topology from its very beginning. Efforts to understand geometry arising from the Stokes formula have led to homology and cohomology theory. On the other hand, differential forms have always played an important role in the study of topological properties of Riemann surfaces.

For a long time, the only algebraic topological information of general validity provided by differential forms has been the de Rham theorem. Recently it has been discovered that differential forms furnish algebraic topological information extending far beyond the de Rham cohomology group in a systematic way.

There are two ways of obtaining additional algebraic topological results from differential forms: the method of iterated path integrals and that of minimal models. The former method takes advantage of the geometric wealth of the path space, which may be regarded as expressing the dynamics of the space in consideration. Usual differential forms are repeatedly integrated to obtain differential forms on the path space. The complex of such path space differential forms possesses a rich structure and yields many topological and geometric results. In the latter method, an algebraic homotopy theory of commutative differential graded algebras (DGA's) is developed, and its correspondence with the topological counterpart is established. Problems in rational homotopy theory are converted to algebraic ones, leading to computation that can then be carried out by using minimal models often in an amazingly simple manner.

This monograph contains an exposition of the minimal model theory due to Dennis Sullivan. The working of the theory can be illustrated by the following example of computing the minimal model of the complex projective space CP^n.

Let $A^*(CP^n)$ be the de Rham complex of CP^n (in either the PL or C^∞ sense). The minimal model $\mathbf{M}^*$ of $A^*(CP^n)$ is a DGA satisfying the following conditions:

(i) $\mathbf{M}^*$ is free as a commutative graded algebra.

(ii) $\mathbf{M}^1 = 0$.

(iii) $d\mathbf{M}^+ \subset \mathbf{M}^+ \wedge \mathbf{M}^+$.

(iv) There is a map ρ: $\mathbf{M}^* \to A^*(CP^n)$ of DGA's inducing isomorphism on cohomology.

There exists a closed 2-form w on CP^n whose cohomology class $[w]$ generates the 2nd de Rham cohomology group. It is not hard to see that essentially the only choice for $\mathbf{M}^*$ has two generators x of degree 2 and y degree $2n+1$, a differential d with $dx = 0$ and $dy = x^n$ and a DGA map ρ: $\mathbf{M}^* \to A^*(CP^n)$ such that $\rho x = w$ and $\rho y = 0$.

The theory then impiles that x and y generate $\mathrm{Hom}(\pi_2(CP^n), Q)$ and $\mathrm{Hom}(\pi_{2n+1}(CP^n), Q)$ respectively. We conclude that

$$\pi_i(CP^n) \otimes Q = \begin{cases} 0, & i \neq 2, 2n+1, \\ Q, & i = 2, 2n+1. \end{cases}$$

Thus, a complicated computation of homotopy groups is transformed into a simple algebraic calculation though at the expense of neglecting torsions.

The book may be divided into two parts:

1. Algebraic topological material leading to the rational homotopy theory.
2. The minimal model theory.

Part 1 consists of the first seven chapters starting from elementary concepts and standard theorems, advancing to the Serre spectral sequences and the obstruction theory, and ending with Q-Postnikov towers and the rational homotopy theory.

Part 2 consists of the remaining chapters covering the following topics: PL de Rham theorem; minimal models; homotopy theory of commutative differential graded algebras; connection between minimal models and Q-Postnikov towers; minimal models for fundamental groups; functorial properties. The usefulness of minimal models is demonstrated in Chapter XIII through examples of computations relating to spheres, complex projective spaces, wedges of spheres, the Booromean ring, symmetric spaces, BU_n and U_n, compact Kähler manifolds, etc.

The monograph is intended as an introduction to the theory of minimal models. Anyone who wishes to learn about the theory will find this book a very helpful and enlightening one. There are plenty of examples, illustrations, diagrams and exercises. The material is developed with patience and clarity. Efforts are made to avoid generalities and technicalities that may distract the reader or obscure the main theme. The theory and its power are elegantly presented. This is an excellent monograph.

It is pointed out that this is a revised and corrected version of a set of informal notes from a summer course taught by the authors together with Eric Friedlander in the summer of 1972. Regarding the origin of these notes, there is an acknowledgement to Dennis Sullivan.

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Real variable methods in Fourier analysis, by M. de Guzmán, Mathematics Studies, no. 46, Notas de Matemática, North-Holland Publishing Company, Amsterdam, 1981, xiii + 392 pp., $44.00. ISBN 0-4448-6124-6

Interpolation of operators and singular integrals, an introduction to harmonic analysis, by Cora Sadosky, Pure and Applied Mathematics, no. 53, Marcel Dekker, New York, 1979, xi + 375 pp., $35.00. ISBN 0-8247-6883-3

In 1948 Anthony Zygmund went to Buenos Aires at the invitation of Gonzales Dominquez. This event has had a number of remarkable consequences, the first of which was Zygmund's meeting A. P. Calderón, and Misha Cotlar. The second, and concomitant consequence was the flowering of classical hard analysis and the problems associated with the Polish school of mathematics in a number of Spanish speaking countries. The two books under review are some of the most recent blooms.

Both Miguel de Guzmán and Cora Sadosky were students of Professor Zygmund in Chicago in the middle sixties. de Guzmán is now the center of a group of young mathematicians working in a variety of hard classical problems in differentiation theory at the Universidad Complutense de Madrid. Sadosky was at universities in Argentina, Uruguay, and Venezuela working on various problems of weights and singular integrals, and is now at Howard University.

A colleague walked into our office, and saw de Guzmán's book on a desk, and made the remark "Another book about those damn little rectangles! Why do people do such things?" This attitude is understandable because many of the theorems about little rectangles are quite messy; it is unfortunate since understanding the geometric interactions of the little rectangles is at the heart of some of the most important results in real analysis. Just as the Vitali covering lemma is at the heart of the proof of Lesbegue's theorem on the differentiation of integrals, so too, many more recondite covering lemmas and