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UNIVERSAL KNOTS

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ABSTRACT. We demonstrate the existence of, and show how to construct, a knot K in S^3 such that every closed oreintable 3-manifold is a branched covering space of S^3 with branch set K.

In January 1982, William Thurston [1] gave an example of a six component link, T in S^3 such that every closed orientable 3-manifold is a branched covering space of S^3 with branch set this link T. He called links with this property "universal" and raised the question as to whether there was a universal knot.

The purpose of this paper is to answer this question in the affirmative. This result follows immediately from Thurston's result and the following theorem.

THEOREM 1. *Let L be any $m+1$-component link in S^3. Then there is a $2m+4$ component link L' in S^3 and a map $p\colon S^3 \to S^3$ such that*

1. *L is a sublink of L'.*
2. *p is a $2m+5$ to 1 branched covering space map.*
3. *The branch set is a knot K.*
4. *$p^{-1}(K) = L'$.*

SKETCH OF THE PROOF. We begin with a $2m+5$ to 1 simple branched covering $p\colon S^3 \to S^3$ branched over a knot $\tilde{K}$. We also assume that the preimage of $\tilde{K}$ is a $2m+4$ component link with one component having index of ramification two and the rest index one. (This is not difficult to arrange.) We designate any m-component sublink having only index of ramification one components, as "important", we call it M and we call the rest N.

Next we study lifts of arcs having both endpoints in the branch set $\tilde{K}$. Certain arcs have as their preimages sets of $2m+3$ arcs, $2m+1$ of which are

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mapped homeomorphically and two of which are mapped by a folding about the midpoint. We can do surgery in a ball neighborhood of such arcs and using this type of surgery it is possible to change any undercrossing in M to an overcrossing. The price we pay is that N is radically altered as is $\tilde{K}$. However $\tilde{K}$ is changed to another knot, not a link.

By a series of such surgeries we gradually change M to L and this completes the proof.

Using this method, and a somewhat different approach than Thurston's to obtain a universal four component link, we obtain a universal knot K. The knot K turns out to be the $(7,6)$ torus knot with clasps.

REFERENCES

1. W. Thurston, *Universal links* (preprint).

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